More About AGM Revision in Description Logics

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1This work was supported by Fapesp Project LogProb, grant 2008/03995-5, São Paulo, Brazil.
1 Introduction

Belief revision studies the dynamics of beliefs defining some operations in logically closed sets (belief sets): expansion, revision and contraction. Revision, in particular deals with the problem of accommodating consistently a newly received piece of information.

Most of the works on belief revision following the seminal paper [1] assume that the underlying logic of the agent satisfies some assumptions. In [5] we showed how to apply revision of belief sets to logics that are not closed under negation. We have, however, assumed that the logic satisfies a property called distributivity. In the present work we show a list of description logics that are not closed under that the logic satisfies a property called distributivity. In the present work we show a list of description logics that are not closed under negation and study which of them are distributive.

1.1 AGM paradigm

The most influential work in belief revision is [1]. In this work the authors defined a number of rationality postulates for contraction and revision, now known as the AGM postulates. The authors then showed constructions for these operations and proved that the constructions are equivalent to the postulates (representation theorem).

Most works in belief revision assume some properties on the underlying logic: Tarskianicity, contraction, deduction and supraclassicality, which we will refer to as the AGM assumptions. The last two together are equivalent to the following two properties together for Tarskian logics:

Definition 1 (distributivity) A logic \( \langle \mathcal{L}, Cn \rangle \) is distributive iff for all sets of formulas \( X, Y, W \in 2^{\mathcal{L}} \), we have that \( Cn(X \cup (Cn(Y) \cap Cn(W))) = Cn(X \cup Y) \cap Cn(X \cup W) \).

Definition 2 (closure under negation) A logic \( \langle \mathcal{L}, Cn \rangle \) is closed under negation iff for all \( A \in 2^\mathcal{L} \) there is a \( B \in 2^\mathcal{L} \) such that \( Cn(A \cup B) = \mathcal{L} \) and \( Cn(A) \cap Cn(B) = Cn(\emptyset) \). The set \( B \) is then called a negation of \( A \).

AGM revision in non-classical logics: In [5] we argued that some description logics are not closed under negation and, hence, do not satisfy the AGM assumptions. Furthermore, the most common way to define revision is via Levi identity \( (K * \alpha) = K - \neg \alpha + \alpha \), which assumes the existence of the negation of \( \alpha \). We proposed then a new construction and a set of postulates for revision for logics that are not closed under negation.

We used two postulates, borrowed from the belief base literature:

(relevance) If \( \beta \in K \setminus K * \alpha \) then there is \( K' \) such that \( K \cap (K * \alpha) \subseteq K' \subseteq K \) and \( K' \cup \{ \alpha \} \) is consistent, but \( K' \cup \{ \alpha, \beta \} \) is inconsistent.

(uniformity) If for all \( K' \subseteq K \), \( K' \cup \{ \alpha \} \) is inconsistent iff \( K' \cup \{ \beta \} \) is inconsistent then \( K \cap K * \alpha = K \cap K * \beta \).

The set of rationality postulates we considered is: closure, success, inclusion, consistency, relevance and uniformity. The following proposition is an evidence that this is a good choice of rationality postulates:

Proposition 3 For logics that satisfy the AGM assumptions, closure, success, inclusion, consistency, relevance and uniformity are equivalent to the original AGM postulates for revision: closure, success, consistency, vacuity and extensionality.

We proposed also a construction inspired in some ideas from [4]:

Definition 4 (Maximally consistent set w.r.t \( \alpha \)) \( \{ X \in K \downarrow \alpha \} \) is consistent if \( X \subseteq K \), \( X \cup \{ \alpha \} \) is consistent and if \( X \subset X' \subseteq K \) then \( X' \cup \{ \alpha \} \) is inconsistent.

Definition 5 (Selection function) \( \langle \mathcal{L}, Cn, \rangle \) A selection function for \( K \) is a function \( \gamma \) such that if \( K \downarrow \alpha \neq \emptyset \), then \( \emptyset \neq \gamma(K \downarrow \alpha) \subseteq K \downarrow \alpha \). Otherwise, \( \gamma(K \downarrow \alpha) = \{ K \} \).

The construction of a revision without negation is defined as \( K * \gamma = \bigcap \gamma(K \downarrow \alpha) + \alpha \).

We proved that, for distributive logics, this construction is completely characterized by the set of rationality postulates we are considering i.e. we proved the representation theorem relating the construction to the set of postulates [5].

1.2 Description Logics

Description logics (DLs) forms a family of formalisms to represent terminological knowledge. The signature of a description logic is a tuple \( \langle N_C, N_R, N_I \rangle \) of concept names, roles names and individual names of the language [2]. From a signature it is possible to define complex concepts via a description language. Each DL has its own description language that admits a certain set of constructors.

The semantic of a DL is defined using an interpretation \( I = \langle \mathcal{T}, \Delta^I \rangle \) such that \( \Delta^I \) is a non-empty set called domain and \( \mathcal{T} \) is an interpretation function. For each concept name the interpretation associates a subset of the domain, for each role name a binary relation in the domain and for each individual an element of the domain. The interpretation is then extended to complex concepts.

A sentence in a DL is a restriction to the interpretation. A TBox is a set of sentences of the form \( C_1 \subseteq C_2 \) that restricts the interpretation of concepts, an ABox is a set of sentences of the form \( C(a), R(a, b) \).

\footnote{Assuming that the logic admits GCI axioms}
The example above depends on the existence of the ABox. In fact, $\mathcal{ALC}$ with empty ABox is distributive:

**Proposition 11** Consider a DL $\langle L, Cn \rangle$ such that for every sentence $\alpha \in L$ there is a sentence $\alpha' \in L$ such that $Cn(\alpha) = Cn(\alpha')$ and $\alpha'$ has the form $\top \subseteq C$ for some concept $C$. Then $\langle L, Cn \rangle$ is distributive.

Since in $\mathcal{ALCO}$ the ABox can be written in terms of the TBox, $\mathcal{ALCO}$ is distributive even in the presence of the ABox.

Finally, if we consider a logic $\langle L, Cn \rangle$ that admits role hierarchy, but does not admit role constructors, then $\langle L, Cn \rangle$ is not distributive. Consider the following example:

**Example 12:** Let $X = \{R \subseteq S_1, R \subseteq S_2\}$, $Y = \{S_1 \subseteq S_3\}$ and $Z = \{S_2 \subseteq S_3\}$. We have that $Cn(Y) \cap Cn(Z) = Cn(\emptyset)$. Hence $R \subseteq S_3 \not\subseteq Cn(X \cup (Cn(Y) \cap Cn(Z)))$, but $R \subseteq S_3 \subseteq Cn(X \cup Y) \cap Cn(X \cup Z)$.

Besides $\mathcal{ALCH}$, the logics behind OWL 1 ($\mathcal{SHOIN}^\mathcal{H}$ for OWL-DL and $\mathcal{SH}^{\mathcal{IF}}$ for OWL-lite), OWL-2 ($\mathcal{SROIQ}$) and the OWL profiles OWL-RL and OWL-QL admit role hierarchy, but do not admit role constructors. None of these logics are distributive.

The following table sums up the results of this section:

<table>
<thead>
<tr>
<th>Description Logic</th>
<th>Negation</th>
<th>Distributivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{ALC}$</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>$\mathcal{ALC}$ without ABox</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>$\mathcal{ALCO}$</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>$\mathcal{ALCH}$, OWL-lite, OWL-DL</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>OWL-QL, OWL-RL and OWL 2</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

## 3 Conclusion and future work

In this work we continued the work started in [5] by showing for which DLs the AGM revision without negation can be applied. We showed that most DLs that admits GCI axioms are closed under negation, but most of them are not distributive. Hence, the representation theorem presented in [5] holds for $\mathcal{ALC}$ with empty TBox and $\mathcal{ALCC}$ are two exceptions. These logics are distributive and not closed under negation. Hence, the AGM postulates and the underlying logic satisfies the AGM assumptions. This is a good evidence that we chose a good set of rationality postulates.

As future work we should look for a construction that can be characterized by this set of postulates (or a similar one) not only in distributive, but in any Tarskian compact logic.

## REFERENCES