Resumos

Abstracts

Sessão: Geometria

Session: Geometry

Organizadores

Organizers

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Instantons on the Exceptional Holonomy Manifolds of Bryant and Salamon

Andrew Clarke

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Resumo

We give a construction of G_2 and $Spin(7)$ instantons on exceptional holonomy manifolds constructed by Bryant and Salamon, by using an ansatz of spherical symmetry coming from the manifolds being the total spaces of rank-4 vector bundles. In the G_2 case, we show that, in the asymptotically conical model, the connections are asymptotic to Hermitian Yang-Mills connections on the nearly Kähler $S^3 \times S^3$.

Invariants of (complex) hyperbolic manifolds

Carlos Henrique Grossi

ICMC - USP São Carlos

Resumo

We study discrete representations of surface groups in the group of complex automorphisms of the unit ball in $\mathbb{C}^2$. We will discuss invariants of these representations and their relationships. In particular, we will formulate a conjectural and far-reaching generalization of Toledo's rigidity theorem that might help explaining the (sometimes enigmatic) behaviour of the mentioned invariants.

I Congresso Brasileiro de
Jovens Pesquisadores em
Matemática Pura e Aplicada

On the umbilicity of complete constant mean curvature spacelike hypersurfaces

Cícero Pedro de Aquino* and Henrique Fernandes de Lima

*UFPI - Teresina, PI

Resumo

The last few decades have seen a steadily growing interest in the study of the geometry of spacelike hypersurfaces immersed in a Lorentz space form. Apart from physical motivations, from the mathematical point of view this is mostly due to the fact that such hypersurfaces exhibit nice Bernstein-type properties, and one can truly say that the first remarkable results in this branch were the rigidity theorems of E. Calabi and S.Y. Cheng and S.T. Yau for hypersurfaces in the Lorentz space. In the case of the de Sitter space, A.J. Goddard conjectured that every complete spacelike hypersurface with constant mean curvature in such ambient space should be totally umbilical. Although the conjecture turned out to be false in its original statement, it motivated a great deal of work of several authors trying to find a positive answer to the conjecture under appropriate additional hypotheses. In this work, we show that a complete spacelike hypersurface immersed with constant mean curvature in the de Sitter space must be totally umbilical, provided that its Gauss mapping has some suitable behavior. In particular, we use an extension of Hopf's maximum principle due to S.T. Yau in order to give new positive answers for the Goddard's conjecture. This is a joint work with Henrique Fernandes de Lima(UFCG).

Lie theory for algebroid 2-representations

Cristian Ortiz*

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Resumo

This talk will discuss Lie algebroids (i.e. geometric structures that unify Lie algebras, regular foliations, Poisson structures, among others) and their representations. Geometrically, a representation of a Lie algebroid is a vector bundle equipped with a flat algebroid connection. For integrable Lie algebroids, i.e. those coming from Lie groupoids, representations can be integrated to groupoid representations. This construction is an extension of the relation between flat connections and parallel transport. We are going to see how this can be done for a more general notion of representation, that of a 2-representation of a Lie algebroid on a 2-term complex of vector bundles, e.g. the adjoint or the coadjoint representations of a Lie algebroid. We will see that any 2-representation integrates to a 2-functor, which plays the role of a higher holonomy of a 2-connection. Then, we will show that the associated semi-direct product is a Lie 2-groupoid, which after truncation, yields to a new construction of topological $\mathcal{VB}$-groupoids "integrating"2-term representations up to homotopy. This is a joint work with Olivier Brahic.

Singular integral affine structures and completely integrable Hamiltonian systems

Rui Loja Fernandes, Daniele Sepe*

*UFF – RJ

Resumo

Classifying and constructing completely integrable Hamiltonian systems are two driving (hard!) questions in Hamiltonian mechanics, which have far reaching consequences and applications in various fields, ranging from symplectic topology to representation theory, algebraic geometry and spectral theory. This talk aims to present a possible framework to tackle both problems (under some mild assumptions) by introducing a differential-geometric notion of singular integral affine structures, which are particular Lagrangian submanifolds of cotangent bundles. This is ongoing work with Rui Loja Fernandes (University of Illinois at Urbana-Champaign).

On the topology and index of minimal surfaces

Davi Maximo*,

*Stanford University

Abstract

In this talk we show that for an immersed two-sided minimal surface in $\mathbb{R}^3$, there is a lower bound on the index depending on the genus and number of ends. Using this, we show the nonexistence of an embedded minimal surface in $\mathbb{R}^3$, of index 2, as conjectured by Choe. Moreover, we show that the index of a immersed two-sided minimal surface with embedded ends is bounded from above and below by a linear function of the total curvature of the surface. This is a joint work with Otis Chodosh.

G_2-instantons over twisted connected sums

Henrique N. Sá Earp*, Thomas Walpuski,
Marcos Jardim, Daniela Prata

*Unicamp, SP

Resumo

I will describe a method to construct G_2–instantons over compact G_2–manifolds arising as the twisted connected sum of a matching pair of asymptotically cylindrical Calabi-Yau 'building blocks', proposed by Kovalev and Corti-Haskins-Nordström-Pacini. It consists on gluing G_2–instantons obtained from holomorphic bundles over the building blocks via the gradient flow method, under boundary conditions 'at infinity' given by a certain notion of 'asymptotic stability'. One requires natural compatibility and transversality conditions which can be interpreted in terms of certain Lagrangian subspaces of a moduli space of stable bundles on a $K3$ surface.

Motivated by this construction, I will present techniques to produce such asymptotically stable bundles over building blocks. The most important tool is a generalisation of Hoppe's stability criterion to bundles over smooth projective varieties X with $\mathrm{Pic(X)} \simeq \mathbb{Z}^\ell$, a result which may be of independent interest.

Time allowing, I will show how linear monads can be used to produce a prototypical model of the curvature blow-up phenomenon along a sequence of asymptotically stable bundles degenerating into a torsion-free sheaf. This effect has been studied in full generality by Uhlenbeck-Yau over 4–manifolds and by Tian over higher dimensional manifolds with special holonomy.

The talk includes material from joint works with Thomas Walpuski (Imeprial College London) and Marcos Jardim and Daniela Prata (Unicamp).

Minimal surfaces with free boundary

Dávi Máximo, Ivaldo Nunes*, Graham Smith

*Universidade Federal do Maranhão - UFMA

Resumo

In this talk we will discuss the free boundary problem for minimal surfaces. Our main goal is to deal with the existence of such surfaces. In particular, we prove that every strictly convex compact domain of $\mathbb{R}^3$ contains a properly embedded free boundary minimal surface which is topologically equivalent to an annulus. This is a joint work with D. Máximo (Stanford University) and G. Smith (Universidade Federal do Rio de Janeiro - UFRJ).

Compact gradient generalized quasi-Einstein metrics with constant scalar curvature

J. N. Gomes*, A. Barros

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Resumo

A gradient generalized m-quasi-Einstein metric on a complete Riemannian manifold (M^n, g) is a choice of a potential function $f : M^n \to \mathbb{R}$ as well as a function $\lambda : M^n \to \mathbb{R}$ such that

$$Ric + \nabla^2 f - \frac{1}{m} df \otimes df = \lambda g, \tag{1}$$

where Ric denotes the Ricci tensor of $(M^n,\, g)$, while $0 < m \leq \infty$ is an integer, ∇^2 and $\otimes$ stand for the Hessian and the tensorial product, respectively.

It is important to point out that if $m = \infty$ and λ is constant, equation (1) reduces to one associated to a gradient Ricci soliton, as well as considering $m = \infty$ and λ not constant we obtain the almost Ricci soliton equation. In addition, if λ is constant and m is a positive integer, it corresponds to m-quasi-Einstein metrics that are exactly those n-dimensional manifolds which are the base of an $(n + m)$-dimensional Einstein warped product. He, Petersen and Wylie was given some classification for m-quasi-Einstein metrics where the base has non-empty boundary. Moreover, they have proved a characterization for m-quasi-Einstein metric when the base is locally conformally flat. We also point out that, Catino have proved that around any regular point of f a generalized m-quasi Einstein metric $\big(M^n,\, g,\, \nabla f,\, \lambda\big)$ with harmonic Weyl tensor and $W(\nabla f, \cdots, \nabla f) = 0$ is locally a warped product with $(n-1)$-dimensional Einstein fibers.

In this lecture we shall show that a compact gradient generalized m-quasi-Einstein metric $\big(M^n,\, g,\, \nabla f,\, \lambda\big)$ with constant scalar curvature must be isometric to a standard Euclidean sphere $\mathbb{S}^n$ with the potential f well determined. This is a joint work with Abdênago Barros (UFC-CE).

On the density and the spectrum of minimal submanifolds in space forms

Luciano Mari*, Barnabé P. Lima,

José Fabio B. Montenegro, Franciane B. Vieira

*UFC - Fortaleza, CE

Resumo

Let $\varphi : M^m \to N^n$ be a properly immersed minimal submanifold in an ambient space close, in a suitable sense, to the space form $\mathbb{N}_k^n$ of sectional curvature $-k \leq 0$. In this talk, I discuss the relationship between the spectrum of M and its density function

$$\Theta(r) = \frac{\mathrm{vol}(M \cap B_r^n)}{\mathrm{vol}(\mathbb{B}_r^m)},$$

where $B_r^n, \mathbb{B}_r^m$ are geodesic balls of radius r in N^n and $\mathbb{N}_k^m$, respectively. In a recent joint work with my colleagues quoted above, we proved that if $\Theta(r)$ grows sub-exponentially ($k > 0$) or sub-polynomially ($k = 0$) along a sequence, then the spectrum $\sigma(M)$ of the Laplace-Beltrami operator of M is the whole half-line $[(m-1)^2 k/4, +\infty)$. Notably, the criterion applies to all Anderson's solutions of Plateau's problem at infinity on the hyperbolic space, independently from their boundary regularity. I will also briefly comment on the relationship between the total curvature of minimal submanifolds and the finiteness of the limit of $\Theta(r)$ at infinity.

A Linear Isoperimetric Inequality and Eigenvalue Estimates in Weighted Manifolds

M. Batista*, M. P. Cavalcante, J. Pyo

*UFAL

Abstract

In this talk we will present an upper bound to the weighted volume of a smooth compact manifold using the weighted mean curvature of its boundary. As consequence we give a linear isoperimetric inequality for such manifolds. We also give an extrinsic upper bound to the first non zero eigenvalue of the drift Laplacian on closed submanifolds of weighted manifolds.

This is a joint work with M. P. Cavalcante (UFAL - Brazil) and J. Pyo (Pusan University, South Korea).

Contact isotropic realizations of Jacobi manifolds

María Amelia Salazar*,

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Abstract

Jacobi manifolds, introduced by Lichnerowicz and Kirillov independently, are analogous (while at the same time generalising) Poisson manifolds, in the sense that the role of symplectic geometry in the latter is played by contact manifolds in the former. Recent work of Crainic and Salazar has provided a new geometric approach to studying Jacobi structures defined on any real line bundle, i.e. not necessarily trivial. Motivated by the theory of integrable Hamiltonian systems on contact manifolds, as well as by the idea of exploring 'compactness' in Jacobi manifolds (analogous to that which Crainic, Fernandes and Martínez-Torres introduced for their Poisson counterparts), this talk presents the classification of some special types of 'desingularisations' of Jacobi structures, which are analogous to those studied by Dazord and Delzant in the Poisson domain. This is joint work with D. Sepe.

On a characterization of vector bundles

Matias del Hoyo

IMPA - Rio de Janeiro, RJ

Abstract

We are concerned with smooth actions of the multiplicative monoid of the real numbers. Examples of such actions are given by the homotheties on a vector bundle, and moreover, any action satisfying a regularity condition arises in this way, leading to a characterization of vector bundles over manifolds. In a joint work with H. Bursztyn (IMPA) and A. Cabrera (UFRJ) we adapt this idea to describe vector bundles over Lie groupoids and Lie algebroids, and their behavior under differentiation and integration. These objects have received much attention lately because of their deep ties with Poisson geometry and with representations up to homotopy. The plan of this talk is to describe the characterization of vector bundles as actions, to outline its application to the realm of groupoids and algebroids, and if time permits, to discuss the non-regular case, on which other interesting geometric structures arise naturally.

Minimal graphs over certain unbounded domains of Hadamard manifolds

Miriam Telichevesky*

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Resumo

Given an unbounded domain Ω of a Hadamard manifold M, it makes sense to consider the problem of finding minimal graphs with prescribed continuous data on its cone-topology-boundary, i.e., on its ordinary boundary together with its asymptotic boundary. In this article it is proved that under the hypothesis that the sectional curvature of M is ≤ -1 this Dirichlet problem is solvable if Ω satisfies certain convexity condition at infinity and if $\partial\Omega$ is mean convex. We also prove that mean convexity of $\partial\Omega$ is a necessary condition, extending to unbounded domains some results that are valid on bounded ones.

Metrics with strongly positive curvature on flag manifolds

Ricardo A. E. Mendes*, Renato G. Bettiol

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Resumo

This work concerns a curvature condition for Riemannian manifolds called "strongly positive curvature", which lies strictly between positive sectional curvature and positive definite curvature operator, and which was introduced by J. Thorpe in the 70s.

We identify the moduli space of homogeneous metrics satisfying this condition on the manifolds W^6, W^{12} and W^{24} of complete K-flags in K^3, where K is the algebra of complex numbers, quaternions and octonions, respectively.

In particular, this finishes the classification of manifolds admitting a homogeneous metric with strongly positive curvature initiated in our previous work. It also provides evidence for a general deformation conjecture. This is joint work with Renato G. Bettiol (U. of Notre Dame, Notre Dame, IN, USA).