Interaction Interfaces in Interactive Geometry Software: A Systematic Mapping of the Literature

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ABSTRACT:

Many research findings indicate that Interactive Geometry (IG) software is an effective tool to support geometry learning. During the learning process, students interact with the software to visualize and manipulate geometric constructions presented dynamically in the graphical user interface (GUI). However, despite the widespread adoption and importance of IG software, the various forms of interaction with it have not been widely investigated. As a first step towards bridging this gap, we carried out a systematic mapping of the literature to synthesize the main research findings available to date regarding the multiple types of interactions offered by GUI of IG software and which forms of input and output are more implemented and investigated by the community. To conduct the literature review, we initially search in five digital libraries to collect 998 papers covering more than 10 years of research in the field of IG software. Then, after a careful analysis of each papers, we verified that 45 papers were specifically related to the development of IG interfaces and only 20 of them satisfied the inclusion and exclusion criteria defined in this work. Finally, we categorized the contributions of these papers according to their goal and research approach. This overview shows that (i) most of the studies focused on the development of desktop interfaces based on keyboard and mouse, with 2D output; and (ii) few efforts have been carried out on the design and development of IG software for devices with multitouch interfaces such as tablets and smartphones. These results provide a valuable contribution of summarizing what has been done by the community and furthermore gives directions of an important venue for further research.

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1. INTRODUCTION

The Interactive Geometry software (IG) was developed with the goal of enabling students to explore geometry through the computer, allowing the learning of geometry through dynamic manipulation of geometric objects (e.g. lines, circles and points) (Isotani and Brandão 2008; Erez and Yerushalmy 2007; Roanes-Lozano 2003). During the learning process, students interact with the software and are able to visualize geometric constructions via the interface. Furthermore, they are able to interact with the features of the software and easily understand the information through these visualizations (Shimomura, Havannber and Hafsteinsson 2013). Baker and colleagues (2001) and Laborde (2007) suggest that the developers of the IG software should be concerned with the pedagogical aspects and design of the interface, with the goal of providing students with natural and intuitive interactions. If the software is designed without considering pedagogical approaches or system usability issues, it will probably fail in providing the adequate support for teaching and learning. This could cause frustration among the students, who struggle to use the software and ultimately do not direct their attention to learning geometry (Schimpf and Spannagel 2001; Kotenkamp and Dohrman 2010). However, despite the importance of educational software interfaces, the various forms of interaction available with the IG software interfaces *have not* been widely explored. There are few studies about exploiting the diversity of interactions and the devices that they run on.

Thus, to better understand this context, a systematic mapping (SM) was conducted, since it is a way to categorize and summarize existing information to answer one or more research questions in an impartial manner. The SM was performed considered to identify the most commonly used forms of interaction (e.g. interaction based on touch, click or pen), the main forms of data input and output in current IG software (e.g. respectively mouse or keyboard and monitor or projector), and for which types of devices IG software has commonly been developed (e.g. desktop computer or tablet). The SM follows a methodology that seeks the nature, extent and quantity of published primary studies[1] in a particular area of interest (Petersen 2008). Secondary studies, similar to SM, use primary studies as a basis for research, and are recommended for the identification of research areas that have few studies (Pretorius and Budgen 2008).

In the following sections, the steps performed to conduct the SM will be presented. The process conducted is described succinctly in Section 1; in Section 2, the research questions that motivated this study are presented; the search for primary studies is detailed in Section 3. The screening of primary studies and the categorization of articles are, respectively, presented in Sections 4 and 5. A description of the resulting categorization is presented in Section 6; a mapping of the primary studies and their analysis is offered in Section 7. A summary of the mapping studies is presented in Section 8; and, finally, threats to validity and final remarks are discussed in Sections 9 and 10.

2. RESEARCH DESIGN AND METHODOLOGY

Frequently, researchers and practitioners have trouble in finding and gathering adequate studies to perform an analysis and summary of current state of the art about a given field, gathering the most important findings produced by the community and identify the research gaps and directions. Often, the review of the literature is conducted using an ad-hoc process, leaving important results behind and making almost impossible to replicate it by others. To overcome these difficulties, systematic reviews/mappings could be performed. According to Kitchenham and Charters (2007), a systematic review of the literature is a process composed by series of reproducible and auditable steps to collect evidences, i.e. published articles, that help to answer defined research questions. In software development for example, this approach can help to summarize the existing evidence about the benefits and limitations of a specific agile method (Kitchenham and Charters, 2007). In the field of educational technology, it can be used to gather information (i.e. empirical evidence) about the maturity and benefits of computational solutions and pedagogical approaches as well as to identify areas that possess insufficient results and need more exploration.

1 Primary studies are individual studies (i.e. not a survey or other similar types of studies) that contribute to gather information necessary to answer the desired research questions (Kitchenham, 2004).

In this study we are using the guidelines proposed by Petersen et al. (2008) to gather information about research conducted on interfaces for GI. These guidelines are composed of five steps: (i) definition of research questions, (ii) search for relevant primary studies, (iii) selection of primary studies, (iv) categorization of articles and (v) mapping and extraction of information that answers the research questions previously established in step (i). We present each step in the next subsections.

This section presents the analysis of the primary studies returned and classified according to the forms of interaction, the forms of input and output, and the devices most frequently adopted for the use of IG software. Most of the selected primary studies were found in Scopus (10 studies) and then in the ACM Digital Library (9 studies). The other digital libraries examined, Springer and IEEExplore, returned one study each. No papers from Elsevier digital library was included since papers from it were also indexed in the previous libraries.

Regarding the types of publication, primary studies published in conferences, journals, and symposiums and as book chapters were analyzed. The largest group of the selected studies (19 papers) were published in conferences (10 papers) and journals (9 papers), while book chapters were the second most common with two studies. It is important to note that when two or more primary studies were returned reporting the same research, only the most recent was included, regardless of the publication type of these studies.

2.1. Definition of Research Questions

IG software have been developed especially for desktop computers, with keyboard and mouse as input devices. However, in recent years, new forms of interaction, such as touchscreens, have emerged. For this reason, this mapping aims to gather information about which forms of interactions with IG emerged in the past few years, what input and output devices have been used, and for which types of devices IG software has been developed for. Thus, the following research questions were formulated:

RQ1: What are the main forms[2] of interaction investigated for use in IG software interfaces?

RQ2: What are the main forms[3] of input and output data investigated for the development of IG software interfaces?

RQ3: What are the key devices[4] for which IG software has been developed?

In addition to the research questions, in the scope of this systematic mapping we also define the population, intervention and expected results of the literature review.

Population: Primary studies on IG software written in English or Portuguese[5]. The authors have chosen to investigate Portuguese studies, due to this language is the mother tongue of them. Furthermore, consolidated digital libraries are more likely to return important studies written in English.

Intervention: Primary studies that discuss the graphical interface of IG software. It is important to mention that it is not necessary that these primary studies present some technical development or practical approach to the proposal, thus allowing the mapping to present a general overview of the graphical interfaces of existing IG software.

Expected outcomes: An overview, presented by categories, of the forms of input and output data are used for learning geometry using IG software. In addition, identification of the forms of interaction and for which types of devices this software has been developed will be another contribution. With this result, we intend to highlight the lack of research in interfaces for IG software.

2.2. Conducting a Search for Primary Studies

2 It means the way how a user interacts with the geometric objects in a IG software.

3 They are related to the way how users insert/construct any geometric objects and how the software present the objects to the user.

4 Desktops, tablets, tabletop, etc.

5 Usually Brazilian journals, when publish article in Portuguese, it also contain meta-data in English (such as title and abstract).

This step comprises of the execution of the search string (Table 1) in digital libraries. In order to obtain a greater number of primary studies. The following databases were selected:

• ACM Digital Library (portal.acm.org);

• IEEExplore (ieeexplore.ieee.org);

• Scopus (www.scopus.com);

• Elsevier – via Science Direct (www.sciencedirect.com) and

• Springer Link – via Science Direct (www.sciencedirect.com).

According do Dyba (2007) the most important publications in Computer Science and related areas (such as Educational Technology) are covered by these digital libraries.

To search for papers in the topic of interfaces in IG software, we created a table with the main keyword and their synonyms as show in Table 1.

Table 1: Search string in English.

Keyword	**Alternative Terms and/or Synonyms**
Interactive geometry	“Dynamic Geometry” OR “Geometry Teaching” OR “Geometry Education”
Interface	HCI OR “Human-Computer Interaction” OR “Man Computer Interaction” OR “Human Factors” OR “User Interface” OR “Computer Interface” OR “Gestural Interface” OR “Human Information Processing” OR gesture OR usability

Then, the general *search string* was constructed as a logical expression with two sentences connected by the logical connector AND, and formatted according to the rules of each digital library, as follows:

(**"Dynamic Geometry"** OR **"Interactive Geometry"** OR **"Geometry Teaching"** OR **"Geometry Education"**) AND (**interface** OR **HCI** OR **"human-computer interaction"** OR **"man computer interaction"** OR **"human factors"** OR **"user interface"** OR **"computer interface"** OR **"gestural interface"** OR **"human information processing"** OR **gesture** OR **usability**)

The first part of the *search string* is related to interactive geometry and geometry teaching, while the second part covers studies related to the interface and studies related to the field of Human-Computer Interaction. The logical operator OR connects the synonyms of the search string, while the AND operator connects the sentences. Since we did not expected a large number of related articles, no particular restriction (filter) was used to limit the search. The string can be extended to other languages, if the words that compose the string must be translated with related meanings.

2.3. Screening of Primary Studies for Inclusion and Exclusion

Inclusion and exclusion criteria are important to the process of finding adequate set of papers to answer the defined research questions. The inclusion criteria determine the scope and validity of systematic mapping studies. And exclusion criteria allow greater precision by removing articles that are not considered relevant. Therefore, during the analysis articles in this work, the following inclusion and exclusion criteria were considered.

2.3.1. Criteria for inclusion

• If multiple articles show very similar studies on the same software, only the most recent is included.

• If there are multiple versions of the same article, e.g. a short article and a complete article, the most complete is included.

• Articles that describe the development or analysis of Interactive Geometry software interfaces on any platform should be included.

2.3.2. Criteria for exclusion

• Articles that have no relationship with Interactive Geometry software interfaces should be excluded.

• Studies not in English or in Portuguese should be excluded.

• Technical reports, documents that are available in the form of short articles or presentations/slides and secondary studies (reviews and systematic mappings of the literature) should be excluded.

2.3.3. Screening Process

The selection of the articles was performed in two steps. The first step involved reading the title, abstract, introduction and conclusion, for the application of the inclusion and exclusion criteria. During the second step, a full reading of selected articles was performed and the inclusion and exclusion criteria again applied in order to categorize the articles.

The first step results in 998 primary studies extracted from the five digital libraries shown in Table 2. The authors of the present study simultaneously carried out the reading of the titles, abstracts, introduction and conclusion of all articles. Then, each one generated a list of selected studies following the first inclusion and exclusion criteria. They compared the two lists to define what studies should be selected. Ultimately, the initial set of 998 studies was reduced to only 45 studies. After this step, the authors carefully read analyzed and categorized each paper to keep only articles that really contribute to answer the defined research questions. Thus, at the end of the process, we had a subset of 21 studies (Table 3).

Papers excluded from the final selection investigated the teaching using Interactive Geometry Software, however the interface was not exploited. For example, the paper "The constraint-based dynamic geometry system" by Marc Freixas et al. (2010), investigates dynamic geometry systems, but only mentions that these systems have interfaces to assist the user in the geometric construction. Another example is "In Pursuit of Desktop Evolution: User Problems and Modern Practices With Desktop Systems" by Ravasio et al. (2004), which mentions about education geometry and interfaces, however is not its research object

Table 2. Number of studies returned by each digital library, total number of candidate studies and final selection.

ACM Digital Library	385
Scopus	338

Elsevier (via Science Direct)	243
Springer Link (via Science Direct)	29
IEEExplore	12
Total	998
Candidates	45
Final Selection	**21**

Table 3. Selected primary studies

Authors	**Title**
Kaufmann and Schmalstieg	Designing Immersive Virtual Reality for Geometry Education
Narboux	A Graphical User Interface for Formal Proofs in Geometry
Iordan et al.	Visual interactive environment for doing geometrical constructions
Liu et al.	PIGP: a pen-based intelligent dynamic lecture system for geometry teaching
Bonnard et al.	Paper interfaces for learning geometry
Blagojevic et al.	Using tangible drawing tools on a capacitive multi-touch display
Reis et al.	Towards Reducing Cognitive Load and Enhancing Usability through a Reduced Graphical User Interface for a Dynamic Geometry System: An Experimental Study
Cai et al.	A Web-Based Mathematical User Interface for E-Science System
Fabre et al.	Constrained gesture interaction in 3d geometric constructions
Banu	Augmented reality system based on sketches for geometry education
Starcic et al.	Design-based research on the use of a tangible user interface for geometry teaching in an inclusive classroom
Schimpf and Spannagel	Reducing the graphical user interface of a dynamic geometry system
Kovárová and Sokolský	Using virtual reality for teaching solid geometry: A case study for a cube section
Blanke and Schneider	TOM: A multi-touch system for learning math

Song and Zhu	A sketch recognition scheme for primary geometry education
Kortenkamp and Materlik	Geometry teaching in wireless classroom environments using Java and J2ME
Shimomura et al.	Haptic cues as a utility to perceive and recognise geometry
Ma et al.	An adaptive sketching user interface for education system in virtual reality
Erez and Yerushalmy	If you can turn a rectangle into a square, you can turn a square into a rectangle ... Young students experience the dragging tool
Mackrell	Design decisions in interactive geometry software

2.4 Categorization

By analyzing each of the 21 studies, we identified six different forms/categories of interactions. A description of each category follows:

- **Based on Pen:** This category makes use of a digital pen for students to perform the construction and manipulation of geometric objects. The digital pen can be used in conjunction with other types of devices, such as glasses for augmented and virtual reality, digital whiteboards, digital tables, and PDA screens.
- **Based on Click:** This category includes all studies in which the IG software demand the use of keyboard and mouse for interactions.
- **Based on Gestures:** This category presents interactions with IG software through gestures (usually captured by cameras, such as Kinect).
- **Based on Touch:** This category includes studies that explore the interfaces with touch interaction, covering the various devices that provide multi-touch and single touch interactions.
- **Haptic:** This category relates to haptic interfaces. The primary goal is to provide interfaces targeted towards the visually impaired.
- **Various techniques:** The primary studies in this category present various other interaction techniques, including augmented reality, 2D interaction, and 3D interaction. A camera is the primary device used to carry out the techniques proposed in each primary study, as well as some other tools such as QR (abbreviation for Quick Response) codes and projectors.

Each primary study was analyzed and categorized by type of contribution. We also identified the categories related to devices utilized to obtain input data (see Table 4). Usually, the devices allow the user to manipulate with the software (which will them generate an output to the user). Categories of devices utilized to obtain input data are listed below:

- Touchscreen
- Digital Pen
- Projector
- Glasses for Augmented and Virtual Reality
- Digital Table
- Gloves
- Haptic Devices
- Whiteboard
- Camera
- Keyboard and Mouse
- Table 4: Categories of devices utilized to obtain input data

Categories
Touchscreen
Digital Pen
Projector
Glasses for Augmented and Virtual Reality
Digital Table
Gloves
Haptic Devices
Whiteboard
Camera
Keyboard and Mouse

As previously mentioned, the GUI allows the user to work with the software, offering communication mechanisms between the user and the computer. This type of interface can be considered a form of output data, in which the user can visually identify the information on the screen. Based on the analysis of the literature we found five types of output based on

2D (bidimensional) and 3D (tridimensional) images, Augmented Reality (AR), and Virtual Reality (VR). We also identified the devices for which IG software had been developed to date: Desktop computers, tablets, tabletops and PDAs.

It is important to mention that among the 21 studies that comprise the final selection, two studies were identified as presenting analysis of general interfaces. These studies present abstract ideas or good practices of development without, however, specifying any technique, method, model or approach for how these interfaces were developed, and therefore no category was created for them.

2.5. Data Extraction and Mapping Process

This section presents the analysis of the primary studies returned and classified according to the forms of interaction, the forms of input and output, and the devices most frequently adopted for the use of IG software. Most of the selected primary studies were found in Scopus (10 studies) and then in the ACM Digital Library (9 studies). The other digital libraries examined, Springer and IEEExplore, returned one study each. No papers from Elsevier digital library was included since papers from it were also indexed in the previous libraries.

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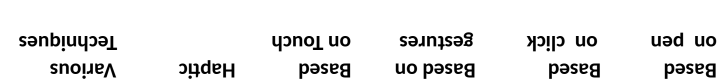
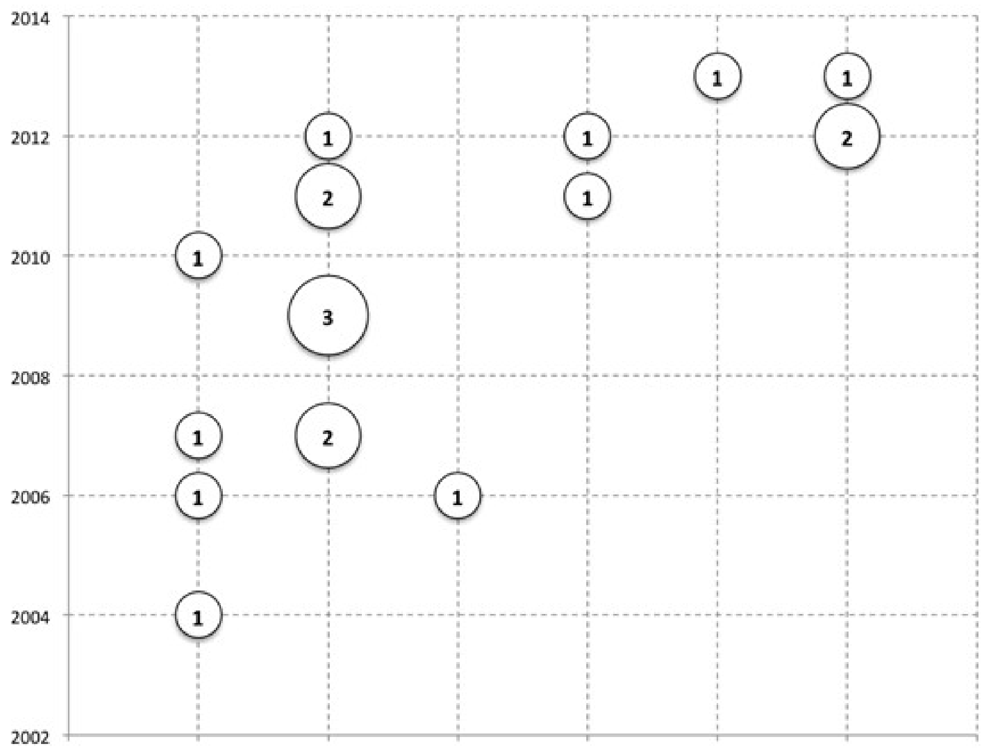
From Figure 1 it can be seen that most of the selected primary studies cover the type of interaction categorized as “Based on Click”. This category showed a larger number of primary studies compared with the other categories, becoming a group of evidence



(areas most focused on according to research conducted). It is most probably that the prevalence of Based on Click is related to the popularity of the mouse technology, that is associated with the popularization of the personal computing. The categories in which the interaction occurs through touch or gestures, or haptically, can be considered evidence deserts (Bailey et al. 2007) due to lack of research in these areas. . According to this evidence, we can answer RQ1 (What are the main forms of interaction investigated for use in IG software interfaces?) and indicate that most IG software use click-based interactions.



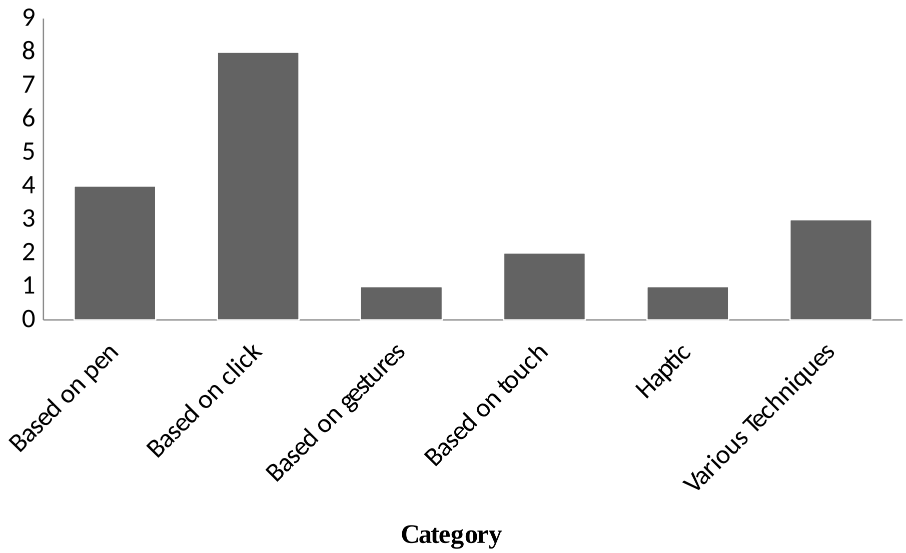


Figure 1. Frequency of studies in each categoryTo verify trends in the area, Figure 2 shows the relationship between the categories of studies (x-axis) and year of publication (y-axis). Despite the greater number of primary studies occurring in the category Based on Click, it can seen that this interest has been decreasing. In the years 2007, 2009 and 2011, this category had three, two and two primary studies, respectively. However, in 2012, this category had only one primary study. More recently, researchers have shown greater interest in the categories Based on Touch and Various Techniques, considering that these two categories emerged during the years 2011 and 2012. In turn, the categories that showed stability in the number of primary studies published are those in Based on Pen and Based on Gestures, with one study each. It appears that there is a study every three years for the pen-based category. In the category Based on Gestures, there was only one study, in 2006.

Figure 2. Distribution of the categories by year

Regard to the types of input we characterize them by the devices that enable the entry of data into the IG software. Figure 3 shows the types of input presented in the analyzed primary studies classified into 10 categories.

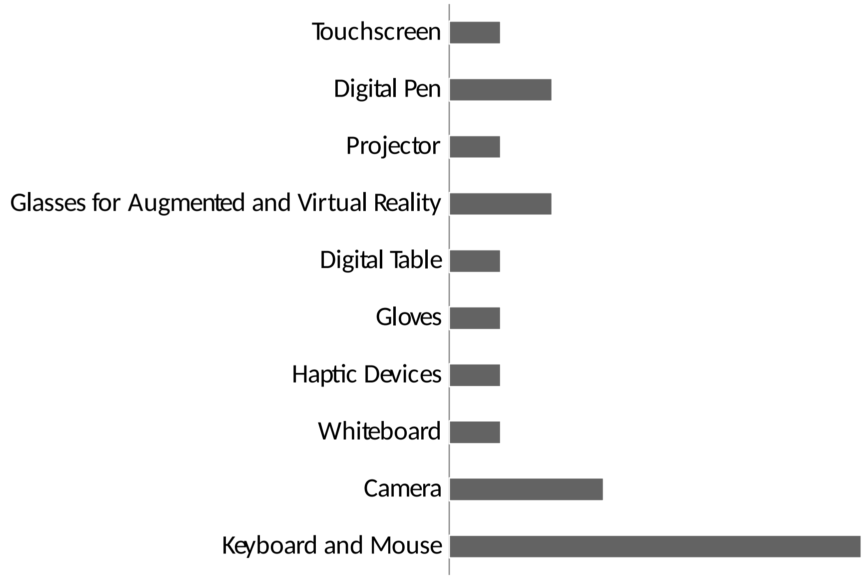


Figure 3: Distribution of the primary studies by data input devices

As noted, the keyboard and mouse are the most commonly used devices for data input, with eight primary studies. Other devices that were also found were cameras, glasses to simulate virtual and augmented reality and digital pens, with three, two and two studies, respectively. It is possible to see that other types of input devices have been little explored by researchers, such as touchscreens, projectors, gloves, digital tables, whiteboards and, finally, haptic devices. In some primary studies, the researchers combined other tools, such as markers, cards and traditional geometric tools, with these input devices.

In addition to the types of input, output types for the IG software were also identified in the literature. In Table 3, it is possible to see that five types of output were mentioned in the primary studies analyzed. The output types most investigated by researchers are 2D interfaces and, subsequently, 3D interfaces. It was observed that these output types are little exploited when combined with other types of output, such as augmented reality (AR) and virtual reality (VR).

Table 3. Distribution of the primary studies by data output devices

2D	12
2D/AR	1

3D	4
3D/AR	1
3D/AR/VR	1

To answer RQ3 (What are the key devices for which IG software has been developed?), Table 4 illustrates the number of primary studies per device type. The device most investigated by researchers was the desktop computer, with 16 primary studies. Other devices such as tablet, tabletop and PDA showed only one study each.

Table 4. Distribution of the primary studies by type of device

Desktop	16
Tablet	1
Tabletop	1
PDA	1

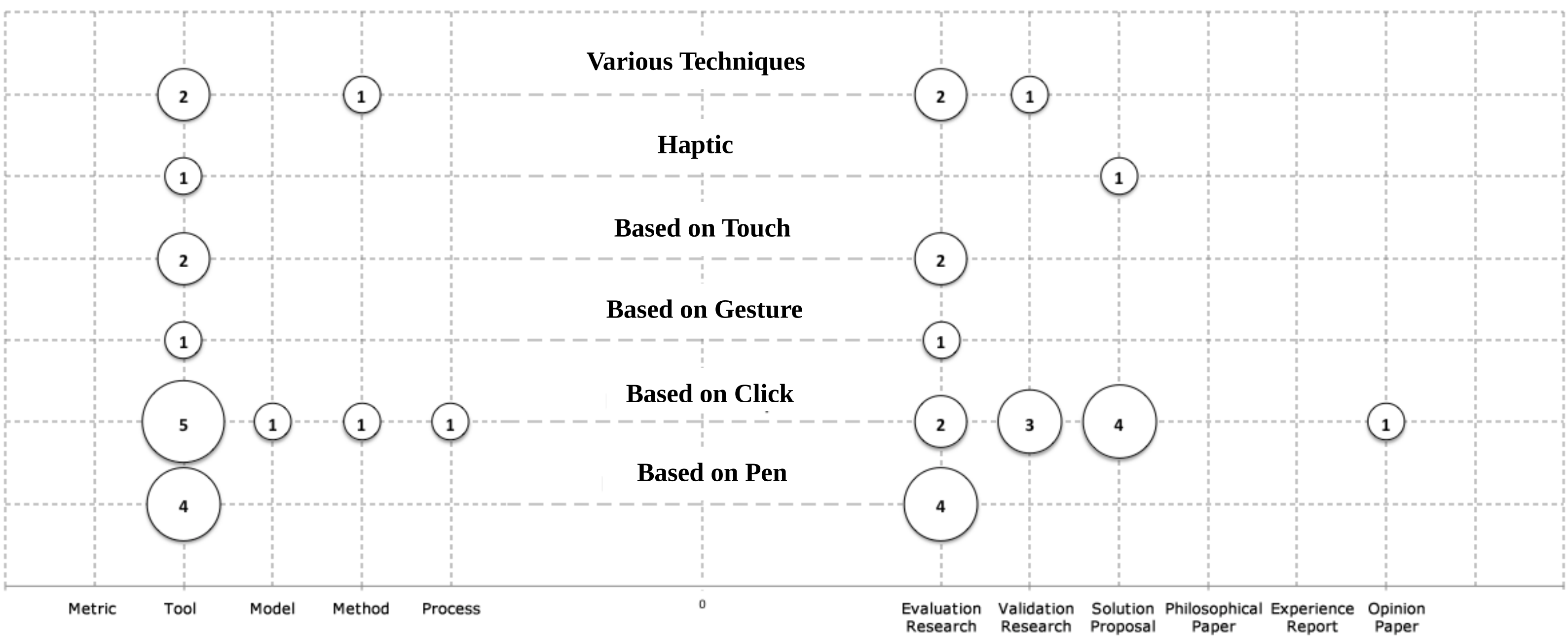
To provide a more detailed analysis of the resultant systematic mapping, showing groups and deserts of evidence, as well as the distribution of primary studies according to categories, we develop a bubble chart (Figure 4) that offers a more accurate view of the frequency of occurrences of the primary studies. These were selected according to the type of contribution and the type of research carried out, in accordance with the definitions proposed by Petersen (2008) and described in Section 1. The y-axis (center) represents the category of the study: on the left are the types of contributions, and on the right are the types of research conducted. The size of each bubble is determined by the number of primary studies classified as belonging to the pairs corresponding to the coordinates of the bubble. This summary provides an overview that allows the identification of existing studies in the area, along with gaps and opportunities for future research. On the Research facet, it can be seen that no articles were found that presented new ideas (Philosophical Papers), nor articles that reporting something done in practice by the author (Experience

Report). The category with the most articles is Evaluation Research, a type of study in which a technique is implemented and the consequences examined. A category was fewer articles was that of the Opinion Paper, which expresses a personal opinion. Along the facet Contribution, no article presented a Metric. The most articles presented a Tool, while in other categories such as Model, Method and Process, few articles were classified.

Figure 4: Map of distribution of studies by category

3. RESULTS

From Figure 1 it can be seen that most of the selected primary studies cover the type of interaction categorized as "Based on Click". This category showed a larger number of



primary studies compared with the other categories, becoming a group of evidence (areas most focused on according to research conducted). It is most probably that the prevalence of Based on Click is related to the popularity of the mouse technology, that is associated with the popularization of the personal computing. The categories in which the interaction occurs through touch or gestures, or haptically, can be considered evidence deserts (Bailey et al. 2007) due to lack of research in these areas.

According to this evidence, we can answer RQ1 (What are the main forms of interaction investigated for use in IG software interfaces?) and indicate that most IG software use click-based interactions.

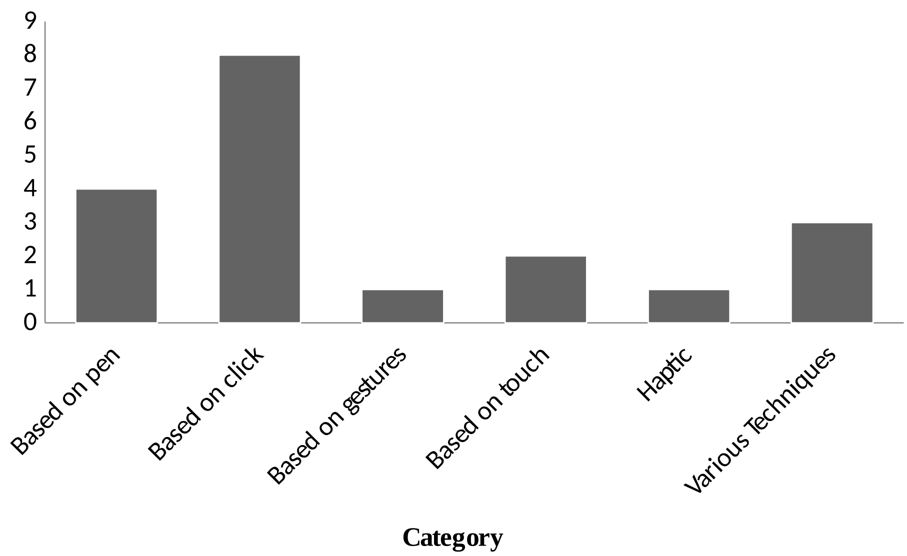


Figure 1. Frequency of studies in each category

Eight of the studies found made use of a keyboard and mouse to provide interactivity for the IG software. Of these eight, two studies investigated the link between the volume of information presented in the interfaces, the use of the cognitive load of students, and the impact of this load on the learning of the users (Reis et al. 2012; Schimpf and Spannagel 2012). Another study involving interaction via keyboard and mouse is presented by Kovárová and Sokolský (2011), who analyzed the problems encountered in the interfaces of Cabri 3D and Archimedes Geo3D software, and used their results to develop software with a 3D interface called Stereo3D.

Some studies used a digital pen instead of a mouse, like that of Kortenkamp and Materlik (2004), who developed a version of Cinderella software for PDA. In addition to this study, three other studies were classified as "Based on Pen." Both Liu et al.'s (2007) and Song and Zhu's (2010) studies used the digital pen to recognize geometric objects built by gestures using the whiteboard. However, in Song and Zhu's study, instead of using the whiteboard, students interacted with a digital table, replacing the mouse on a desktop computer. In both software programs, it is possible to manually draw geometric objects, while the system does the job of automatically recognizing what is being drawn.

Instead of using a digital pen or a mouse, studies classified "Based on Touch" used a touchscreen. Only two studies were selected for this category. In Blagojevic et al.’s (2012) study, the drawing of geometric objects is explored using traditional tools such as a ruler, compass or protractor on the surface of the tablet screen. Although this study was not exclusively about IG software, the authors emphasize that this tool can be used for teaching geometry. In addition to this study, Blanke and Schneider (2011) developed software for a tabletop in which students can interact with objects by means of a multitouch interface.

Only one study was classified as "Based on Gestures." In this type of interaction, each geometric object requires a different type of gesture for construction and manipulation. Fabre et al. (2006) believe that the approach of gestures for the construction of geometric objects is intuitive and natural for students to use in IG software, but if the software includes several types of gestures and a high degree of freedom, learning it can be confusing.

Three primary studies that were classified as "Various Techniques" presented various different interaction techniques, such as augmented reality, 2D and 3D. A camera is the primary device used to support software use in each of these studies, as well as the use of some other tools such as QR codes and projectors. The implementation of Banu’s (2012) proposed approach explores the use of cameras to recognize QR codes (and correspondingly project the appropriate geometric objects by means of augmented reality in 3D, thus teaching students spatial geometry. However, Starcic et al.’s (2012) study uses physical objects with the shape of geometric objects so that students can interact alone or collaboratively. This interaction is recognized by means of a camera positioned in front of students at the time of the activities, and instructions are computed and processed by the table computer. Bonnard et al.’s (2013) study also makes use of a camera to identify what instructions should be executed. Students spread cards containing a description of the object and a QR code on a table. The camera reads this QR code, processes the instruction and projects the geometric object via the projector through augmented reality.

Finally, two studies were not classified into any of the aforementioned categories, as they did not present methods of interaction in the interfaces for the software. Erez and Yerushalmy (2007) explored how the manipulation of geometric objects by "dragging the mouse" on screen can affect the perception of geometric concepts and the learning of geometry. Mackrell's (2011) study presents an analysis of how the components of a screen, such as the toolbar, should be developed.

4. THREATS TO VALIDITY

To ensure an unbiased selection, the research questions and the inclusion and exclusion criteria were defined at the beginning of the mapping. However, a threat related to assessing the quality of the included studies can not be ruled out, as the studies were selected without assigning scores.

Another possible threat identified is the possibility that some relevant articles were not included due to the use of a limited collection of digital libraries. The classification system and elaborate categories also represent a threat to validity. As shown by Pretorius and Budgen (2008), the best way to sort and categorize results is only obtained at the end of the selection. Furthermore, the grouping of studies in more than one category can represent a potential threat to assessing frequency and considering statistics in this mapping study.

4. CONCLUDING REMARKS

This paper presented a systematic mapping conducted in order to determine (i) what forms of interaction are most investigated in the development of IG software, (ii) what forms of input and output are most explored, and (iii) which devices are most commonly used to run the IG software. During the execution of the procedures of this mapping, 998 articles were found, of which 45 articles were selected in the first step and 21 articles in the final step.

4.1. Interpretation of the results

According to the results obtained, the forms of interaction, the forms of input and output and the devices used for implementing IG software are still not frequently explored. However, based on the data obtained in our analysis, it was possible to answer the question RQ1 after to perform the categorization of research in this area. This categorization, based on the forms of interaction, resulted in the following six categories: Based on Pen, Based on Click, Based on Gesture, Based on Touch, Haptic, and Various Techniques. They are also listed in Table 3.

The category Based on Click contained most primary studies in our review of the literature, and it can be considered the approach most frequently researched and implemented. In contrast, the categories with the fewest studies were those based on gesture and touch. Furthermore, as part of this study, the forms of interaction related to input and output in IG software were identified. The form of interaction most investigated was that of keyboard and mouse with 2D output, answering question RQ2. Other forms of input and output were also presented. Forms of input included camera, whiteboard, gloves, digital table, glasses for augmented and virtual reality, projector, digital pen, haptic devices, and touchscreens. Forms of output, apart from the 2D approach, included software that addressed 3D output, augmented reality and virtual reality.

The types of devices on which this software is run were also investigated in this mapping (Table 6). The device most explored by researchers is the desktop computer, corresponding to 84% of the primary studies, answering question RQ3. On the other hand, mobile devices such as tablets and tabletops have not been widely researched, with only one primary study for each type of device.

Therefore, considering this mapping project, it becomes apparent that most IG software has been developed for desktop use, i.e. it is based on click interaction and run on desktop computers. However, this type of interaction is becoming obsolete with the increased sales of smartphones and tablets, which feature the tap- and gesture-based interaction types. Due to increased sales of these devices and their ease of use, the governments of various countries are introducing the use of these devices into the classroom to support teaching activities, resulting in a need for IG environments to be developed for these types of devices.

4.2. Limitations of the study

To ensure an unbiased selection, the research questions and the inclusion and exclusion criteria were defined at the beginning of the mapping. However, a threat related to assessing the quality of the included studies can not be ruled out, as the studies were selected without assigning scores.

Another possible threat identified is the possibility that some relevant articles were not included due to the use of a limited collection of digital libraries. The classification system and elaborate categories also represent a threat to validity. As shown by Pretorius and Budgen (2008), the best way to sort and categorize results is only obtained at the end of the selection. Furthermore, the grouping of studies in more than one category can represent a potential threat to assessing frequency and considering statistics in this mapping study.

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