- O número e (limite de uma sequência)
- Derivada de f(x)=x^r, r uma constante real
- Derivada de h(x)=f(x)^g(x)

$f(x) = a^x, \quad x \in \mathbb{R} \qquad a>0, a \neq 1$

$g(x) = \log_a x, \quad x \in \mathbb{R}^+$

$(f \circ g)(x) = (g \circ f)(x) = x$

base a tq $f'(0) = 1$.

se existir tal base é e.

$$1 = f'(0) = \lim_{h \to 0} \frac{e^h - 1}{h}$$

$u = e^h - 1$

$\Downarrow$

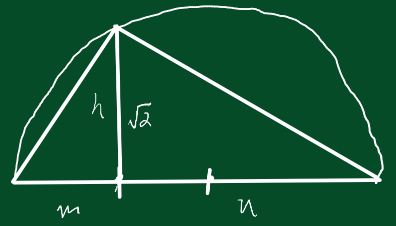
$e^h = u + 1$

$\ln e^h = \ln(u+1)$

$h = \ln(1+u)$

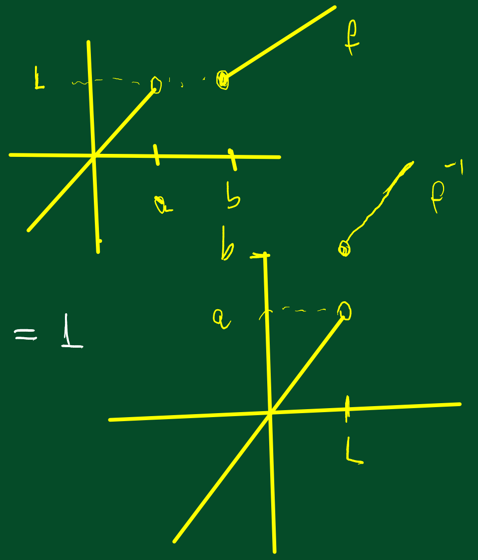
$u = e^h - 1, \quad h \to 0 \Leftrightarrow u \to 0$

$$1 = \lim_{u \to 0} \frac{u}{\ln(1+u)}$$



$h^2 = m.n$

$$1 = \lim_{u \to 0} \frac{1}{\frac{1}{u} \cdot \ln(1+u)} = \lim_{u \to 0} \frac{1}{\ln(1+u)^{1/u}} \quad (*)$$



$$\Rightarrow \lim_{u \to 0} \ln(1+u)^{1/u} = 1 \Rightarrow \ln\left(\lim_{u \to 0} (1+u)^{1/u}\right) = 1$$

$$\Rightarrow \boxed{e = \lim_{u \to 0} (1+u)^{1/u}}$$

$(*) = \dfrac{\lim_{u \to 0} 1}{\lim_{u \to 0} \ln(1+u)^{1/u}}$

$x = \frac{1}{u}: \quad u \to 0^+ \Rightarrow x \to +\infty$

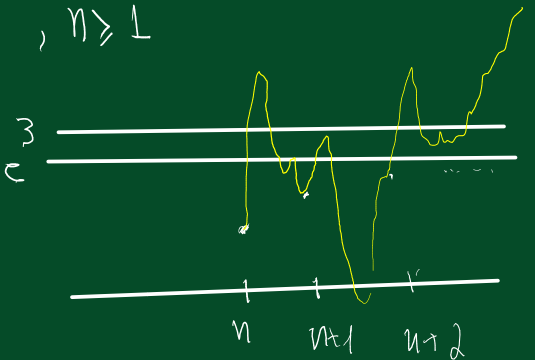
$\qquad\quad u \to 0^- \Rightarrow x \to -\infty$

$$\Rightarrow e = \lim_{x \to +\infty} (1 + 1/x)^x \quad \text{ou} \quad e = \lim_{x \to -\infty} \left(1 + \frac{1}{x}\right)^x$$

$\sqrt{2} \notin \mathbb{Q}$, mas é raiz de $p(x) = x^2 - 2$ com coef. racionais (algébrico)

$e \notin \mathbb{Q}$ e não existe $p(x)$ com coef. em $\mathbb{Q}$ tq $p(e) = 0$ (Transcendente)

$X_n = (1 + 1/n)^n \quad , n \geq 1$



$X_1 = 2$

$X_2 = \frac{9}{4} = 2,25$

$X_3 = \frac{64}{27} = \dots$

⋮

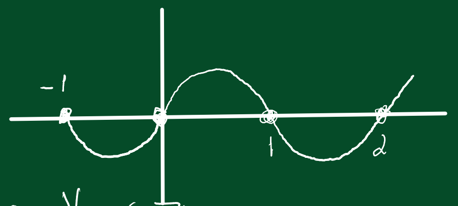
Mostra-se que:

- $2 \leq x_n \leq 3$
- X_n é crescente $(x_{n+1} > x_n)$

$\Rightarrow X_n$ converge!

$\not\Rightarrow \lim_{x \to \infty} (1 + 1/x)^x$ existe (mas mostramos que sim, via confronto)

$f(x) = \text{sen}(\pi x)$



$f(n) = \text{sen}(n\pi) = 0, \ \forall n \in \mathbb{Z}$

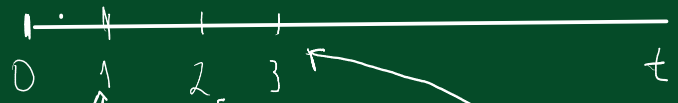
Juros compostos:

C = inicial

i = juros no período em

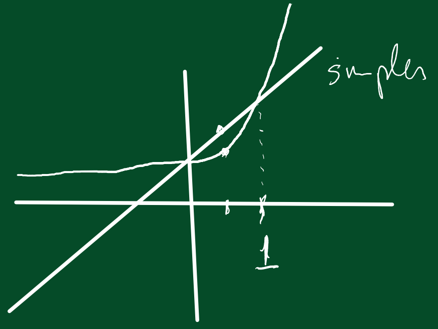
$M = C.(1+i)^n$

$C \quad C.(1+i) \quad C.(1+i).(1+i) \quad C(1+i)(1+i)(1+i)$ compostos

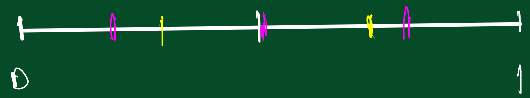


$C \quad C.(1+n) \quad C(1+i) + iC \quad C(1+i) + iC + iC$ simples

$C + niC = C.(1+ni)$



$i = 100\% = 1$



$(1+1)^1 = 2$

$(1 + 1/2)^2$

$(1 + 1/3)^3$

$(1 + 1/4)^4 \quad \dots \quad (1 + 1/n)^n, \ n \to \infty$

$$f(x) = e^x \Rightarrow f'(x) = e^x \qquad a^b = e^{b \cdot \ln a}$$

$$g(x) = \ln x \Rightarrow g'(x) = \frac{1}{x} \qquad \log_b a = \frac{\log_c a}{\log_c b}$$

$$f(x) = a^x \Rightarrow f'(x) = \ln a \cdot a^x$$

$$g(x) = \log_a x \Rightarrow g'(x) = \frac{1}{(\ln a) \cdot x}$$

$$f(x) = x^r \Rightarrow f'(x) = \,? \quad , \; r \in \mathbb{R}$$

$$\Downarrow$$

$$f(x) = e^{r \cdot \ln x} \qquad x \xrightarrow{r \cdot \ln(\ )} r \cdot \ln x \xrightarrow{e^{(\ )}} e^{r \ln x}$$

$$\Downarrow$$

$$f'(x) = e^{r \cdot \ln x} \cdot r \cdot \frac{1}{x} = x^r \cdot r \cdot \frac{1}{x} = r \cdot x^{r-1}.$$

$$h(x) = f(x)^{g(x)} = e^{g(x) \ln f(x)}$$

$$h'(x) = e^{g(x) \ln f(x)} \cdot \left[g'(x) \cdot \ln f(x) + g(x) \cdot \frac{1}{f(x)} \cdot f'(x) \right]$$

$$= f(x)^{g(x)-1} \cdot \left[f(x)\, g'(x) \ln f(x) + g(x) f'(x) \right]$$

Exemplo: $f(x) = x^x \quad , \; x > 0 \qquad 0^0 = \,?$

$$f(x) = e^{x \cdot \ln x} \Rightarrow f'(x) = e^{x \ln x} \cdot \left[1 \cdot \ln x + x \cdot \frac{1}{x} \right]$$

$$= x^x (\ln x + 1)$$

(a) $f(x) = \dfrac{\ln\left(x^3 + 2^{x^3}\right)}{x^2 + e^{\cos x}}$

$$\left[\ln\left(\frac{f(x)}{g(x)}\right)\right]' = \left(\ln f(x)\right)' - \left(\ln g(x)\right)'$$

$$= \frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)}$$

$$f'(x) = \frac{\left(\ln\left(x^3 + 2^{x^3}\right)\right)' \cdot \left(x^2 + e^{\cos x}\right) - \ln\left(x^3 + 2^{x^3}\right) \cdot \left(x^2 + e^{\cos x}\right)'}{\left(x^2 + e^{\cos x}\right)^2}$$

$$= \frac{\dfrac{1}{x^3 + 2^{x^3}} \cdot \left(3x^2 + 2^{x^3}(\ln 2) \cdot 3x^2\right) \cdot \left(x^2 + e^{\cos x}\right) - \ln\left(x^3 + 2^{x^3}\right) \cdot \left(2x - e^{\cos x} \cdot \operatorname{sen} x\right)}{\left(x^2 + e^{\cos x}\right)^2}$$

$h(x) = \ln\left(x^3 + 2^{x^3}\right)$

$f_1(x) = \ln x \Rightarrow f_1'(x) = \dfrac{1}{x}$

$f_2(x) = 2^x \Rightarrow f_2'(x) = (\ln 2) \cdot 2^x$

$f_3(x) = x^3 \Rightarrow f_3'(x) = 3x^2$

$h(x) = f_1\left(f_3(x) + f_2\left(f_3(x)\right)\right)$

$$h'(x) = f_1'\left(f_3(x) + f_2\left(f_3(x)\right)\right) \cdot \left(f_3(x) + f_2\left(f_3(x)\right)\right)'$$

$$= f_1'\left(f_3(x) + f_2\left(f_3(x)\right)\right) \cdot \left(f_3'(x) + f_2'\left(f_3(x)\right) \cdot f_3'(x)\right)$$

$$= \frac{1}{x^3 + 2^{x^3}}\left(3x^2 + (\ln 2) \cdot 2^{x^3} \cdot 3x^2\right)$$

(n) $f(x) = (3 + \cos x)^{\operatorname{tg}(x^2)} = e^{\operatorname{tg}(x^2) \cdot \ln(3 + \cos x)}$

$$f'(x) = (3 + \cos x)^{\operatorname{tg}(x^2)}\left[\sec^2(x^2) \cdot 2x \cdot \ln(3 + \cos x) + \operatorname{tg}(x^2) \cdot \frac{-\operatorname{sen} x}{3 + \cos x}\right]$$