The orbit structure of the Gelfand-Zeitlin integrable system

Mark Colarusso (University of Notre-Dame, USA)

We discuss our recent work, which describes and constructs polarizations of regular adjoint orbits for certain classical groups. This research generalizes results of Kostant and Wallach. Kostant and Wallach construct polarizations of regular adjoint orbits in the Lie algebra of $n \times n$ complex matrices $\mathfrak{gl}(n)$. They accomplish this by integrating a completely integrable system derived from Gelfand-Zeitlin theory to a global action of $\mathbb{C}^{\binom{n}{2}}$ on $\mathfrak{gl}(n)$. We explicitly describe the orbit structure of this Gelfand-Zeitlin action on an open, dense subset of $\mathfrak{gl}(n)$ and generalize the construction to complex orthogonal Lie algebras $\mathfrak{so}(n)$. In the case of $\mathfrak{gl}(n)$, we obtain complete descriptions of polarizations of all regular adjoint orbits. For $\mathfrak{so}(n)$, we construct polarizations of certain regular semisimple adjoint orbits.