Hochschild-Mitchell (co)-Homology of $G - k$-categories over a ring, Galois Coverings and Skew Categories

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Abstract

Let $\mathcal{C}$ be category, over commutative ring k, provided with an action of a group G. Its Hochschild-Mitchell homology and cohomology are denoted by $HH_*(\mathcal{C})$ and $HH^*(\mathcal{C})$. Let $\mathcal{C}[G]$ be the skew-category. If the action of G is free on the objects of $\mathcal{C}$, and if the coinvariants and invariants functor are exact, we obtain isomorphisms $HH_*(C)_G \simeq HH_*^{\{1\}}(\mathcal{C}[G])$ and $[HH(\mathcal{C})]^G \simeq HH^*_{\{1\}}(\mathcal{C}[G])$, where $\{1\}$ is the trivial conjugacy class of G.

Using an auxiliary category $M_G\mathcal{C}$ we show that these isomorphisms also holds if the action in not free, in particular they hold for Hochschild (co)homology of a k-algebra provided with an action of G by automorphisms.

Hence $HH^*(\mathcal{C})^G$ is a canonical direct summand of $HH^*(\mathcal{C}[G])$.