On total chromatic number of circulant graphs

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Total Coloring and Circulant Graph

A k-total coloring of a graph G is an assignment of k colors to the elements of G such that adjacent elements have different colors. The total chromatic number $\chi''(G)$ is the smallest integer k for which G has a k-total coloring. Clearly, $\chi''(G) \geq \Delta + 1$, and the Total Coloring Conjecture (TCC) states that for any simple graph G, $\chi''(G) \leq \Delta + 2$, where Δ is the maximum degree of G [1]. Graphs with $\chi''(G) = \Delta(G) + 1$ are called Type 1, and graphs with $\chi''(G) = \Delta(G) + 2$ are called Type 2.

A circulant graph $C_n(d_1, d_2, \cdots, d_l)$ with $1 \leq d_1 < \cdots < d_l \leq \lfloor \frac{n}{2} \rfloor$ has vertex set $V = \{v_0, v_1, \cdots, v_{n-1}\}$ and edge set $E = \bigcup_{i=1}^{l} E_i$ where $E_i = \{e_0^i, e_1^i, \cdots, e_{n-1}^i\}$ and $e_j^i = (v_j, v_{j+d_i})$ where the indexes of the vertices are considered modulo n. An edge of E_i is called edge of length d_i.

The main goal of this current work is to determine the total chromatic number of all circulant graphs $C_n\left(1, \left\lfloor \dfrac{n}{2} \right\rfloor\right)$. It is well known by Chetwynd [2] that when n is even the graph is Type 2. So, we focus on the odd case.

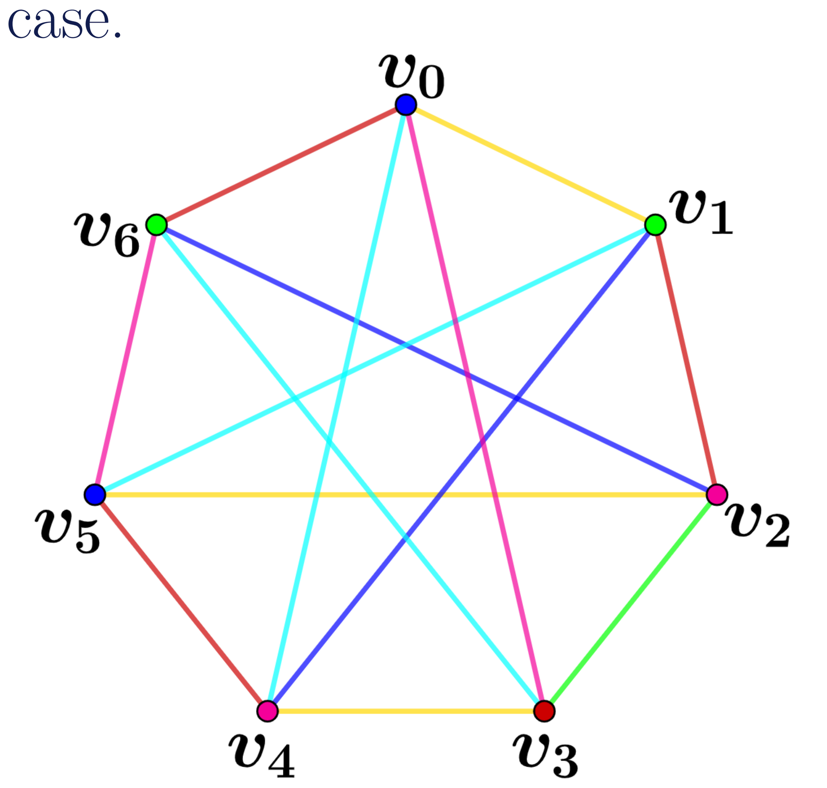


Figure 1:Example of circulant graph $C_7(1, 3)$

Auxiliar Result

We prove that $C_7(1, 3)$ is Type 2, by using the following theorem to prove that this graph does not have any 5-total coloring.

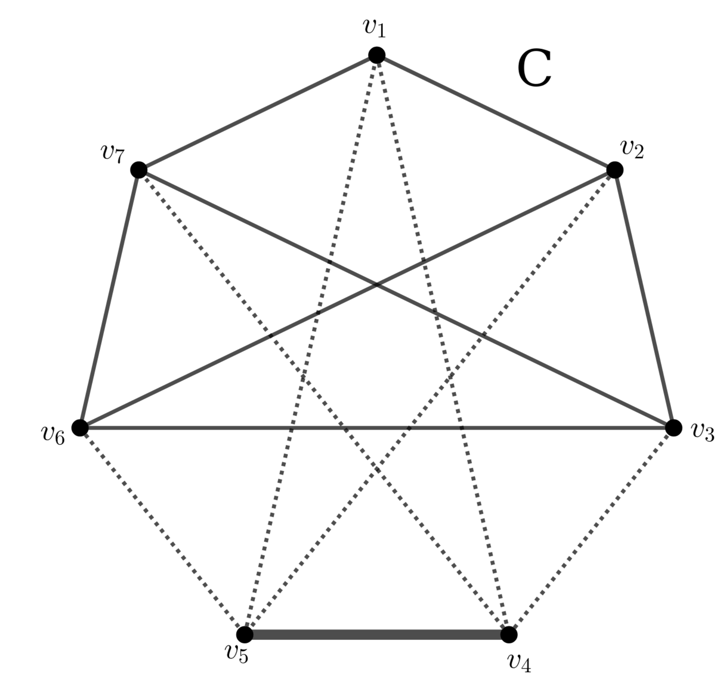
Theorem 1.[Sasaki [4], 2013] If a k-regular graph G does not have maximal macthing of maximum lenght $\left\lfloor \dfrac{|E|}{k+1} \right\rfloor$, then does not exists $(k+1)$-total coloring of G

Main Result

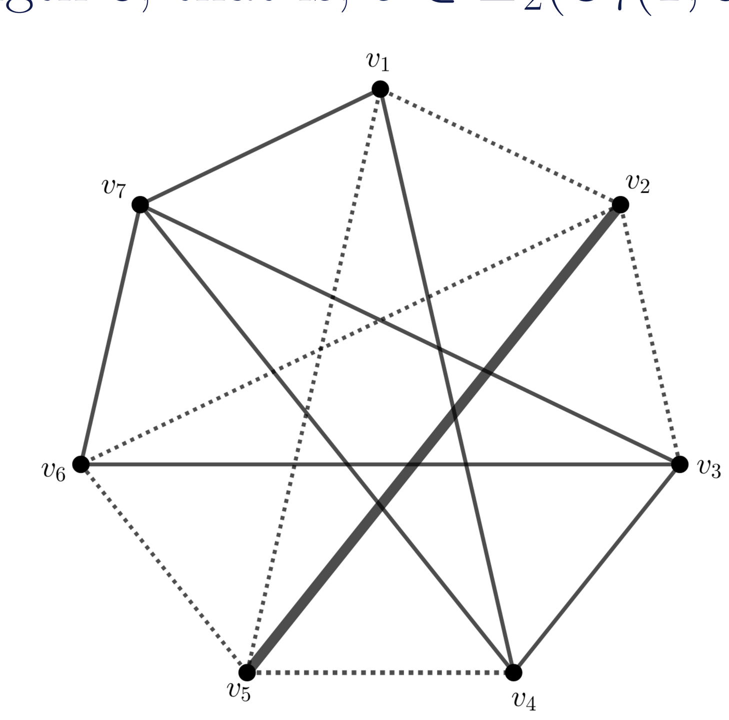
Theorem 2. The graph $C_7(1, 3)$ is Type 2.

Supposing that $C_7(1, 3)$ has a maximal matching M of lenght $\left\lfloor \dfrac{14}{5} \right\rfloor = 2$. Let $e \in M$, we can consider two cases:

- if an edge has length 1, that is, $e \in E_1(C_7(1, 3))$



- if an edge has length 3, that is, $e \in E_2(C_7(1, 3))$



In any case, we do not have a maximal matching of lenght 2 for $C_7(1, 3)$. Concluding that there is no $5-$total coloring of $C_7(1, 3)$.

Agradecimentos

CAPES

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