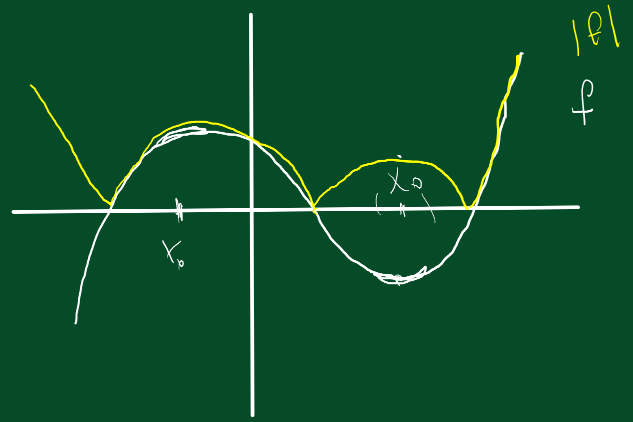
3.14.a) f é derivável em x_0; $|f|$ é derivável em x_0?

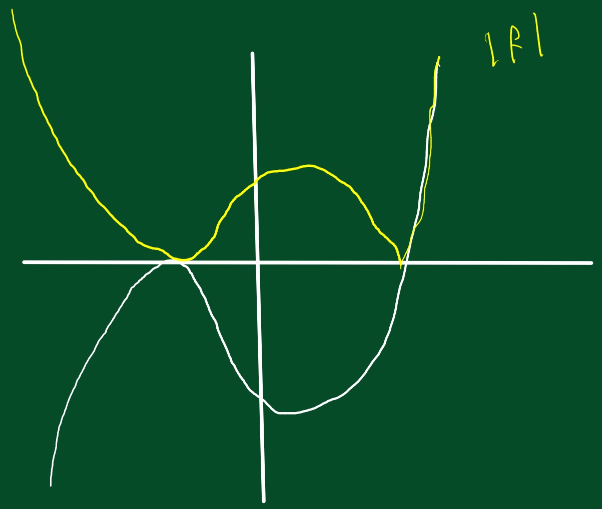


$$|f(x)| = \begin{cases} f(x), & \text{se } f(x) \geq 0 \\ -f(x), & \text{se } f(x) < 0 \end{cases}$$

f derivável em $x_0 \Rightarrow f$ é contínua em x_0

$f(x_0) \neq 0 \Rightarrow \exists\, I \subset \mathbb{R}$, intervalo aberto, $x_0 \in I$

onde $f(x) \neq 0$ (pos. se $f(x_0) > 0$ e neg. se $f(x_0)$))



c. $f(x) = x \cdot g(x)$, g é contínua em $x_0 = 0$

$f'(x_0) = 1 \cdot g(x_0) + x_0 g'(x_0)$ (errado!)

$$\lim_{x \to 0} \frac{f(x) - f(x_0)}{x - 0} = \lim_{x \to 0} \frac{x \cdot g(x) - 0 \cdot g(0)}{x} =$$

$$= \lim_{x \to 0} g(x) = g(0).$$

$\therefore f'(0) = g(0).$

$$h(x) = \frac{f(x)+g(x)}{2} + \frac{|f(x)-g(x)|}{2}$$

$$\left.\begin{array}{l} \cdot)\ f(x) \geq g(x) : h(x) = \frac{f(x)+g(x)}{2} + \frac{f(x)-g(x)}{2} = f(x) \\ \cdot)\ f(x) \leq g(x) : h(x) = \frac{f(x)+g(x)}{2} - \frac{f(x)-g(x)}{2} = g(x). \end{array}\right\} \Rightarrow h(x) = \max\{f(x), g(x)\}$$

Taxas relacionadas

$x \longleftrightarrow y$

mede variação de y

em termos da variação de x

$$\begin{tikzcd} x \arrow[rr, leftrightarrow] & & y \\ & t \arrow[ul] \arrow[ur] & \end{tikzcd}$$

$x(t) \rightarrow x'(t)$

$y(t) \rightarrow y'(t)$

Notação de Leibniz

$$f'(x_0) = \frac{df}{dx}(x_0) \text{ ou } \left.\frac{df}{dx}\right|_{x=x_0}$$

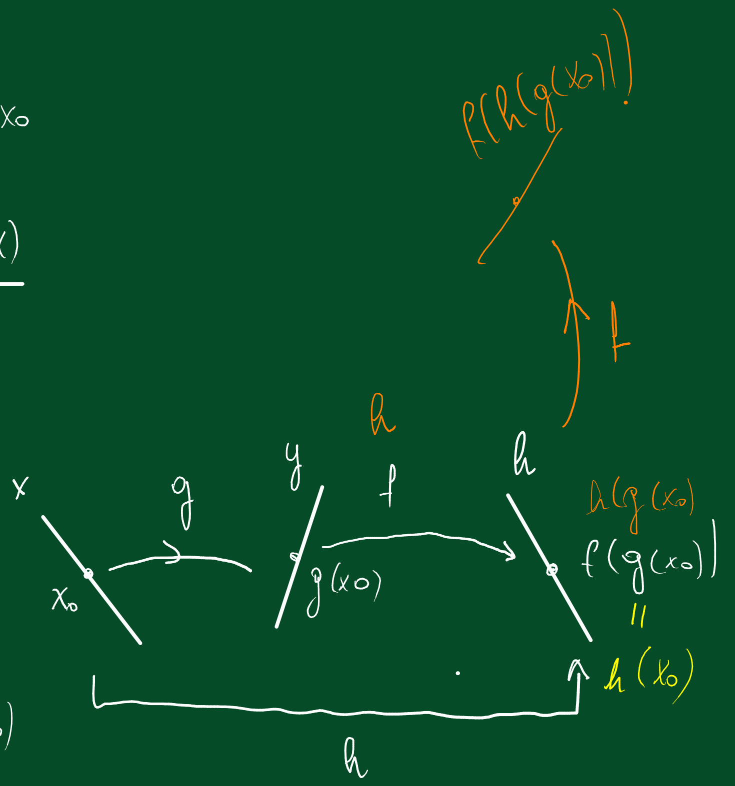
$$\| $$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$h(x) = (f \circ g)(x) = f(g(x))$$

$$h'(x_0) = f'(g(x_0)) \cdot g'(x_0)$$

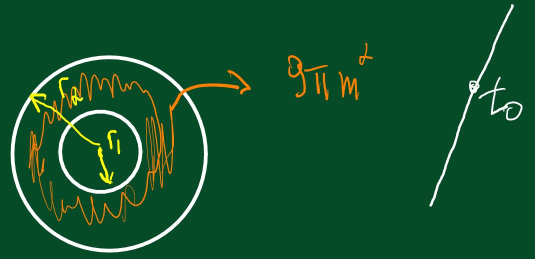
$$\frac{dh}{dx}(x_0) = \frac{d\overset{f}{h}}{dy}(g(x_0)) \cdot \frac{dg}{dx}(x_0)$$



$$\frac{df}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$$

$$\frac{df}{dx} = \frac{df}{dh} \cdot \frac{dh}{dg} \cdot \frac{dg}{dx}$$

4.3]



$$A_1(r_1) = \pi r_1^2 \qquad A_2(r_2) = \pi r_2^2 \Rightarrow A_2(r_2) - A_1(r_1) = 9\pi$$

$$\pi r_2^2(t) - \pi r_1^2(t) = 9\pi$$

$$\pi\cdot 2 r_2(t)\cdot r_2'(t) - \pi\, 2\cdot r_1(t)\cdot r_1'(t) = 0$$

$$r_1'(t_0) = \frac{r_2(t)\cdot r_2'(t)}{r_1(t)} \overset{(!)}{=} \frac{5\cdot 1}{4} = 5/4.$$

$$\frac{dA_2}{dt}(t_0) = 10\pi$$

$$\|$$

$$\frac{dA}{dr}(r_2(t_0))\cdot r_2'(t_0) \quad (*)$$

t_0 é tal que: $A_1(t_0) = 16\pi = \pi r_1^2(t_0) \Rightarrow \boxed{r_1(t_0) = 4}$

$$\Downarrow$$

$$A_2(t_0) = 25\pi = \pi r_2^2(t_0) \Rightarrow \boxed{r_2(t_0) = 5}$$

$$\begin{tikzcd} t_0 \arrow[r, "r_2"] & r_2(t_0) \arrow[r, "A_2"] & A(r_2(t_0)) \\ t \arrow[rr, bend right, "A_2(t)"'] & r_2 & A_2 \end{tikzcd}$$

$$\frac{dA_2}{dt}(t_0) = \frac{dA_2}{dr_2}(r_2(t_0))\cdot \frac{dr_2}{dt}$$

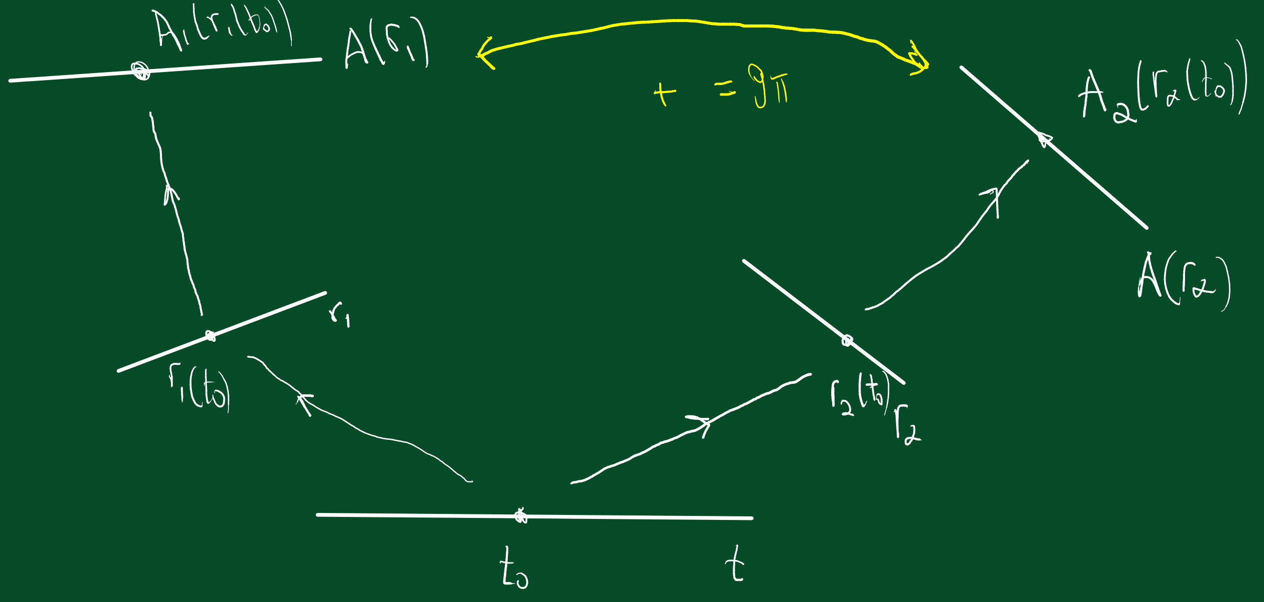
$$\frac{dA_1}{dt} = \frac{dA_1}{dr_1}\cdot \underbrace{\frac{dr_1}{dt}}_{?}$$

(!) em t_0.

$$(*)\ \frac{dA}{dr}(r(t_0))\cdot \frac{dr_2}{dt}(t_0) = 10\pi$$

$$2\pi r_2(t_0)\cdot \frac{dr_2}{dt}(t_0) = 10\pi$$

$$10\pi \frac{dr_2}{dt}(t_0) = 10\pi \Rightarrow \boxed{r_2'(t_0) = 1}$$

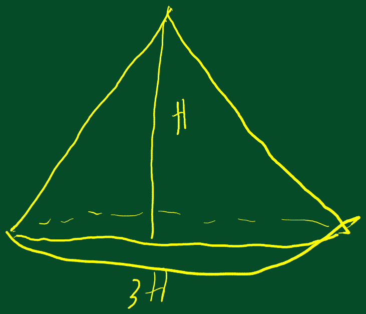


$$s(t) = 5t + s_0 \to \frac{ds}{dt} = 5$$

X

$$\frac{ds}{dt}(t_0) = 5, \ \forall t_0$$

$$\frac{\dot{d}s}{dt}(t_1) = ?$$

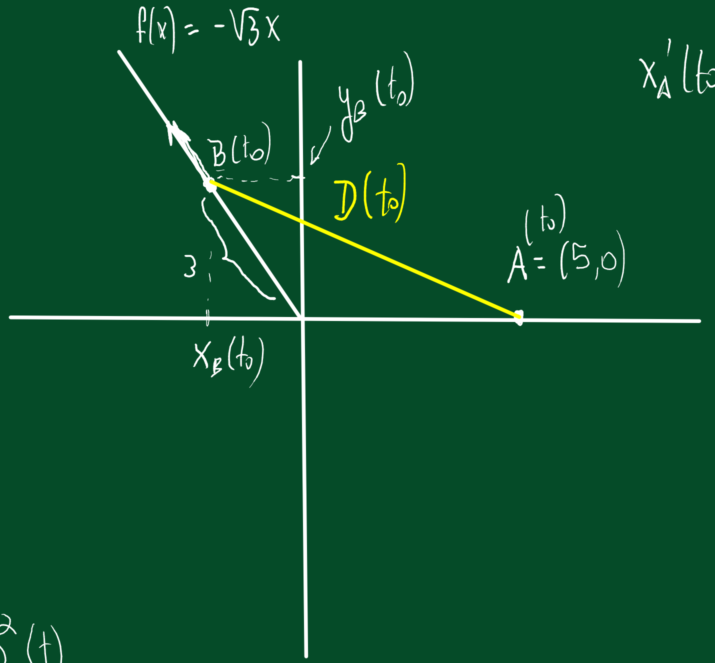


$$V = \frac{\pi}{3}\left(\frac{3H}{2}\right)^2 \cdot H$$

$$\frac{dV}{dt} = \frac{dV}{dH} \cdot \frac{dH}{dt}$$

$$V'(t) = V'(H(t)) \cdot H'(t)$$

4.4



$$x_A'(t_0) = \frac{dA}{dt}(t_0) = 3$$

$$\frac{dB}{dt}(t_0) = 4$$

$$\frac{dD}{dt}(t_0) = ?$$

$$A(t) = (x_A(t), 0)$$

$$B(t) = (x_B(t), \overbrace{-\sqrt{3}\,x_B(t)}^{y_B(t)})$$

$$D^2(t) = (x_B(t) - x_A(t))^2 + 3x_B^2(t)$$

$$\underbrace{2D(t_0)}_{D_0} \cdot \underbrace{\frac{dD}{dt}(t_0)}_{(*)} = 2(\underbrace{x_B(t_0)}_{-3/2} - \underbrace{x_A(t_0)}_{5}) \cdot (\underbrace{x_B'(t_0)}_{-2} - \underbrace{x_A'(t_0)}_{3}) + 6\underbrace{x_B(t_0)}_{-3/2} \cdot \underbrace{x_B'(t_0)}_{-2}$$

$$\Rightarrow \frac{dD}{dt}(t_0) = \frac{83}{14}$$

$$D^2(t_0) = \left(-\tfrac{3}{2} - 5\right)^2 + 3\cdot\left(-\tfrac{3}{2}\right)^2 = D_0$$

$$t \xrightarrow{D} D(t) \xrightarrow{(\ )^2} D^2(t)$$

$$x_A(t_0) = 5$$

$$x_B^2(t_0) + y_B^2(t_0) = 9.$$

$$4x_B^2(t_0) = 9 \rightarrow$$

$$x_B(t_0) = -3/2$$

$$y_B(t_0) = 3\sqrt{3}/2$$

$$D_B^2(t) = x_B^2(t) + y_B^2(t) = 4x_B^2(t)$$

$$\underbrace{2D_B(t_0)}_{3} \cdot \underbrace{\frac{dD_B}{dt}(t_0)}_{4} = 8 \cdot \underbrace{x_B(t_0)}_{-3/2} \cdot \underbrace{\frac{dx_B}{dt}(t_0)}_{?}$$

$$\Downarrow$$

$$x_B'(t_0) = \frac{dx_B}{dt}(t_0) = -2.$$