Resumos

Abstracts

Sessão: Física Matemática e Teoria Ergódica

Session: Mathematical Physics and Ergodic Theory

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Open quantum random walks and the recurrence problem

Carlos Felipe Lardizabal

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Abstract

In this seminar we discuss a model of open quantum random walk on $\mathbb{Z}$, and there we examine a notion of quantum recurrence. Combining some classical and quantum ideas, one can also analyze positive recurrence. The calculation of probabilities is typically non-commutative (via trace functionals), but globally the walk presents certain classical properties. Joint work with Rafael Rigão Souza (UFRGS).

Series of metastable states for Reversible Probabilistic Cellular Automata

Cristian Spitoni

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Abstract

Metastable states are very common in nature and are typical of systems close to a first order phase transition. Classical examples are the supersaturated vapor and the magnetic hysteresis. The full mathematical description of metastability is quite recent and still incomplete. In this framework, Probabilistic Cellular Automata pose challenging problems and show unexpected behaviors. Probabilistic Cellular Automata (PCA) are discrete-time dynamics consisting of cells interacting with each other according to a stochastic rule. They have been introduced as a stochastic generalization of Cellular Automata (CA), which are characterized by a deterministic evolution rule.

In this talk we study the metastability properties of a class of PCA with multiple (not necessarily degenerate) metastable states. In the presence of such deep wells, we prove an addition formula for the exit times from the metastable states in the case they form a series. With this expression we mean that the structure of the energy landscape is such that the system has two not degenerate metastable states and the system, started at the one with highest energy, must necessarily pass through the second one before relaxing to the stable state. This is a joint work with E.N.M. Cirillo (Rome University, Italy) and F.R. Nardi (TU Eindhoven, The Netherlands).

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Quantitative derivation of the Gross-Pitaevskii equation

Gustavo de Oliveira

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Abstract

Starting from the many-body Schroedinger equation we show that the dynamics of Bose-Einstein condensates can be described, in the Gross-Pitaevskii limit, by the time-dependent Gross-Pitaevskii equation. Furthermore, we give a bound on the rate of convergence for this approximation. Our results hold for a class of modified coherent states in Fock space which model a condensate in a trap. To construct these initial states we use Bogoliubov transformations. We present the main steps to obtain these results. This work was done in collaboration with N. Benedikter and B. Schlein.

Entropy, pressure and duality

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Abstract

The duality of Fenchel-Rockafellar can be useful when we are interested in maximizing or minimizing some linear function defined on a convex and compact set. We wish to use this duality in order to analyze the problem of maximizing the expression $\int A d\pi + H(\pi)$ over a convex set of probabilities, where the entropy function H is concave. The convex set can be, for example, the invariant probabilities of some dynamic system or a convex subset of this. In some cases where we have simple constraints we will explicitly determine the dual problem associated. We will apply these ideas in the study of entropy and pressure when considering holonomic probabilities associated to an iterated function system (IFS).

Critical line for a Potts model coupled to causal dynamical triangulations

José Javier Cerda Hernández

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Abstract

We introduce the (annealed) Potts model coupled to two-dimensional causal dynamical triangulations. Using duality on a torus (periodic boundary condition) we provide a relation between the free energy of the Potts model coupled to CDTs and its dual. This duality relation comes from the FK representation for the Potts model. We use the duality relation, the FK representation and the high-temperature expansion for determine a region in the quadrant of parameters where the critical curve for the Potts model coupled to CDTs and Potts model coupled to dual CDTs can be located. This is done by outlining a region where the infinite-volume Gibbs measure exists and is unique and a region where the finite-volume Gibbs measure has no weak limit (in fact, does not exist if the volume is large enough).

Variational description of Gibbs-non-Gibbs dynamical transitions for spin-flip systems

Julián F. Martínez

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Abstract

We discuss the concept of Gibbs/ non-Gibbs measure in the lattice together with its extension to the mean field / local-mean field context, and the emergence of dynamical Gibbs-non-Gibbs transitions under independent spin-flip (”infinite-temperature”) dynamics. We show that these dynamical transitions are equivalent to bifurcations in the set of global minima of the large-deviation rate function describing optimal conditioned trajectories of the empirical density. Possible bifurcation scenarios are fully determined in the mean field case, yielding a full characterization of passages from Gibbs to non-Gibbs -and vice versa- with sharp transition times. Based on joint work with Roberto Fernández and Frank den Hollander.

Some Problems in Analytic Number Theory for Polynomials over Finite Fields

Júlio Andrade

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Abstract

In this talk I will explore some traditional problems of analytic number theory in the context of function fields over a finite field. Several such problems which are currently viewed as intractable can, in the function field scenario, be attacked with vastly different tools than those of traditional analytic number theory. The resulting theorems in the function field setting can be used to check existing conjectures in the classical case, and to generate new ones. The problems I will discuss include: the twin prime conjecture, the additive divisor problem, moments of L-functions and connections with random matrix theory.

Frenkel-Kontorova model and Gibbs measures

Maël Mevel

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Abstract

First, we will introduce the one-dimensional Frenkel-Kontorova model and give some of its basic properties. We will then introduce the notion of ground and minimizing configurations, see if it exists (and eventually are unique) in this particular model, and explain the link with the notion of effective potential (Chou-Griffiths). We will define the Lax-Oleinik operators, and give several results about the model (rotation numbers, Aubry theory).

In a second part, we will look at this model from a different point of view (pure dynamic systems); defining Gibbs (or equilibrium) measures for a specific temperature, we will relate those to ground configurations and effective potentials, when the temperature goes to infinity.

In a last part, we will talk about the case when the potential is not coercive, and define a new type of Lax-Oleinik operator.

Phase transition in ferromagnetic Ising model with a cell-board external field

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Abstract

In this paper we show the presence of a first-order phase transition for a ferromagnetic Ising model on $\mathbb{Z}^2$ with a periodical external magnetic field (proposed by Maruani et al. [4]). The external field takes two values $\pm h$, with $h > 0$, composing a cell-board configuration with rectangular cells of sides $L_1 \times L_2$ sites, such that total value of the external field is zero. Formally, for each n, m integers we define

$$C(n,m) = \{(t_1,t_2) \in \mathbb{Z}^2 : \quad nL_1 \le t_1 < (n+1)L_1, \\ mL_2 \le t_2 < (m+1)L_2\},$$

then

$$\mathbf{Z}_+ = \bigcup_{\substack{n,m: \\ n+m \text{ is even}}} C(n,m), \qquad \mathbf{Z}_- = \mathbb{Z}^2 \setminus \mathbf{Z}_+.$$

Let $\sigma \in \Omega = \{-1,+1\}^{\mathbb{Z}^2}$ be a configuration on $\mathbb{Z}^2$. We study the model with a formal Hamiltonian defined for any $\sigma \in \Omega$ as

$$H(\sigma) = -J \sum_{\langle t,s \rangle} \sigma(t)\sigma(s) - \sum_s h_s \sigma(s),$$

where $J > 0$, the symbol $\langle t,s \rangle$ denotes nearest neighbours $s, t \in \mathbb{Z}^2$, that is $|t-s| = 1$, and

$$h_s = \begin{cases} h, & \text{if } s \in \mathbf{Z}_+, \\ -h, & \text{if } s \in \mathbf{Z}_-. \end{cases}$$

The phase transition holds if $h < \frac{2J}{L_1} + \frac{2J}{L_2}$. Our result is related with Nardi et al. [5], we prove a phase transition in a more general

model that the considered by them. Moreover, the phase transition in antiferromagnetic Ising model with constant external field (see [2]) holds true as a corollary of our proof.

We use an approach based on the technique of reflection positivity [1]. Particularly, we apply a certain key inequality which is usually referred to as the chessboard estimate. This tool allows us to construct a sort of the Peierls arguments evaluating the contour probabilities. This is a joint with Eugene Pechersky and Anatoly Yambartsev, both from IME-USP.

References

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[5] Nardi, F.R., Olivieri, E., Zahradník, M.: *On the Ising model with strongly anisotropic external field*. Journ. Stat. Phys., **97**: 87-144 (1999).

Zeons and Combinatorial identities

Petrus H. R. dos Anjos

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Abstract

We show that the ordinary derivative of a real analytic function of one variable can be realized as a Grassmann-Berezin-type integration over the Zeon algebra, the Z-integral. As a by-product of this representation, we give show how zeons can be employed to obtain properties of combinatorial objects. Particularly, we give new proofs of the Faà di Bruno formula and Spivey's identity. The approach described here is suitable to accommodate new Z-integral representations including Stirling numbers of the first and second kind, Bell, central Delannoy, Euler, Fibonacci, and Genocchi numbers, and the special polynomials of Bell, generalized Bell, Bernoulli, Hermite, and Laguerre.

The algoritmic Lovász Local Lemma and the Hard Core lattice gas

Rogério Gomes Alves

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Abstract

In this talk we will point out a relation between the Moser-Tardos algorithmic version of the Lovász Local Lemma and the cluster expansion of the Hard Core lattice gas in statistical mechanics. Through this relation we conclude that the Moser-Tardos algorithm is successful in a polynomial time if the cluster expansion converges. Joint work with Aldo Procacci.

Stochastic processes with long memory

Sandro Gallo

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Abstract

We introduce two main approaches to stochastic chain with long memory. 1) The "process approach", where $\mathbb{Z}$ is interpreted as time and 2) the "random field" approach where $\mathbb{Z}$ is interpreted as space. In both frameworks a notion of continuity is present: regularity in the former, and Gibbsianity in the later. We will explain what is known concerning the relation between both approaches, and what are the main open questions in the field. All along the talk, I will try to mention the relation to a third approach: Ergodic Theory.

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Quasiballistic and quasilocalized Schrödinger operators are generic

Silas L. Carvalho

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Abstract

We derive sufficient conditions for the presence of generic sets of discrete Schrödinger operators on $l^2(\mathbb{Z}^d)$, $d \geq 1$, with both quasilocalized and quasiballistic dynamics, and apply them to three operator spaces, that is, with uniformly bounded, analytic quasiperiodic and unbounded potentials. It is concluded, for these spaces, that the dynamics is typically (from the topological viewpoint) nontrivial, whereas quantum intermittency is exceptional.

Phase Transition in a Bidimensional Random Polymers Model

Simone Vasconcelos da Silva

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Abstract

Phase transition issues are addressed for random polymers on $\mathbb{Z}^2$ with self-repulsive interactions. It is shown that, in the absence of drift and with power law interactions, the polymer exhibits transition from diffusive to a ballistic behavior. When non-null drifts are added and positive translation invariant interactions are considered, the polymer presents a ballistic behavior. We also derive a Central Limit Theorem for the model.

How do we know that the creations of worlds are not determined by falling grains of sand?

Wioletta Magdalena Ruszel

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Abstract

The concept of self-organized criticality was introduced by Bak, Tang and Wiesenfeld in 1987 as an explanation for the existence of a certain type of noise and power-law behaviour in power spectra in particular physical systems. It captures the idea that certain physical systems can drive themselves into a critical state which shares several properties of equilibrium systems at the critical point such as power-law decay of correlations. Models of self-organized criticality provide a mechanism which can be used to explain the emergence of complexity in many natural phenomena. A toy model which displays this behaviour is the so-called sandpile model One of the most exciting applications of the sandpile dynamics is to model neuronal comunication in the brain. Here we study the abelian sandpile model on a random trees.

It was proven that for the full binary tree (and Bethe lattice) the probability that an avalanche is of size k decays as a power-law with mean-field exponent 3/2.

For the binary and binomial tree we prove exponential decay of correlations, and in a small supercritical region (i.e., where the branching process survives with positive probability) exponential decay of avalanche sizes. This shows a phase transition phenomenon between exponential decay and power law decay of avalanche sizes.

Finally we discuss our work in progress about self-organized criticality on Galton-Watson trees.

This is joint work with Antal Jarai (U Bath), Frank Redig (TU Delft) and Ellen Saada (U Paris 5).