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Abstracts

Lattices and Quasialgebras

Helena Albuquerque
Universidade de Coimbra, Portugal

Joint work with Rolf Sören Kraußhar.

Let G be a finite group and $\mathbb{R}G$ its group algebra defined over $\mathbb{R}$. If we define in G a 2-cochain F, we can consider the algebra $\mathbb{R}_F G$ which is obtained from $\mathbb{R}G$ deforming the product, $x._F y = F(x,y)xy, \forall_{x,y \in G}$. Examples of $\mathbb{R}_F(\mathbb{Z}_2)^n$ algebras are Clifford algebras (like complex numbers and quaternions) and Cayley algebras (like octonions).

An n-dimensional lattice in $\mathbb{R}^n$ is a set of points of the form

$$\Omega = \mathbb{Z}\omega_1 + \cdots + \mathbb{Z}\omega_n$$

where $\omega_1, \ldots, \omega_n$ are some $\mathbb{R}$-linear independent vectors from $\mathbb{R}^n$.

If we consider a group G with n elements, say $G = \{g_1, \ldots, g_n\}$, then we can identify each element $a_1 g_1 + \ldots + a_n g_n, a_i \in \mathbb{R}$ of the algebra $\mathbb{R}_F G$ with the element $(a_1, \ldots, a_n) \in \mathbb{R}^n$. With this identification the multiplication defined in $\mathbb{R}_F G$ will introduce a special multiplication on $\mathbb{R}^n$. This is called the multiplication of $\mathbb{R}^n$ induced by the group G using the cochain F. In this case we say that $\mathbb{R}^n$ is embedded in $\mathbb{R}_F G$.

The simplest non-trivial examples of this theory are lattices defined in $\mathbb{R}^2$ with complex multiplication, that are very well studied by numerous authors. In this talk we will present a generalization of this case studying n-dimensional lattices in the vector space $\mathbb{R}^n$ embedded in some twisted group algebras $\mathbb{R}_F G$.

Quadratic Malcev superalgebras

Elisabete Barreiro
Universidade de Coimbra, Portugal

(joint work with Helena Albuquerque and Said Benayadi)

A quadratic Malcev superalgebra is a Malcev superalgebra M endowed with a non-degenerate, supersymmetric, even, and M-invariant bilinear form. In this talk we describe inductively quadratic Malcev superalgebras. In [1], H. Albuquerque and S. Benayadi described the quadratic Malcev superalgebras with reductive even part and the action of the even part on the odd part completely reducible. In [3] we present an description of quadratic Malcev superalgebras with reductive even part, using the notion of double extension of Malcev superalgebras exposed in [1], and transferring to Malcev superalgebras the concept of generalized double extension given in [5] for Lie superalgebras. This description generalize to Malcev superalgebras the work done in Lie case [2, 4, 6].

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Derived tame local and two-point algebras

Viktor Bekkert
UFMG, Brazil

We determine representation type of the bounded derived category of finitely generated modules over finitely generated complete local and two-point algebras.

This is a joint work with Yuriy Drozd (Institute of Mathematics, Ukraine) and Vyacheslav Futorny (Universidade de São Paulo, Brazil).

Commutative n-ary Leibniz algebras

Pilar Benito
Universidad de La Rioja, Spain

(joint work with Sara Madariaga and José María Pérez-Izquierdo)

An n-ary Leibniz algebra is a linear space endowed with a n-multilinear product $[x_1, \ldots, x_n]$ satisfying $[[x_1, \ldots, x_n], y_2, \ldots, y_n] = \sum_{i=1}^{n} [x_1, \ldots, [x_i, y_2, \ldots, y_n], \ldots, x_n]$. In case the product is totally symmetric, the algebra is said to be commutative.

It is easily checked that any simple binary Leibniz algebra (arbitrary field and dimension) is a Lie algebra and therefore is no commutative. Ternary Leibniz algebras are exactly the so called balanced symplectic algebras introduced in 1972 by J.R. Faulkner and J.C. Ferrer [2] and renamed in [1] as null-symplectic triples. From two copies of a null-symplectic triple $\mathcal{T}$ we can get a 3-graded Lie algebra by declearing $\mathcal{T}$ as ± 1-components and the inner derivation algebra of $\mathcal{T}$ as 0-component. In this way, any null-symplectic triple provides a (linear) *Jordan pair*.

Using representation theory of Lie algebras, in 2003 A.P. Pojidaev [4] proved that, over fields of characteristic zero, there are no finite-dimensional commutative simple n-ary Leibniz algebras. Our goal in this talk is to show that Pojidaev's Theorem can be stablished for $n \geq 3$ and arbitrary dimension over fields of characteristic $> n$. We use two different approximations to prove the assertion. The notion of *absolute zero divisors* in Jordan pairs let us get the result for $n = 3$. In case $n \geq 4$, the result follows using the notion of *covers of thin sandwiches* introduced in [3]. Both proofs are based on results given in [5] concerning local nilpotency of associative enveloping algebras of linear Lie algebras.

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Free field realizations of modules for the Lie algebra of vector fields on a torus

Yuly Billig
Carleton University, Canada

In this talk we introduce generalized Wakimoto modules $W(\alpha)$ for the loop Lie algebra $sl_2 \otimes \mathbb{C}[t, t^{-1}]$ and show that they admit the action of a much larger Lie algebra $\mathcal{D}$ of vector fields on a 2-dimensional torus.

For $\alpha \notin \mathbb{Z}$ we prove that the modules $W(\alpha)$ are irreducible over $\mathcal{D}$, while for $\alpha \in \mathbb{Z}$ these modules form a complex

$$\ldots \to W(-1) \to W(0) \to W(1) \to W(2) \to \ldots$$

with non-trivial kernels and images.

This is a joint work with V.Futorny.

Complexity and Module Varieties for classical Lie superalgebras

Brian D. Boe
University of Georgia, USA

(Joint work with Jonathan R. Kujawa and Daniel K. Nakano)

Let $\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}$ be a classical Lie superalgebra and $\mathcal{F}$ be the category of finite dimensional $\mathfrak{g}$-supermodules which are semisimple over $\mathfrak{g}_{\bar{0}}$. We investigate the homological properties of the category $\mathcal{F}$. In particular $\mathcal{F}$ is self-injective in the sense that all projective supermodules are injective. Moreover all supermodules in $\mathcal{F}$ admit projective resolutions with polynomial rate of growth and, hence, one can study complexity in $\mathcal{F}$. If $\mathfrak{g}$ is a Type I Lie superalgebra we introduce support varieties which detect projectivity and are related to the associated varieties of Duflo and Serganova. If in addition $\mathfrak{g}$ has a (strong) duality then the conditions of being tilting or projective are equivalent.

Functional identities and their applications to graded algebras

Matej Brešar
University of Ljubljana and University of Maribor, Slovenia

We will first give a brief informal survey of the theory of functional identities, and then present some of its recent applications. Specifically, we will consider the question whether a grading of a Lie or a Jordan algebra is induced by a grading of its associative envelope. The latter is a joint work with Yu. Bahturin and I. Shestakov.

Linear Groups of Automorphisms of the Binary Tree

Andrew M. Brunner
University of Wisconsin-Parkside, USA

This is joint work with Said N. Sidki.

Let A is a state-closed abelian torsion group acting on the one-rooted binary tree. Then A is conjugate to a subgroup of the topological closure $\overline{D}$ of the infinitely generated elementary abelian 2-group $D = \left\langle \sigma^{(n)} | n \geq 0 \right\rangle$. Here $\sigma^{(n)}$ is the automorphism of the tree active only at vertices on the n-th level of the tree.

For each integer m there is an affine group $\overline{D} \rtimes U(m, \mathbb{Z}_2(t))$ where $U(m, \mathbb{Z}_2(t))$ is a certain subgroup of finite index in the general linear group $GL(m, \mathbb{Z}_2(t))$. Here $\mathbb{Z}_2(t)$ denotes the function field in an indeterminate t over $\mathbb{Z}_2$ the field of two elements. $U(m, \mathbb{Z}_2(t))$ is isomorphic to a group of finite-state automorphisms of the binary tree.

The only torsion in $U(m, \mathbb{Z}_2(t))$ consists of elements of order 2. For $m > 1$, the groups $U(m, \mathbb{Z}_2(t))$ contain in a natural way the free product of infinitely generated elementary abelian 2-groups. In fact, there are simple 3-generator subgroups which contain the free product of infinitely generated elementary abelian 2-groups.

As an application of the representation, let $\gamma = (\gamma\sigma, \gamma)$ and $\mu^{-1} = (\mu\sigma, \mu)$ be recursively defined automorphisms of the one-rooted binary tree. The group generated by γ, μ is 2-dimensional; elementary calculations with matrices show that the group has a presentation $\langle \gamma, \mu | \gamma^2\mu = \mu\gamma^2 \rangle$.

Álgebras hereditárias e suas representações

Flavio Ulhoa Coelho
IME-USP, Brazil

Este é um minicurso introdutório à chamada Teoria de Representações de Álgebras. Por conveniência, nossas álgebras serão sempre álgebras de dimensão finita sobre um corpo algebricamente fechado. Como objeto de nossa discussão, vamos nos concentrar nas chamadas álgebras hereditárias. Lembramos que uma álgebra A é *hereditária* se o seu radical é um A-módulo projetivo. A partir de uma tal álgebra A iremos associar um grafo orientado Q_A que irá *representar* não só a própria álgebra como também os seus módulos finitamente gerados.

Rational Conformal Field Theories and Modular Categories

Alexei Davydov
University of Sidney, Australia

We give an overview of the recent development of the theory of modular categories and their applications to theoretical physics. In particular, we discuss relations between internal algebraic structures in modular categories and boundary RCFT, full RCFT (modular invariants), and extensions of chiral algebras.

Computing with rational functions and applications to symmetric functions, invariant theory and PI-algebras

Vesselin Drensky
Bulgarian Academy of Sciences, Bulgaria

This is a joint project with Francesca Benanti, Silvia Boumova, Georgi K. Genov, and Plamen Koev.

Let K be a field of characteristic 0 and let $K[[X]] = K[[x_1, \dots, x_d]]$ be the algebra of formal power series in d variables. We consider series $f(X) \in K[[X]]$ which can be presented as rational functions with denominators products of binomials of the form $1 - X^a = 1 - x_1^{a_1} \cdots x_d^{a_d}$. Following Berele, we call such functions nice rational functions. Nice rational functions play a key role in the theory of linear systems of homogeneous diophantine equations. They appear also as Hilbert series of symmetric algebras of finitely generated graded modules of finitely generated graded commutative algebras and Hilbert series of finitely generated relatively free algebras in varieties of associative, Lie and Jordan algebras. Nice rational symmetric functions $f(X) \in K[[X]]^{S_d}$ can be presented as formal series

$$f(X) = \sum_{\lambda} m(\lambda) S_\lambda(X), \quad m(\lambda) \in K,$$

where $S_\lambda(X)$ is the Schur function associated with the partition $\lambda = (\lambda_1, \dots, \lambda_d)$. Recently Berele established that if $f(X)$ is a nice rational symmetric function, then the generating function

$$M(f) = \sum_{\lambda} m(\lambda) X^\lambda$$

of its multiplicities $m(\lambda)$ is also a nice rational function. We give a direct proof of this result using ideas of Elliott from 1903. Our proof also provides two easy algorithms which compute the function $M(f)$.

We apply our algorithms to several problems in the theory of symmetric functions, invariant theory, and algebras with polynomial identities. Starting with the Hilbert series of the pure and mixed trace algebras of three generic 3×3 matrices computed by Berele and Stembridge, we calculate the generating function of the multiplicities $m(\lambda_1, \lambda_2, \lambda_3)$ of the pure and mixed trace cocharacter sequences for 3×3 matrices. Our method allows easily to calculate the Hilbert series of the invariants of the special linear group $SL_d(K)$ and the unitriangular group $UT_d(K)$ acting on a polynomial $GL_d(K)$-module of small dimenion. We also calculate Hilbert series of invariants of $SL_d(K)$ and $UT_d(K)$ acting on relatively free algebras $K\langle X\rangle/T(R)$, provided that we know the Hilbert series of $K\langle X\rangle/T(R)$ or the cocharacter sequence of the T-ideal $T(R)$ of the polynomial identities of the PI-algebra R.

Partial projective representations of groups and their factor sets

Michael Dokuchaev
IME-USP, Brazil

Partial representations were introduced in the theory of C^*-algebras by R. Exel [4] and J. Quigg and I. Reaburn [7] as an important working tool when dealing with algebras generated by partial isometries on a Hilbert space. This concept is intimately related to that of a partial action, which was defined by R. Exel in [4], and which serves, in particular, to introduce more general crossed products (see [3] and [1]). Since then, several relevant classes of C^*-algebras have been described as crossed products by partial actions [6], as well as a wide class of Hecke algebras [5]. The generalized crossed products, which appeared in this way, are based on the concept of a twisted partial action. In classical crossed products the twisting is a 2-cocycle, so it is natural to wonder if there exists an adequate cohomological theory, which would agree with the twisting, incorporated in partial crossed products. In [2] the first step was made towards the investigation of this problem. More precisely, partial projective representations of groups were defined and studied as well as the corresponding factors sets. Moreover, the interaction between the partial projective representations and partial actions were examined, showing, in particular, that our new factors sets are exactly the so-called K-valued partial twistings.

In our talk we shall communicate some of the recent results obtained in collaboration with Boris Nóvikov on this subject.

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Jordan gradings on exceptional simple Lie algebras

Alberto Elduque
Universidad de Zaragoza, Spain

Models of all the gradings on the exceptional simple Lie algebras induced by Jordan subgroups of their groups of automorphisms are provided. Most of these models are obtained through simpler gradings on symmetric composition algebras.

A few comments about compact quantum groups

Walter Ferrer
Universidad de la Republica, Uruguay

In this talk, after presenting a short introduction to the basic definitions concerning compact quantum groups in terms of a o–structure (a la Manin) instead of in terms of a *–operation, we introduce the concept of positive antipode and exhibit some of the basic properties of this operator that is an automorphism of compact quantum group. We finish the talk with some constructions for the finite dimensional case.

Globalization of partial actions on semiprime rings

Miguel Ferrero
UFRGS, Brazil

In this talk we will give conditions for a partial action on a semiprime ring to have a globalization. In the case that this globalization exists there exists also a globalization which is a semiprime ring and this one is unique unless equivalence. This results are contained in a joint paper by the author and Wagner Cortes.

How inner ideals can help to understand Lie algebras

Esther Garcia
Universidad Rey Juan Carlos, Spain

In this talk we will review different notions related to inner ideals, such us socle, descending chain conditions on inner ideals, complementation, weak complementation, etc, and the structure theories for Artinian simple Lie algebras and for simple Lie algebras with an essential Artinian socle. The talk is mainly based on two papers:

- An Artinian theory for Lie algebras, Journal of Algebra, 319 (2008), 938-951 (by A. Fernández López, E. Garcia, M. Gómez Lozano).

- Inner ideal structure of nearly Artinian Lie algebras, Proceedings of the American Mathematical Society ,137 (2009), 1-9 (by A. Fernández López, E. Garcia, M. Gómez Lozano).

Bass cyclic units as factors in a free product in integral group ring units
Jairo Zacarias Gonçalves
IME-USP, Brazil

Joint work with Angel del Rio. Let $U(\mathbb{Z}G)$ be the group of units of the integral group ring $\mathbb{Z}G$, of the finite group G over the ring of integers $\mathbb{Z}$. Let G be a group of odd order, and let a be a noncencentral element of prime order of G. We prove that if u is a Bass cyclic unit based on the element a , then there exists either a Bass cyclic unit or a bicyclic unit v, such that $< u, v^n >$ is a nonabelian free group for n sufficiently large.

The bar-radical of generalized standard baric algebras
Henrique Guzzo Jr
IME-USP, Brazil

Joint work with J.C.M. Ferreira.

Let U be an algebra over F not necessarily associative, commutative or finite dimensional. If $\omega : U \longrightarrow F$ is a nonzero homomorphism of algebras, then the ordered pair (U, ω) will be called a *baric algebra* over F and ω its *weight function* or simply its *weight*. For $x \in U$, $\omega(x)$ is called *weight* of x. If $e \in U$ has weight 1 and $e^2 = e$, then e will be called *idempotent of weight 1*.

When B is a subalgebra of U and $B \not\subseteq ker\omega$, then B is called a *b-subalgebra* of (U, ω). In this case, (B, ω_B) is a baric algebra, where $\omega_B = \omega|_B : B \longrightarrow F$.

Let B be a b-subalgebra of (U, ω). Then the subset $\mathrm{bar}(B) = \{x \in B \mid \omega(x) = 0\}$ is a two-side ideal of B of codimension 1, called *bar ideal* of B. If B is a b-subalgebra of U and $\mathrm{bar}(B)$ is a two-side ideal of $\mathrm{bar}(U)$, then B is called *normal b-subalgebra* of (U, ω). If $I \subseteq \mathrm{bar}(B)$ is a two-side ideal of B, then I is called *b-ideal* of B.

A baric algebra (U, ω) is called *b-simple* if for all normal b-subalgebra B of U, $\mathrm{bar}(B) = (0)$ or $\mathrm{bar}(B) = \mathrm{bar}(U)$. When (U, ω) has an idempotent of weight 1, then (U, ω) is b-simple if, and only if, its only b-ideals are (0) and $\mathrm{bar}(U)$.

Let (U, ω) be a baric algebra. We define the *bar-radical* of U, denoted by $\mathrm{rad}(U)$, as: $\mathrm{rad}(U) = (0)$, if (U, ω) is b-simple, otherwise as $\mathrm{rad}(U) = \cap\mathrm{bar}(B)$, where B runs over the maximal normal b-subalgebra of U. Of course, $\mathrm{rad}(U)$ is a b-ideal of U.

We say that U is *b-semisimple* if $\mathrm{rad}(U) = (0)$.

Let us denote the *associator* by $(x, y, z) = (xy)z - x(yz)$, for $x, y, z \in U$, the *commutator* by $[x, y] = xy - yx$, for $x, y \in U$ and let us write

$$H(x, y, z) = (x, y, z) + (y, z, x) + (z, x, y), \text{ for } x, y, z \in U.$$

For any element $x \in U$, we denote by R_x the *right multiplication* $R_x : a \longmapsto ax = aR_x$ for all $a \in U$ and the *left multiplication* $L_x : a \longmapsto xa = aL_x$ for all $a \in U$. A *derivation* D of U is a linear operator on U satisfying $(xy)D = (xD)y + x(yD)$ for all $x, y \in U$.

We define a *generalized standard algebra* over F to be a nonassociative algebra U over F in which the following five conditions are satisfied, see [5]:

(i) $(x, y, x) = 0$, for all $x, y \in U$;

(ii) $H(x, y, z)x = H(x, y, xz)$, for all $x, y, z \in U$;

(iii) $(x, y, wz) + (w, y, xz) + (z, y, xw) = [x, (w, z, y)] + (x, w, [y, z])$, for all $x, y, z \in U$;

(iv) if F has characteristic 3, then $(x^2, y, x) = 0$, for all $x, y \in U$;

(v) the linear operator on U, $D_{x,y} = [L_x, L_y] + [L_x, R_y] + [R_x, R_y]$ is a derivation of U, for all $x, y \in U$.

All alternative algebras or Jordan algebras are generalized standard algebras and all generalized standard algebras are power-associative algebras.

In this work we prove that if (U, ω) is a finite dimensional generalized standard baric algebra with unity element over a field F of characteristic $\neq 2$, then $rad(U) = R(U) \bigcap (bar(U))^2$, where $R(U)$ is the nilradical (maximal nilideal) of U. We present an example showing that the condition on the existence of the unity element is absolutely necessary.

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Big Projective Modules over Noetherian Semilocal Rings

Dolors Herbera

Universitat Autònoma de Barcelona, Spain

Let R be a ring. The set $V^*(R)$ of isomorphism classes of countably generated projective right R-modules has a structure of commutative monoid with the sum induced by the direct sum of countably generated projective modules. For example, if $D_1, \dots, D_k$ denote division rings, then for a semisimple artinian ring $R \cong M_{n_1}(D_1) \times \cdots \times M_{n_k}(D_k)$ such monoid is isomorphic to $(\mathbb{N}_0^*)^k$, where $\mathbb{N}_0^* = \{0, 1, 2, \dots\} \cup \{\infty\}$.

A ring R is semilocal provided $R/J(R)$ is semisimple artinian (here $J(R)$ stands for the Jacobson radical of the ring R). Recent results of Pavel Příhoda allow us to see $V^*(R)$ as a submonoid of $V^*(R/J(R))$. We determine which monoids $V^*(R)$ can be realized when R is a semilocal noetherian ring.

We prove that for a noetherian semilocal ring R, with exactly k isomorphism classes of simple right (or left) modules, the monoid $V^*(R)$ viewed as a submonoid of $V^*(R/J(R))$, is isomorphic to the monoid of solutions in $(\mathbb{N}_0^*)^k$ of a system consisting of congruences and homogeneous diophantine linear equations. The converse also holds, that is, if M is a submonoid of $(\mathbb{N}_0^*)^k$, containing an order unit $(n_1, \dots, n_k)$ of $(\mathbb{N}_0^*)^k$, and which is the set of solutions of a system of congruences and linear homogeneous diophantine equations then it can be realized as $V^*(R)$ for a noetherian semilocal ring such that $R/J(R) \cong M_{n_1}(D_1) \times \cdots \times M_{n_k}(D_k)$ for suitable division rings $D_1, \dots, D_k$.

This is a report on joint work with P. Příhoda.

Representations of quantum affine algebras at roots of unity

Dijana Jakelic
University of North Carolina, USA

The category of finite-dimensional representations of quantum affine algebras is one of the most studied topics within representation theory of quantum groups. This is not a semisimple category, but its simple objects are highest-weight in an appropriate sense. For generic values of the quantization parameter, a result of Chari and Kashiwara provides a useful way of obtaining indecomposable but generally reducible objects by giving sufficient conditions for a tensor product of simple objects to be highest-weight. In particular, a tensor product of fundamental representations can always be reordered in such a way that these conditions are satisfied. This consequence is one of the essential ingredients used to describe the block decomposition of the category. In this talk, we will focus on a joint work with A. Moura where we consider the setting when the quantization parameter is a root of unity. We prove an analogue of Chari's version of the aforementioned result on tensor products of simple modules. However, it is not difficult to find examples of tensor products of fundamental representations that cannot be reordered so that they are highest-weight. We will then discuss techniques to overcome the lack of this property when studying the blocks in the root of unity setting.

Free Groups in Orders of Quaternion Algebras

Orlando Stanley Juriaans
IME-USP, Brazil

We show the existence of free groups in quaternion algebras using Pell's and other algebraic equations. This is joint work with A.C. Souza FIlho.

An estimate of a dimension of a variety of non-associative algebras

Iryna Kashuba
IME-USP, Brazil

Let Ω be an arbitrary family of non-isomorphic n-dimensional alternative algebras over algebraically closed field $\mathbf{k}$ that depends continuously on a certain set of parameters $p_1, \dots, p_N$. Then the asymptotic of dimension of Ω, i.e. the largest possible number N of parameters, when n is fixed is $\frac{4}{27}n^3 + O(n^{8/3})$. We also formulate a conjecture for the asymptotic of a number of parameters to define irreducible family of n-dimensional Jordan algebras, namely $N = \frac{1}{6\sqrt{3}}n^3 + O(n^{8/3})$, and will prove it for some closed subvariety of the variety of Jordan algebras. This is joint result with I.Shestakov.

Right coideal subalgebras in $U_q^+(\mathfrak{so}_{2n+1})$

V.K. Kharchenko
UNAM, Mexico

We give a complete classification of right coideal subalgebras that contain all grouplike elements for the quantum group $U_q^+(\mathfrak{so}_{2n+1})$, provided that q is not a root of 1. If q has a finite multiplicative order $t > 4$, this classification remains valid for homogeneous right coideal subalgebras of the small Lusztig quantum group $u_q^+(\mathfrak{so}_{2n+1})$. As a consequence, we determine that the total number of right coideal subalgebras that contain the coradical equals $(2n)!!$, the order of the Weyl group defined by the root system of type B_n.

Isomorphisms of graded simple algebras

Plamen Koshlukov
UNICAMP, Brazil

(joint work with M. Zaicev)

Let G be an abelian group and F an algebraically closed field such that the order of any finite subgroup of G is nonzero in F. Assume that A and B are two finite dimensional G-graded simple F-algebras. (Here A and B are simple as graded algebras.) Our main result is that A and B are isomorphic as G-graded algebras if and only if they satisfy the same G-graded identities.

We recall that when G is the trivial group then A and B are matrix algebras, and the above statement is just the Amitsur—Levitzki theorem. Furthermore, for Lie algebras and for representations of Lie algebras analogous results were obtained by Kushkulei and Razmyslov, for Jordan algebras by Drensky and Racine, and for split Jordan pairs by Neher.

The central polynomials for the Grassmann algebra

Alexei Krasilnikov
UnB, Brazil

This is joint work with Antônio Pereira Brandão Jr., Plamen Koshlukov and Élida Alves da Silva.

We describe the central polynomials for the infinite-dimensional unitary Grassmann algebra G over an infinite field F of characteristic $\neq 2$. More precisely, we exhibit a set of polynomials that generates the vector space $C(G)$ of the central polynomials of G as a T-space. Using a deep result of Shchigolev we prove that if $char F = p > 2$ then the T-space $C(G)$ is not finitely generated. Moreover, over such a field F, $C(G)$ is a limit T-space, that is, $C(G)$ is not a finitely generated T-space but every larger T-space $W \supsetneqq C(G)$ is.

We obtain similar results for the infinite-dimensional non-unitary Grassmann algebra H as well.

The Lie color algebra of skew elements of a graded associative algebra

Jesús Laliena
Universidad de La Rioja, Spain

For any abelian group G with a given anti-symmetric bicharacter ϵ, a G-graded associative algebra can be made into an (ϵ, G)-Lie color algebra using a commutator twisted by ϵ.

If we consider in A a ϵ-involution, that is a graded linear map $*\colon A \to A$ such that $(a_\alpha)^{**} = a_\alpha$ and $(a_\alpha a_\beta^{)} * = \epsilon(\alpha, \beta) a_\beta^* a_\alpha^*$ for every $a_\alpha \in A_\alpha$, $a_\beta \in A_\beta$, we have that $K = \{a \in A : a^* = -a\}$ and also $[K, K]$ are (ϵ, G)- Lie color algebras. We are interested in study the ideals of K when A is prime or semiprime as G-graded associative algebra.

Towards a characterization of the Kostrikin radical in Lie Algebras

Miguel Angel Gomez Lozano
Universidad de Malaga, Spain

We prove that, under certain conditions, a nondegenerate Lie algebra is a subdirect product of strongly prime Lie algebras, or equivalently, that the Kostrikin radical of a Lie algebra L is the intersection of all strongly prime ideals of L.

Simple Lie triples, involutorial Lie algebras and related modules

Sara Madariaga
Universidad de La Rioja, Spain

(joint work with Pilar Benito and José María Pérez-Izquierdo)

Following the general idea of the so called split extension construction of Lie algebras from Lie modules, in 1961 B. Harris [1] introduced the notion of module for Lie triple systems (Lts for short). Alternative notions of this type of modules can be found in [3] and [4], however Harris' notion is a suitable choice from a categorical point of view.

Given a Lts $\mathcal{T}$, the universal enveloping algebra $(\mathcal{L}_u(\mathcal{T}), \sigma)$ of $\mathcal{T}$, embeds the Lts $\mathcal{T}$ into the involutorial Lie algebra $\mathcal{L}_u(\mathcal{T})$ as the (-1)-eigenspace respect to the involution σ. According to [1], the categories of $\mathcal{T} - modules$ and $(\mathcal{L}_u(\mathcal{T}), \sigma) - modules$ (modules for the Lie algebra $\mathcal{L}_u(\mathcal{T})$ with σ-compatible action) are closely related by means of *Harris functor*. In 2002, L. Hodge and B.J. Parshall [2] used this functor to prove that $\mathcal{T}$-modules is a quotient category of $(\mathcal{L}_u(\mathcal{T}), \sigma) - modules$. They also proved that the functorial relationship stablished by this functor preserves irreducibility. In this way, irreducible modules for Lts can be related to irreducible modules for Lie algebras. The aim of this talk is to describe irreducible modules for simple Lts using *Harris functor* and irreducible modules of simple involutorial Lie algebras. Some examples in low ranks and several module dimension formulas will be also shown.

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Groups with prescribed element orders

Victor Mazurov
Sobolev Institute of Mathematics, Russia

For a periodic group G, denote by $\omega(G)$ the *spectrum*, i.e. the set of element orders, of G.

We discuss the following conjecture (see Problem 12.39 in [1]):

Let G be a finite simple group. If H is a group such that $|H| = |G|$ and $\omega(H) = \omega(G)$ then H is isomorphic to G.

The work is supported by Russian Foundation of Basic Research (projects no. 08-01-00322) and by the State Maintenance Program for the Leading Scientific Schools of the Russian Federation (grant NSh–344.2008.1).

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Jordan Algebras: What? and Why?

Kevin McCrimmon
University of Virginia, USA

Part I.

I will discuss the origin of Jordan algebras in Pascual Jordan's search for a foundation for quantum mechanics, and how his serendipitous mistakes in formulation led to mathematical situations where Jordan algebras played an unexpected role. I will also discuss the related algebraic systems of Jordan triples and pairs, and stress the increasing role played by the quadratic approach to Jordan theory.

Part II.

I will discuss the stages in our understanding of Jordan structure theory, leading to the final claassification by Efim Zelmanov. I will mention some of the standard tools in Jordan theory: Peirce decompositions, inner ideals, assorted radicals, and absorbers.

Minimal Ideals in Jordan Algebras

Kevin McCrimmon
University of Virginia, USA

Joint work with Teresa Cortes and Jose Anquela.

We show that all minimal ideals of a quadratic Jordan algebra are either simple or trivial as algebras, in the sense that all Jordan products vanish (squares, circle products, triple products, and U-products). Previous work had shown that minimal ideals were either simple or were trivial as triple systems; starting in the latter case from the vanishing of triple products, we show (after some delicate manipulations in the multiplication algebra) that all squares and circles vanish as well.

$\mathbb{Z}_2^3$-graded identities of octonions

Fernando Henry Meirelles
IME-USP, Brazil

The algebra of octonions O has a natural $\mathbb{Z}_2^3$-grading that comes from the Cayley-Dickson process. Here we study graded identities of octonions with respect to this grading. Let $x = x_g$ be a homogeneous element of O that belongs to O_g, then we will write $g(x) = g$. For homogeneous elements $x, y, z \in O$ we define

$\alpha(x, y) = 1$ if $g(x) = g(y)^{\pm 1}$ and $\alpha(x, y) = -1$ otherwise;

$\beta(x, y, z) = -1$ if $gr\langle g(x), g(y), g(z)\rangle = Z_2^3$ and $\beta(x, y, z) = 1$ otherwise.

Then all the $\mathbb{Z}_2^3$-graded identities of O follow from the identities of two types:

$$xy - \alpha(x, y)xy = 0;$$

$$(xy)z - \beta(x, y, z)x(yz) = 0.$$

The work is done under the supervising of Prof. I.Shestakov

On anti-structurable algebras

Daniel Mondoc
Royal Institute of Technology, Sweden

In this talk there will be introduced anti-structuable algebras and examples will be connected with $(-1,-1)$-Freudenthal Kantor triple systems and the construction of certain Lie superalgebras by the standard embedding method (joint work with N. Kamiya).

On minimal affinizations of quantum affine algebras

Adriano Moura
UNICAMP, Brazil

Let $\mathfrak{g}$ be a finite-dimensional simple Lie algebra over the field of complex numbers. Consider its loop algebra $\mathfrak{g}$ and the corresponding quantized enveloping algebras $U_q(\mathfrak{g})$ and $U_q(\tilde{\mathfrak{g}})$. It is known that, unless $\mathfrak{g}$ is of type A, there is no quantum group analogue of the evaluation maps $\tilde{\mathfrak{g}} \to \mathfrak{g}$. In particular, the concept of evaluation representations is not available in the context of the quantum affine algebra $U_q(\tilde{\mathfrak{g}})$ in general. Chari and Pressley introduced and studied the concept of minimal affinizations which play the role of evaluation modules in the sense that their the simplest of irreducible representations. We plan to discuss a few results on the characters minimal affinizations and Kirillov-Reshetikhin modules.

The Hilbert series of Hopf-invariants of free algebra

Lucia Murakami
IME-USP, Brazil

We look into the problem of determinig the Hilbert series of the subalgebra of invariants of a free associative algebra of finite rank under a linear action of a finite dimensional Hopf algebra. The case of group gradings will we treated as a special case in comparison with the already established result of Dicks and Formanek (1982) for the case of group actions by automorphisms. This is a joint work with Vitor Ferreira.

Representations of commutative power associative algebras

Pablo S. M. Nascimento
IME-USP/UFPA, Brazil

We investigate the structure of irreducible commutative and power associative modules for simple algebras. Except in some minor cases, simple commutative and power associative algebras are Jordan. The existence of non-Jordan commutative power associative modules for simple Jordan algebras will be discussed. Joint work with Ivan Shestakov and Lucia Murakami.

Comutatividade fraca entre grupos isomorfos

Ricardo Nunes de Oliveira
UFG, Brazil

The notion of weak commutativity between groups was introduced by Said Sidki (1980), by introducing the group of weak commutativity

$$\chi(H) = \left\langle H, H^{\psi} | \; [h, h^{\psi}] = 1, \; \forall h \in H \right\rangle,$$

where $\psi : H \to H^{\psi}$ an isomorphism. It was shown that χ, seen as an operator on the class of groups, preserves a number of properties such as finiteness, nilpotency and solubility. In a recent joint paper with Sidki (to appear in the Bulletin of SBM) we consider a more general form of weak commutativity, where given two isomophic groups H, $\dot{H}$ and a bijection $f : H \setminus \{e\} \to \dot{H} \setminus \{e\}$, we define

$$\chi(H, f) = \left\langle H, \dot{H} \mid [h, f(h)] = 1, \; \forall h \in H \right\rangle .$$

We produce results which support the conjecture that $\chi(H, f)$preserves nilpotency. Finiteness follows from a general theorem in the same paper of 1980.

The question of extending the finiteness criterion and other properties to a group involving three or more copies of H, leads us to consider quotients of the group

$$\chi(H, n) = \left\langle H, H^{\psi}, \dots, H^{\psi^{n-1}} \mid [h^{\psi^i}, h^{\psi^j}] = 1, \; \forall h \in H, \; 1 \leqslant i, \; j \leqslant n-1 \right\rangle$$

where $\psi^n = 1$. We will give both theoretical and computational results in this direction.

Nonhomogeneous Subalgebras of Lie and Special Jordan Superalgebras

Luiz A. Peresi

(Joint work with Murray R. Bremner, University of Saskatchewan.)

We consider polynomial identities satisfied by nonhomogeneous subalgebras of Lie and special Jordan superalgebras: we ignore the grading and regard the superalgebra as an ordinary algebra. The Lie case has been studied by Volichenko and Baranov: they found identities in degrees 3, 4 and 5 which imply all the identities in degrees ≤ 6. We simplify their identities in degree 5, and show that there are no new identities in degree 7. The Jordan case has not previously been studied: we find identities in degrees 3, 4, 5 and 6 which imply all the identities in degrees ≤ 6, and demonstrate the existence of further new identities in degree 7. Our methods use the representation theory of the symmetric group, the Hermite normal form of an integer matrix and the Lenstra-Lenstra-Lovász algorithm for lattice basis reduction.

Matrix models for infinite dimensional Lie algebras

Vladimir Pestov
University of Ottawa, Canada

Connes' Embedding Conjecture states that every von Neumann algebra admitting a faithful finite trace embeds into a tracial ultraproduct of a family of finite dimensional matrix algebras. While the conjecture remains open, attempts to solve it have led to the following concept. A group Γ is *hyperlinear* (or: *admits matrix models*) if Γ embeds, as a subgroup, into a metric ultraproduct of a family of unitary groups $U(n)$ of finite rank, equipped with the normalized Hilbert-Schmidt distance. Connes' conjecture would imply that every group is hyperlinear, so finding an example of a non-hyperlinear group is one possible way to disprove the conjecture.

In this talk we bring attention to a possibility of using with the same purpose Lie algebras instead of groups. A Lie algebra $\mathfrak{g}$ is *metrized* (or *orthogonal*, or else *quadratic*) if it admits a nondegenerate invariant bilinear form. This concept is receiving a considerable amount of attention, especially in the case $\dim \mathfrak{g} < \infty$. By a construction of Bordemann, this linear form can be extended to a trace on the universal enveloping algebra $U(\mathfrak{g})$ which, under some additional conditions, leads to a representation of $\mathfrak{g}$ in a finite von Neumann algebra. Say that a metrized Lie algebra $\mathfrak{g}$ is *hyperlinear,* or *admits matrix models,* if it embeds into a metric ultraproduct of finite dimensional metrized Lie algebras. We will discuss this notion and various results and examples surrounding it.

On right alternative superalgebras
Juaci Picanço da Silva
UFPA, Brazil

It is known that irreducible unital right alternative bimodules of dimension 4 for matrix algebras of order 2 form an infinite set depending on three parameters. We classify right alternative superalgebras having matrix algebras of oder 2 as even part and a irreducible unital right alternative bimodule of dimension 4 as odd part. They still form an inifinte set, however, depending only on a single parameter. This is a joint work with I. Shestakov and L. Murakami.

Normal subgroups of Poincaré duality groups
Aline Pinto
UnB, Brazil

The result we shall present states that the existence of a finitely generated normal subgroup of infinite index in a pro-p group G of Poincaré duality of dimension 3 gives rather strong consequences for the structure of G. Joint work with Pavel Zalesski.

Códigos de grupos
Cesar Polcino Milies
IME-USP, Brazil

Conceitos básicos da teoria de códigos: distância de Hamming, distância mínima e capacidade de correção de erros. Códigos Lineares, matriz de codificação e de verificação. Códigos cíclicos com ideais de anéis quociente de anéis de polinômios. Códigos cíclicos como ideais de álgebras de grupo. Códigos de grupos. Semisimplicidade. Geradores idempotentes. Cálculos explícitos de idempotentes a partir da estrutura de grupos cíclicos e abelianos.

Classification of Simple Finite Dimensional Structurable and Noncommutative Jordan Superalgebras

Aleksandr P. Pozhidaev
IM SB RAS, Russia

Joint work with Ivan Shestakov.

Classification of simple finite dimensional structurable algebras was obtained by I. Kantor, B. Allison, and O. Smirnov. We consider the superalgebra case. Recall that a superalgebra $(A, ^-)$ with a superinvolution $^-$ is called *structurable* if

$$[T_z, V_{x,y}] \;=\; V_{T_z(x),y} - (-1)^{xz} V_{x,T_{\bar{z}}(y)},$$

where $T_x(z) = xz + (-1)^{xz} z(x - \bar{x})$, $V_{x,y}(z) = (x\bar{y})z + (-1)^{x,y,z}(z\bar{y})x - (-1)^{xz+yz}(z\bar{x})y$.

We describe the simple Lie superalgebras arising from the simple unital finite dimensional structurable superalgebras of characteristic 0 and construct four series of the unital simple structurable superalgebras of Cartan type. We give a classification of simple finite dimensional structurable superalgebras of Cartan type over an algebraically closed field F of characteristic 0. Together with the Faulkner theorem on the classification of classical such superalgebras, it gives a classification of the simple finite dimensional structurable superalgebras over F.

Nonsupercommutative Jordan superalgebras of capacity $n \geq 2$

Aleksandr P. Pozhidaev
IM SB RAS, Russia

Joint work with Ivan Shestakov.

Classification of simple finite dimensional noncommutative Jordan algebras of characteristic 0 was mainly obtained in the works of R. D. Schafer, R. H. Oehmke, and K. McCrimmon. We consider the superalgebra case. Recall that a superalgebra U is called a *nonsupercommutative Jordan superalgebra* provided that

$$[R_{x\circ y}, L_z] + (-1)^{x(y+z)}[R_{y\circ z}, L_x] + (-1)^{z(x+y)}[R_{z\circ x}, L_y] = 0,$$
$$[R_x, L_y] = [L_x, R_y]$$

hold for all $x, y, z \in U$. We prove the Coordinatization Theorem for the nonsupercommutative Jordan superalgebras of capacity $n \geq 3$, describing such algebras. We prove that the symmetrized Jordan superalgebra for a simple finite dimensional nonsupercommutative Jordan superalgebra of characteristic 0 and capacity $n > 1$ is simple. Modulo a "nodal" case, we classify the simple central finite dimensional nonsupercommutative Jordan superalgebras of characteristic 0.

Subgroup theorems and wreath products

Luis Ribes
Carleton University, Canada

I will describe the use of wreath products to prove some classical subgroup theorems like Nielsen-Schreier, Kurosh, as well as some more recent ones, both for abstract and profinite groups.

A tutorial on quantum Clifford algebras

Roldão da Rocha
UFABC, Brazil

There is a correspondence between quantum Clifford algebras and Clifford algebras defined by an arbitrary (not necessarily symmetric) quadratic form. It is shown that the Chevalley isomorphism theorem cannot be generalized to algebras if the $\mathbb{Z}_n$-grading or other structures are added. Clifford algebras of the same quadratic but different bilinear forms are non-isomorphic as graded algebras, and the Atiyah-Bott-Shapiro mod 8 periodicity theorem is extended by a deformed tensor product, using the Witt theorem. Some applications and examples are provided.

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Projective resolutions in Borel Schur algebras: a new approach

Ana Paula Santana
Universidade de Coimbra, Portugal

In 1981, S. Donkin proved that the Schur algebra for the general linear group, $S(\mathbb{K}, n, r)$, has finite global dimension. This led to the problem of describing explicit projective resolutions of the Weyl modules for $S(\mathbb{K}, n, r)$.

Let $\mathfrak{U}_n^+(\mathbb{K})$ denote the Kostant form over the field $\mathbb{K}$ of the universal enveloping algebra of the Lie algebra of *nxn* complex nilpotent upper triangular matrices.
In this talk I will focus on the role of $\mathfrak{U}_n^+(\mathbb{K})$ on recent developments in this subject.

This is joint work with I. Yudin.

On ternary $[R_x, R_y]$-derivation algebras

Paulo Saraiva
Universidade de Coimbra, Portugal

Joint work with A.P. Pozhidaev.

J. M. Osborn in 1965 introduced the class $\mathcal{A}$ of Lie triple algebras, which are commutative algebras satisfying

$$R_{(x,y,z)} = \left[R_y, [R_x, R_z]\right], \tag{1}$$

where $(.,.,.)$ stands for the associator, $[.,.]$ is the commutator, and R_x is the right multiplication operator. This class includes Jordan algebras; in particular, J. M. Osborn considered a Peirce decomposition of such an algebra and proved that simple algebras in $\mathcal{A}$ containing an idempotent are Jordan algebras.

Given an algebra A, under commutativity the identity (1) is equivalent to the following property:

$$D_{x,y} = \left[R_x, R_y\right] \in Der\,(A)\,. \tag{2}$$

J. R. Faulkner in 1967 proved that, in the Jordan algebras, all operators $D_{x,y}$ are derivations (*inner derivations*).

In the present communication, we consider a generalization of the identity (2) to n-ary algebras ($n > 2$) and propose the following definition: given an n-ary algebra A with multilinear multiplication $\{., \dots, .\} : \times^n A \to A$, we say that A is an $[R_x, R_y]$-*derivation algebra* if, for every $x = (x_2, \dots, x_n)\,, y = (y_2, \dots, y_n)$ $(x_i, y_i \in A)$ the operator

$$D_{x,y} = \left[R_{x_2,\dots,x_n}, R_{y_2,\dots,y_n}\right]$$

(where $zR_{(x_2,\dots,x_n)} = \{z, x_2, \dots, x_n\}$) is a derivation of A.

We give a first example of these algebras in the ternary case, namely, by defining the multiplication

$$\{x, y, z\} = (y, z)\,x + (x, z)\,y + (x, y)\,z$$

on a linear space V equipped with a symmetric bilinear form $f = (.,.)$; denote such an algebra by V_f. Further, we describe some properties of V_f (namely, its simplicity, the identities of degrees 1 and 2, and the derivation algebra of V_f). Also, we present other examples of ternary algebras satisfying the ternary version of (2); namely, those which arise from defining a certain multiplication on the algebras obtained by the Cayley-Dickson process.

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On Kac Wakimoto conjecture about dimension of simple representation of Lie superalgebra

Vera Serganova
University of California, Berkeley, USA

Let $\mathfrak{g}$ be a simple classical Lie superalgebra. Some time ago Kac and Wakimoto conjectured that a simple finite dimensional $\mathfrak{g}$ module has a non zero superdimension if and only if its degree of atypicality equals the defect of $\mathfrak{g}$. We will give a proof of this conjecture in some cases.

Automorfismos de polinômios

Ivan Shestakov
IME-USP, Brazil

Os polinm̂ios são os objetos mais antigos estudados na álgebra. Mesmo assim, a estrutura do anel de polinm̂ios, de seus subanéis, derivações e automorfismos têm vários problemas abertos.

Neste curso, nós vamos falar sobre os automorfismos de polinômios.

Seja $A_n = F[x_1, \ldots, x_n]$ um anel de polinômios sobre um corpo F de números (racionais, reais ou complexos) nas variáveis $x_1, \ldots, x_n$. Um automorfismo de A_n é uma aplicação bijetora $\varphi : A_n \to A_n$ tal que para quaisquer polinômios $f, g \in A_n$ e para qualquer $\alpha \in F$ tem-se

$$\varphi(f+g) = \varphi(f) + \varphi(g),\ \varphi(fg) = \varphi(f)\varphi(g),\ \varphi(\alpha f) = \alpha\varphi(f).$$

É claro que os elementos $\varphi(x_1), \ldots, \varphi(x_n)$ geram de novo o anel A_n, isto é, $A_n = F[\varphi(x_1), \ldots, \varphi(x_n)]$. Portanto, o problema de descrição de todos os automorfismos de A_n é equivalente ao problema de descriçãao de todas n-uplas de polinômios $f_1, \ldots, f_n$ que geram A_n. Este problema é muito importante e tem várias aplicações em álgebra e geometria.

Um exemplo evidente de automorfismo é uma aplicação do seguinte tipo: $\phi : (x+1, \ldots, x_i, \ldots, x_n) \mapsto (x_1, \ldots, \lambda x_i + f, \ldots, x_n)$, onde $0 \neq \lambda \in F$ e o polinômio f não contem x_i. Os automorfismos desse tipo chamam-se *elementares*.

No ano de 1942, H.W.E.Jung provou que, no caso $n = 2$, qualquer automorfismo de $A_2 = F[x, y]$ pode ser obtido como uma composição de automorfismos elementares. Mas o problema semelhante para $n \geq 3$ ficou em aberto.

Em 1972, M.Nagata construiu um automorfismo σ do anel A_3 para o qual ele conjeturou a impossibilidade de ser composto por automorfismos elementares.

Em 2004, I.Shestakov e U.Umirbaev confirmaram a conjetura de Nagata.

Nós vamos apresentar algumas idéias e métodos da demostração desta conjetura, além de falar sobre outros problemas e resultados relacionados com a estrutura de automorfismos de polinômios, incluindo a famosa *Conjetura de Jacobian*.

On the Fitting height of a finite group

Pavel Shumyatsky
UnB, Brazil

It is well-known that a finite group G is nilpotent if and only if every pair of conjugate elements generate a nilpotent subgroup. Recently Guest proved that G is solvable if and only if every pair of conjugate elements generate a solvable subgroup. Independently, this was also proved by Gordeev, Grunewald, Kunyavskii and Plotkin. It is interesting to see if other classes of finite groups admit similar characterization. We prove that a group is solvable with Fitting height at most h if and only if so is every subgroup generated by a pair of conjugates.

State-closed groups

Said N. Sidki
UnB, Brazil

Automorphisms of one-rooted regular trees $\mathcal{T}(Y)$ indexed by finite sequences from a finite set Y of size $m \geq 2$, have a natural interpretation as automata on the alphabet Y, and with states which are again automorphisms of the tree. A subgroup of the group of automorphisms $\mathcal{A}(Y)$ of the tree is said to be *state-closed*, in the language of automata (or *self-similar* in the language of dynamics) of degree m provided the states of its elements are themselves elements of the same group.

There is a very large variety of types of state-closed groups. Normally groups in this class are considered under additional conditions such as having finite number of generators, being finite-state, or satisfying certain identities. Some of the examples are the torsion groups of Grigorchuk and of Gupta-Sidki , the torsion-free BSV and Basilica weakly branch groups, all finitely generated and finite-state. There are also the faithful representations of affine groups over the integers and of free groups as finite-state and state-closed groups.

State-closed free abelian groups of degree 2 and having finite rank were characterized by Nekrashevych-Sidki in 2004 and finitely generated state-closed torsion-free nilpotent groups of degree $m \geq 2$ were studied by Berlatto-Sidki in 2007.

We will review in this lecture known results and recent work by the author and Andrew Brunner on state-closed general abelian groups without restrictions on finite generation or degree.

Group-graded identities of PI-algebras

Irina Sviridova
UnB, Brazil

We will consider associative algebras over a field F of zero characteristic graded by a finite abelian group G, and satisfying a nontrivial non-graded polynomial identity. Let us assume that F is a splitting field of the group G.

We will say a G-graded finite dimensional F-algebra A is GPI-reduced if $A = (C_1 \times \cdots \times C_p) \oplus J$ with $C_1 J C_2 J \cdots J C_p \neq 0$, where C_i ($i = 1, \ldots, p$) are simple G-graded finite dimensional algebras, and J is the Jacobson radical of A.

Then we have the next generalizations of the theorems of A.R.Kemer [1] for the case of algebras graded by an abelian group.

Theorem 1. *If G is an abelian group, F is a splitting field of G then the ideal of graded identities of a G-graded finitely generated PI-algebra over F coincides with the ideal of graded identities of some finite dimensional G-graded F-algebra.*

We can suppose here that the finite dimensional algebra is the direct sum of GPI-reduced algebras.

It is clear that an ideal of G-graded identities Γ is the ideal of graded identities of a finitely generated (or finite dimensional) G-graded PI-algebra over F if and only if Γ contains a Capelli polynomial. In general we have

Theorem 2. *If G is an abelian group, F is a splitting field of G then the ideal of graded identities of an associative G-graded PI-algebra A over F coincides with the ideal of G-graded identities of the direct sum of the Grassmann envelopes of some finite dimensional $(G \times Z_2)$-graded $(G \times Z_2)$PI-reduced algebras.*

Corollary 1. *If G is a finite abelian group then the ideal of graded identities of an associative G-graded PI-algebra over a field of zero characteristic is finitely generated as a GT-ideal.*

The similar results take place for ideals of identities with automorphisms.

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Enveloping algebras of Lie super-algebras satisfying non-matrix polynomial identities

Hamid Usefi
University of British Columbia, Canada

It is known that if the enveloping algebra $U(L)$ of a Lie algebra L satisfies a polynomial identity then L is forced to be abelian (over a field of characteristic zero). We are interested to investigate the same problem for Lie super-algebras. In this talk we shall show how the structure of Clifford algebras can be used to characterize when the enveloping algebra of a Lie super-algebra satisfies a non-matrix polynomial identity. In particular, we are able to determine when $U(L)$ is Lie solvable, Lie nilpotent, or Lie super-nilpotent. (joint work with David Riley and Jeff Bergen).

Kostant form of the universal enveloping algebra of sl_n^+

Ivan Yudin
Universidade de Coimbra, Portugal

In our work arXiv:0803.4382 with A.P. Santana it was proved that the construction of (minimal) projective resolutions for one-dimensional modules over Borel-Schur algebra $S^+(n,r)$ is essentially equivalent to the construction of (minimal) projective resolution for the one-dimensional module $\mathbb{K}_{\mathfrak{U}}$ over Kostant form $\mathfrak{U}_{\mathbb{K}}(sl_n^+)$ of $\mathfrak{U}(sl_n^+)$.

In my talk I will present our recent results, based on Groebner basis approach, towards construction of minimal projective resolution for $\mathbb{K}_{\mathfrak{U}}$. In particular, I will describe Groebner basis of $\mathfrak{U}_{\mathbb{K}}(sl_n^+)$ and first three steps of minimal projective resolution for $\mathbb{K}_{\mathfrak{U}}$.

Spherical functions associated to the three dimensional sphere

Ignacio Zurrian
Universidad Nacional de Cordoba, Argentina

Given a symmetric pair (G,K) the irreducible spherical functions on G associated to any K-type π are obtained from an irreducible representation of the group G. An important property of this special functions is that they are eigenfunctions of every element in the algebra $D(G)^K$ of all differential operators on G which are invariant under left and right multiplication by elements in G and K respectively.

In this work, we consider the pair $(G,K) = (\mathrm{SO}(4), \mathrm{SO}(3))$ and we determine all the matrix valued irreducible spherical functions. This is accomplished by associating to every one of them a vector valued function $H = H(u)$ of a real variable u, which is analytic at $u = 0$ and its components are solutions of two coupled systems of ordinary differential equations. By an appropriate conjugation involving Hahn polynomials we uncouple one of the systems then this is taken to a system of hypergeometric equations, leading to express the entries of H in terms of Gegenbauer's polynomials. Finally, we identify those simultaneous solutions and use the representation theory of $\mathrm{SO}(4)$ to characterize all irreducible spherical functions.