Regras de L'Hospital:

Teorema: Sejam $f, g: I \to \mathbb{R}$, $a \in I$.

Se $\lim_{x\to a}\frac{f'(x)}{g'(x)} = L$, então $\lim_{x\to a}\frac{f(x)}{g(x)} = L$,

caso $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$

Extensões: vale p/ $a = \pm\infty$ ou se $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = \pm\infty$ ou ainda $L = \pm\infty$

e também para laterais.

Exemplos: 1) $\lim_{x\to 1}\frac{x^5-6x^3+8x-3}{x^4-1}$ $=$ $\lim_{x\to 1}\frac{5x^4-18x^2+8}{4x^3} = \frac{-5}{4}$

$f(x) = x^5-6x+8x-3$

$g(x) = x^4-1$

2) $\lim_{x\to 0^+} x.\ln x = "0.(-\infty)"$ $=$ $\lim_{x\to 0^+}\frac{\ln x}{1/x} = \lim_{x\to 0^+}\frac{1/x}{-1/x^2} = \lim_{x\to 0^+} -x = 0.$

$x\ln x = \frac{\ln x}{\frac{1}{x}} = \frac{x}{\frac{1}{\ln x}}$

3) $\lim_{x\to+\infty}\sqrt{x}-\sqrt{x+1} = \lim_{x\to+\infty}\sqrt{x}-\sqrt{x^2-1}\cdot\frac{(\sqrt{x}+\sqrt{x^2-1})}{\sqrt{x}+\sqrt{x^2-1}} = \lim_{x\to+\infty}\frac{x-x^2+1}{\sqrt{x}+\sqrt{x^2+1}}$

$\lim_{x\to+\infty}\frac{-2x+1}{\frac{1}{2\sqrt{x}}+\frac{2x}{\sqrt{x^2+1}}} = -\infty.$

$\frac{2x}{x\sqrt{1+\frac{1}{x^2}}}$

$f(x) = -2x+1 \Rightarrow \lim_{x\to+\infty} f(x) = -\infty$

$g(x) =$ $\Rightarrow \lim_{x\to+\infty} g(x) = 2$

$\mu = 1/x \Rightarrow \lim_{\mu\to+\infty}\frac{e^{\mu}}{\mu^2} = +\infty.$

4) $\lim_{x\to 0^+} x^2.e^{1/x} = "0.\infty"$ $\quad \lim_{x\to 0^+}\frac{e^{1/x}}{1/x^2} = \lim_{x\to 0^+}\frac{-1/x^2\, e^{1/x}}{-2/x^3} = \lim_{x\to 0^+}\frac{x\,e^{1/x}}{2}$

$= \lim_{x\to 0^+}\frac{e^{1/x}}{2/x} = \lim_{x\to 0^+}\frac{-1/x^2 . e^{1/x}}{-2/x^2} = \lim_{x\to 0^+}\frac{e^{1/x}}{2} = +\infty$

$\lim_{x\to 0^-} x^2.e^{1/x} = 0.$

5) $\lim\limits_{x \to 0^+} x^x = "0^0"$

$= \lim\limits_{x \to 0^+} e^{x \cdot \ln x} = e^{\overset{"0}{\lim\limits_{x \to 0} x \cdot \ln x}} = e^0 = 1$

6) $\lim\limits_{x \to 0} (1+x)^{1/x} = "1^{\pm\infty}"$

$= \lim\limits_{x \to 0^\pm} e^{\frac{1}{x} \cdot \ln(x+1)} = e^{\lim\limits_{x \to 0^+} \frac{\ln(x+1)}{x}} = e^1 = e$

$\lim\limits_{x \to 0^+} \frac{\ln(x+1)}{x} = \lim\limits_{x \to 0} \frac{1}{x+1} = 1$

Lista 2:

30-k) $\lim\limits_{x \to 0} \frac{1}{1-\cos x} - \frac{2}{x^2} = \lim\limits_{x \to 0} \frac{x^2 - 2 + 2\cos x}{x^2(1-\cos x)}$

$\lim\limits_{x \to 0} \frac{2x - 2\operatorname{sen} x}{2x(1-\cos x) + x^2 \operatorname{sen} x} = \lim\limits_{x \to 0} \frac{2 - 2\cos x}{2(1-\cos x) + 2x\operatorname{sen} x + 2x \operatorname{sen} x + x^2 \cos x} =$

$\lim\limits_{x \to 0} \frac{2\operatorname{sen} x}{2\operatorname{sen} x + 4\operatorname{sen} x + 4x\cos x + 2x\cos x - x^2 \operatorname{sen} x} =$

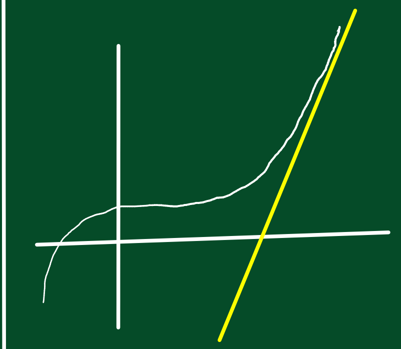
$\lim\limits_{x \to 0} \frac{2\cos x}{6\cos x + 6\cos x - 6x\operatorname{sen} x - 2x\operatorname{sen} x - x^2\cos x} = \frac{2}{12} = \frac{1}{6}$

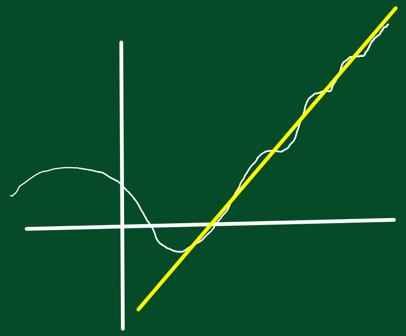
Esboço de gráficos

1) Determinar Dom f
2) Crescimento (sinal de f')
3) Concavidade (sinal de f'')
4) limites pertinentes (extremos do domínio)
5) Interceptos com eixos
6) Assíntotas.

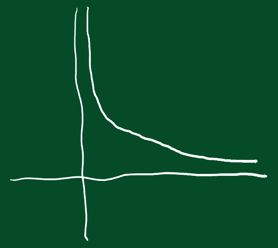
Assíntota:

$y = ax + b$ é assíntota de f em $\pm\infty$ se $\lim_{x\to\pm\infty} f(x) - (ax+b) = 0$





$f(x) = 1/x$



$a = \lim_{x\to+\infty} \frac{1/x}{x} = 0$

$b = \lim_{x\to\infty} f(x) - 0 \cdot x = 0$

$y = 0$ é assíntota de f em $+\infty$.

$$\lim_{x\to+\infty} f(x) - (ax+b) = 0 \Rightarrow \lim_{x\to\infty} \frac{f(x)}{x} - a + \overset{0}{\frac{b}{x}} = 0 \Rightarrow a = \lim_{x\to+\infty} \frac{f(x)}{x}$$

se existe esse $a \in \mathbb{R}$ então $b = \lim_{x\to+\infty} f(x) - ax$

Ex.: $f(x) = \ln x$

$$a = \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\ln x}{x} = \lim_{x\to+\infty} \frac{1}{x} = 0.$$

$$b = \lim_{x\to+\infty} f(x) - 0 \cdot x = \lim_{x\to+\infty} \ln x = +\infty \notin \mathbb{R}$$

Logo ñ tem assíntota.

Exemplo: $f(x) = \frac{x^3}{x^2+1}$

$\frac{-3\sqrt{3}}{4}$

1) $D_f = \mathbb{R}$

2) $f'(x) = \frac{3x^2(x^2+1) - 2x\,x^3}{(x^2+1)^2} = \frac{x^4+3x^2}{(x^2+1)^2} > 0$

f é crescente.

3) $f''(x) = \frac{x(-2x^2+6)}{(x^2+1)^3}$

	$-\sqrt{3}$		0		$\sqrt{3}$	
f''	$+$		$-$	$+$		$-$
f	$\cup$		$\cap$	$\cup$		$\cap$

4) $\lim_{x \to +\infty} f(x) = +\infty$

5) $f(0) = 0$

$f(x) = 0 \iff x = 0$

6) Assíntotas:

·) $+\infty$: $a = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{x^2}{x^2+1} = 1$

$\Rightarrow y = x$ é assint. em $+\infty$

$b = \lim_{x \to +\infty} f(x) - x = \lim_{x \to +\infty} \frac{-x}{x^2+1} = 0$