Lista 2: 1.f, 1.g, 1.q e 1.k

$$(f\cdot g\cdot h)' = f'gh + fg'h + fg\cdot h'$$
$$\parallel$$
$$((f\cdot g)\cdot h)'$$

$1 + tg^2 x = sec^2 x$

1.f) $f(x) = \ln(e^x+1)$

$$x \xmapsto{e^{(\ )}} e^x \xrightarrow{(\ )+1} e^x+1 \xrightarrow{\ln(\ )} \ln(e^x+1)$$

$$f'(x) = \frac{1}{e^x+1}\cdot e^x = \frac{e^x}{e^x+1}$$

$\ln^2 x$

1.g) $f(x) = (\ln x)^2 + (1+2^{x^3})^x$

$e^{x\ln(1+2^{x^3})}$

$$\boxed{\begin{array}{l} f(x) = a^x \\ f'(x) = a^x\cdot\ln a \\ \text{se } a\in\mathbb{R}. \end{array}}$$

$$f'(x) = 2(\ln x)\cdot\frac{1}{x} + e^{x\ln(1+2^{x^3})}\cdot\left[1\cdot\ln(1+2^{x^3}) + x\cdot\frac{(\ln 2)\cdot 2^{x^3}\cdot 3x^2}{1+2^{x^3}}\right]$$

$$= \frac{2\ln x}{x} + (1+2^{x^3})^x\left[\ln(1+2^{x^3}) + \frac{3x^3\cdot 2^{x^3}\cdot\ln 2}{1+2^{x^3}}\right]$$

1.k) $f(x) = \ln(\operatorname{arctg} x)$

$$f'(x) = \frac{1}{\operatorname{arctg} x}\cdot\frac{1}{1+x^2} = \frac{1}{(1+x^2)\operatorname{arctg} x}.$$

1.q) $f(x) = \ln\left(\sqrt{\frac{x+1}{x-1}}\right)$

$$f'(x) = \sqrt{\frac{x-1}{x+1}}\cdot\frac{1}{2}\sqrt{\frac{x-1}{x+1}}\cdot\left[\frac{x-1-x-1}{(x-1)^2}\right]$$

$$= \frac{-1}{(x+1)(x-1)} = \frac{-1}{x^2-1}.$$

ou

$$f(x) = \ln\left(\left(\frac{x+1}{x-1}\right)^{1/2}\right) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$$

$$= \frac{1}{2}\left[\ln(x+1) - \ln(x-1)\right]$$

$$\Rightarrow f'(x) = \frac{1}{2}\left[\frac{1}{x+1}\cdot 1 - \frac{1}{x-1}\cdot 1\right] = \frac{1}{2}\cdot\frac{-2}{x^2-1}$$

$g(x) = \operatorname{arctg} x$
$h(x) = tg\, x$
$(h\circ g)(x) = x$

$$\Downarrow$$

$$h'(g(x))\cdot g'(x) = 1$$

$$\Downarrow$$

$$g'(x) = \frac{1}{h'(g(x))}$$

$$= \frac{1}{sec^2(\operatorname{arctg} x)}$$

$$= \frac{1}{1+tg^2(\operatorname{arctg} x)}$$

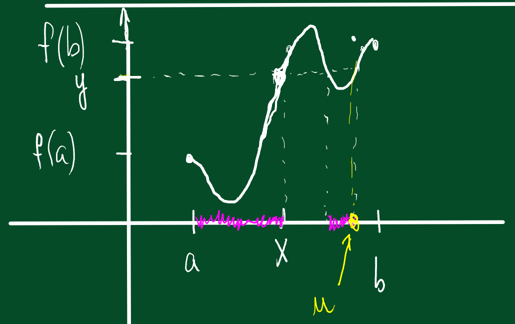
$$= \frac{1}{1+x^2}.$$

$$h(x) = \frac{sen\, x}{\cos x}$$

$$h'(x) = \frac{\cos^2 x + sen^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = sec^2 x$$

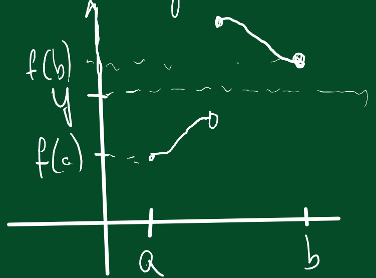
Teorema do Valor Intermediário (TVI)



$f(x) = y$

$S = \{ x \in [a,b] : f(x) \leq y \}$

$u = \sup S \Rightarrow f(u) = y$



Teo: Sejam $f:[a,b] \to \mathbb{R}$ contínua e y entre $f(a)$ e $f(b)$. Então existe $x \in [a,b]$ tal que $f(x) = y$.

Aplicação: garantir a existência de raízes p/ equações.

Exemplo: A equação $x^3 + 3x^2 - 2x = 7$ tem solução?

$f(x) = x^3 + 3x^2 - 2x$ é contínua.

$f(0) = 0 \leq 7$

$\boxed{f(2) = 16 \geq 7} \Rightarrow f(0) \leq 7 \leq f(2)$

pelo TVI, existe $0 \leq x \leq 2$ tal que $f(x) = 7$

$\boxed{f(1) = 2 \leq 7} \Rightarrow \exists x \in [1,2]$ tq $f(x) = 7$.

$f(3/2) = 7 + \frac{1}{8} = \frac{57}{8} \geq 7 \Rightarrow$ ñ ~~existe raiz~~ entre $3/2$ e 2? falso!

$\Downarrow$

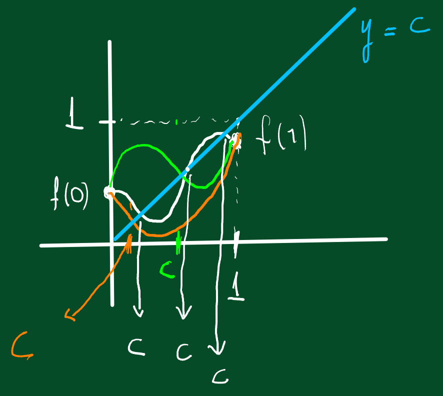
$\exists x \in [1, 3/2]$ tq $f(x) = 7$

$f(5/4) = \ldots$

Lista 2]

2. $f:[0,1] \to \mathbb{R}$ contínua, $0 \leq f(x) \leq 1$ $\Big\}$ $\exists\, c \in [0,1] : f(c) = c$.

$f(c) - c = 0$ ← $\underbrace{f(x) - x}_{g(x)} = 0$ (quero solução p/ isso!)



$g(x) = f(x) - x$ é contínua

$g(0) = f(0) \geq 0$

$g(1) = f(1) - 1 \leq 0$

$\Downarrow$

do TVI, existe $c \in [0,1]$

$g(c) = 0 \Rightarrow f(c) - c = 0$

$\Rightarrow f(c) = c$. (c é ponto fixo de f)