Maximum acyclic subgraph under conflict constraints on bounded degree graphs*

Leonardo de Abreu, Manoel Campêlo, and Ana Karolinna Maia

Universidade Federal do Ceará, Campus do Pici, CEP 60440-900, Fortaleza, Brazil
{leonardo.abreu,mcampelo}@lia.ufc.br, karolmaia@ufc.br

Abstract. We study the maximum acyclic subgraph under conflict constraints problem. Conflict constraints state that the two elements of certain pairs of arcs cannot appear simultaneously in a solution. We manage to characterize a perfect dichotomy between polynomial and NP-Hard cases of this problem based on the maximum degrees of the input digraph and the conflict graph, which represent conflict constraints. We present the first two approximation algorithms for this problem that runs in polynomial time when the conflict graph has bounded degree.

Keywords: Acyclic subgraph · Disjunctive constraints · Computational complexity · Tractability.

The maximum acyclic subgraph under conflict constraints (also called negative disjunctive constraints) problem (MASCC) consists in, given a directed graph $D = (V, A)$ and a simple graph $G = (A, E)$, finding a set $A' \subseteq A$ of maximum cardinality so that the graph induced by A' in D is acyclic and A' is an independent set of G. An edge in the conflict graph G identifies two arcs of D that cannot be both in a solution. This problem is known to be NP-Hard [6].

The MASCC was introduced in [6] where some 1/2-approximation algorithms were presented. However, the maximum independent set of G is assumed to be known on these algorithms, even though finding it is also an NP-Hard problem. In other words, those algorithms are polynomial only for the class of conflict graphs where the maximum independent set problem is polynomial-time solvable.

We derive complexity and approximation results for the problem by bounding the maximum degrees of D and G.

Complexity

If $E = \emptyset$, the MASCC on (D, G) is equivalent to finding the maximum acyclic subgraph of D, which is NP-Hard if $\Delta(D) \geq 3$ and polynomial-time solvable otherwise [7]. If D is already acyclic then the MASCC of (D, G) is equivalent to finding the maximum independent set of G, which is also NP-Hard if $\Delta(G) \geq 3$ and polynomial otherwise [2]. Starting from this, we could get a perfect dichotomy: the MASCC on (D, G) is NP-Hard when $\min\{\Delta(D), \Delta(G)\} \geq 2$ or $\max\{\Delta(D), \Delta(G)\} \geq 3$, and it is polynomial-time solvable otherwise. For, we show the following results.

* Partially supported by CNPq and FUNCAP.

Theorem 1. *The MASCC on (D, G) is NP-Hard, even if $\Delta(D) = \Delta(G) = 2$.*

Idea of proof. By a reduction from 3-SAT, we show that deciding if there is a set $A' \subseteq A$ so that $|A'| \geq |A|/2$, $D[A']$ is acyclic and A' is independent in G is NP-Complete. The graphs obtained by the reduction are union of disjoint cycles.

Theorem 2. *The MASCC on (D, G) is polynomial-time solvable if $\Delta(D) = 2$ and $\Delta(G) = 1$.*

Approximability and FPT

We developed a $(\frac{2.5}{3+\Delta(G)} + \epsilon)$-approximation algorithm for the MASCC. The algorithm consists in a combination of any of the known $\frac{1}{2}$-approximation algorithms for the maximum acyclic subgraph cited in [6] and the $(\frac{5}{3+\Delta} + \epsilon)$-approximation algorithm presented in [3] and improved in [5]. We checked that the same strategy, but applying the AEKS-SR algorithm presented in [5] instead, yields an approximation algorithm with ratio $\frac{\log\log\Delta}{O(\Delta)}$.

When $\Delta(G) = 2$, the approximation ratio of the first algorithm is $1/2$, as well as for the algorithms in [6], as expected. We observe that, under the Unique Games Conjecture, we cannot expect a factor better than $1/2$, even if G is empty [4]. On the other hand, unless $P = NP$, there is no constant approximation factor (for general G) even if D is already acyclic [1].

A closely related problem to the MASCC is, given (D, G), finding a set $F \subseteq A$ with minimal cardinality so that $D[A \setminus F]$ is acyclic and F is a vertex cover of G. This is the feedback arc set under positive disjunctive constraints problem (FASPDC). Note that, if $A' \subseteq A$ is a solution to MASCC, $A \setminus A'$ is a solution to FASPDC. We show that this problem is FPT on the size of the solution by a reduction to the classical feedback vertex set.

References

1. Bazgan, C., Escoffier, B., Paschos, V.T.: Completeness in standard and differential approximation classes: Poly-(D)APX- and (D)PTAS-completeness. Theoretical Computer Science **339**(2), 272 – 292 (2005)
2. Berman, P., Fujito, T.: On approximation properties of the independent set problem for low degree graphs. Theory of Computing Systems **32**(2), 115–132 (1999)
3. Berman, P., Fürer, M.: Approximating maximum independent set in bounded degree graphs. In: Proceedings of SODA '94. pp. 365–371 (1994)
4. Guruswami, V., Manokaran, R., Raghavendra, P.: Beating the random ordering is hard: Inapproximability of maximum acyclic subgraph. In: 49th Annual IEEE Symposium on Foundations of Computer Science. pp. 573–582 (2008)
5. Halldórsson, M.M., Radhakrishnan, J.: Improved approximations of independent sets in bounded-degree graphs. In: Schmidt, E.M., Skyum, S. (eds.) Algorithm Theory — SWAT '95. pp. 195–206 (1994)
6. Mapa, S.M.S., Urrutia, S.: On the maximum acyclic subgraph problem under disjunctive constraints. Information Processing Letters **115**(2), 119–124 (2015)
7. Newman, A.: Approximating the maximum acyclic subgraph. Ph.D. thesis, Massachusetts Institute of Technology (2000)