Resumos

Abstracts

Sessão: Conjuntos, Topologia e Espaços de Banach

Session: Set Theory, Topology and Banach Spaces

Organizadores

Organizers

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Invited speakers

Set theory, topology and Banach spaces

Measures on Suslinean spaces

Piotr Borodulin Nadzieja

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Resumo

A compact Hausdorff space is Suslinean if it is ccc and non-separable. We will overview some classical examples of small Suslinean spaces and discuss the problem when a Suslinean space can serve as a support for a measure.

Katetov order on MAD families

Carlos Martinez Ranero

Universidad de Concepción, Chile

Resumo

Katetov ordering on almost disjoint families was introduced by Garcia-Ferreira and Hrusak in an attempt to classify them. We answered one of the basic questions by consistently constructing a MAD family maximal in this order. This is an ongoing project with many fundamental problems open.

Large Lindelöf spaces with points G_δ

Toshimichi Usuba

Kobe University, Japan

Resumo

We introduce a simple construction of Lindelöf spaces with points G_δ. Using this construction, we prove the following: Suppose either (1) there exists a regular Lindelöf P-space of pseudocharacter $\leq \omega_1$ and size $> 2^\omega$, (2) CH holds and there exists a Kurepa tree, or (3) CH and $\square(\omega_2)$ hold. Then there exists a regular Lindelöf space with points G_δ and size $> 2^\omega$. This means that the non-existence of large regular Lindelöf spaces with points G_δ is a large cardinal property.

Contributed talks

Set theory, topology and Banach spaces

Between paracompactness and the D-property

Robson A. Figueiredo*, Lúcia R. Junqueira and Santi Spadaro

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Resumo

A space X is a D-space if whenever a neighborhood $N(x)$ of x, for each $x \in X$, is given, then there is a closed discrete subset D of X such that $\{ N(x) : x \in D \}$ covers X. It is a famous open question asked by van Douwen and Pfeffer in [3] whether for the regular spaces any of the standard covering properties, such as Lindelöf or paracompact, imply the D-property. In this talk we introduce a new class of topological spaces that is stronger than both the class of paracompact spaces and the class of D-spaces: the D-paracompact spaces. We also investigate the relationship between the D-paracompactness and other properties like Menger and metrizability as well as its behavior under the usual topological operations.

Referências

[1] G. Gruenhage, *A survey of D-spaces*, Contemporary Mathematics **533** (2011), 13–28.

[2] L. F. Aurichi, *D-spaces, topological games, and selection principles*, Topology Proceedings **36** (2010), 107–122.

[3] Eric K. van Douwen and Washek F. Pfeffer, *Some properties of the Sorgenfrey line and related spaces*, Pacific Journal of Mathematics **81** (1979), 371–378.

CH implies a compact space K is metrizable if $K^2 \setminus \Delta$ is dominated by the irrationals

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Resumo

A space X has a $\mathbb{P}$-diagonal if $X^2 \setminus \Delta$ is covered (dominated) by a family of compact sets $\{K_f : f \in \omega^\omega = \mathbb{P}\}$ satisfying that $K_f \subset K_h$ whenever $f \leq h$ (coordinatewise).

In their paper, Cascales, Orihuela and Tkachuk proved that under $\mathrm{MA}(\omega_1)$ a compact space X has a $\mathbb{P}$-diagonal iff it is metrizable. We will prove the following:

CH implies that every compact space with a $\mathbb{P}$-diagonal is metrizable.

Referências

[1] T. Eisworth, *Countable compactness, hereditary π-character, and the continuum hypothesis*, Topology Appl. **153**:18 (2006), 3572–3597.

[2] B. Cascales, J. Orihuela and V. V. Tkachuk, *Domination by second countable spaces and Lindelöf Σ-property*, Topology Appl. **158**:2 (2011), 204–214.

Characterization of linearly Lindelöf topological spaces through family of discrete sets

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Resumo

Let (X, τ) be a topological space, we say that X is linearly Lindelöf, if every open and increasing covering of X admits a countable subcover. Let $\mathscr{A}$ be the family of discrete subsets of X. For any $A \in \mathscr{A}$, we denote: $A^{\perp} = \{x \in X \setminus A : A \cup \{x\} \notin \mathscr{A}\}$. If $A^{\perp} \neq \emptyset$ we can choose a discrete $\emptyset \neq A_1 \subset A^{\perp}$, in general if $\alpha = \theta + 1$ is a successor ordinal and $A_{\theta}^{\perp} \neq \emptyset$ we can choose a discrete $\emptyset \neq A_{\alpha} \subset A_{\theta}^{\perp}$, if κ is a limit ordinal and $\bigcap_{\alpha<\kappa} A_{\alpha}^{\perp} \neq \emptyset$ we can choose a discrete $\emptyset \neq A_{\kappa} \subset \bigcap_{\alpha<\kappa} A_{\alpha}^{\perp}$. If we continue this procedure until an ordinal μ, we have a discrete chain starting at A: $C_A = \{A_{\kappa} : \kappa < \mu\}$. We say that C_A collapses if $\bigcap_{\kappa<\mu} A_{\kappa}^{\perp} = \emptyset$, we also say that μ is the length of the chain. For all well ordered discrete sets $D = \{d_{\alpha} : \alpha < \theta\}$ with $cf(\theta) \geq \omega_1$, we denote $D_{\gamma} = \{d_{\alpha} : \gamma \leq \alpha < \theta\}$, for all $\gamma < \theta$. In this work we characterize linearly Lindelöf topological spaces, via discrete chains, as follows: let X be a topological space T_1, then X is linearly Lindelöf if, and only if, all discrete chain such that the cofinality of its length is greater or equal than ω_1 does not collapse and for all well ordered discrete sets $D = \{d_{\alpha} : \alpha < \theta\}$ with $cf(\theta) \geq \omega_1$, we have $\bigcap\limits_{\gamma<\theta} D_{\gamma}^{\perp} = \emptyset$.

Referências

[1] V. V. Tkachuk, *Spaces that are projective with respect to classes of mappings*, Transactions of Moscow Mathematical Society, **50** (1988) 139–156.

[2] A. Dow, M.G. Tkachenko, V. V. Tkachuk and R. G. Wilson, *Topologies generated by discrete subspaces*, Glasnik Math. J. **37** (2002) 189–212.

[3] O. T. Alas, L. R. Junqueira and R. G. Wilson, *The degree of weakly discretely generated spaces*, Acta Math. Hungar., **143** (2) (2014), 453–465.

[4] A. Bella and P. Simon, *Spaces which are generated by discrete sets*, Topology Appl., **135** (2004), 87–99.

[5] Petra Staynova, *A comparison of Lindelof-type covering properties of topological spaces*, Rose-Hulman Undergraduate Mathematics Journal, volume **12** (2011), arXiv: 1212.2863v1 [math.GN] 12 Dec 2012.

Products of free spaces and applications

Pedro Kaufmann

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Resumo

In recent years, much attention has been dedicated to the so-called *free spaces* over a metric space. These Banach spaces are natural isometric preduals to spaces of Lipschitz functions and encode important geometric properties of the original metric space, in particular concerning optimal transport. Despite of their simple definition, many basic questions on free spaces remain unanswered. In this exposition, we show that the free space over a Banach space X, denoted by $\mathcal{F}(X)$, is isomorphic to the ℓ_1-sum of countable copies of $\mathcal{F}(X)$. As applications, we deduce a non-linear version of Pełczyński's decomposition method for free spaces and identify the free space over any n-dimensional compact riemannian manifold with $\mathcal{F}(\mathbb{R}^n)$, up to isomorphism.

Reflection theorems for local cardinal functions

Alberto Marcelino Efigênio Levi* and Lúcia R. Junqueira

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Resumo

We say that a cardinal function ϕ reflects an infinite cardinal κ, if given a topological space X with $\phi(X) \geq \kappa$, there exists $Y \in [X]^{\leq\kappa}$ with $\phi(Y) \geq \kappa$. We investigate in [6] some problems, discussed by Hodel and Vaughan in [3] and Juhász in [4], related to the reflection for the cardinal functions χ (character) and ψ (pseudocharacter). Among other results, we present some new equivalences with CH, and we use the theory of character and convergence spectra developed in [5] to obtain some results about reflection of ψ in compact Hausdorff spaces.

Referências

[1] F. Casarrubias-Segura and A. Ramírez-Páramo, *Reflection theorems for some cardinal functions*, Topology Proceedings **31** (2007), 51-65.

[2] A. Dow, *An introduction to applications of elementary submodels to topology*, Topology Proceedings **13** (1988), 17-72.

[3] R. E. Hodel and J. E. Vaughan, *Reflection theorems for cardinal functions*, Topology and its Applications **100** (2000), 47-66.

[4] I. Juhász, *Cardinal functions and reflection*, Topology Atlas Preprint n^o 445, 2000.

[5] I. Juhász and W. A. R. Weiss, *On the convergence and character spectra of compact spaces*, Fundamenta Mathematicae **207** (2010), 179-196.

[6] L. R. Junqueira and A. M. E. Levi, *Reflecting character and pseudocharacter*, submitted.

[7] L. R. Junqueira and F. D. Tall, *The topology of elementary submodels*, Topology and its Applications **82** (1998), 239-266.

[8] L. R. Junqueira, *Upwards preservation by elementary submodels*, Topology Proceedings **25** (2000), 225-249.

Non-universality of the group of isometries of the Urysohn-Katětov metric spaces

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Resumo

One of the central observations about the Urysohn universal metric space $\mathbb{U}$ is Uspenskij's result stating that the group $Iso(\mathbb{U})$ is a universal Polish group: every second-countable topological group is isomorphic with a suitable topological subgroup of $Iso(\mathbb{U})$. The question of existence of a universal topological group of a given uncountable weight $\mathfrak{m} > \aleph_0$ remains open.

In this connection, it is rather natural to begin by examining the group of isometries of a non-separable version of the Urysohn space $\mathbb{U}_{\mathfrak{m}}$ constructed by Katětov for every cardinal cardinal $\mathfrak{m}$ such that: $\sup\{\mathfrak{m}^{\mathfrak{n}} \colon \mathfrak{n} < \mathfrak{m}\} = \mathfrak{m}$. We observe that in contrast with Uspenskij's result the group $Iso(\mathbb{U}_{\mathfrak{m}})$ is not a universal group of weight $\mathfrak{m}$ for $\mathfrak{m}$ uncountable.

Automatic continuity for isometry groups

Marcin Sabok

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Resumo

We present a general framework for automatic continuity results for groups of isometries of metric spaces. In particular, we prove automatic continuity property for the groups of isometries of the Urysohn space and the Urysohn sphere, i.e. we show that any homomorphism from either of these groups into a separable group is continuous. This answers a question of Melleray. As a consequence, we get that the group of isometries of the Urysohn space has unique Polish group topology and the group of isometries of the Urysohn sphere has unique separable group topology. Moreover, as an application of our framework we obtain new proofs of the automatic continuity property for the group $\mathrm{Aut}([0,1],\mu)$, due to Ben Yaacov, Berenstein and Melleray and for the unitary group of the infinite-dimensional separable Hilbert space, due to Tsankov. The results and proofs are stated in the language of model theory for metric structures.

Adding pathological exhaustive submeasures

Omar Selim

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Resumo

Maharam's problem is a problem concerning the existence of a very easily described function on the Cantor algebra. It was first asked in 1947 by Dorothy Maharam and turned out to be very difficult. The problem was interesting because it cropped up in many different areas of mathematics. Following sixty years of consistent effort, by many mathematicians, Michel Talagrand settled this problem in the negative. Talagrand's solution is also very difficult! It is still a mystery as to what exactly is the theory concerning the functions considered by Maharam. Consequently, trying to find alternative solutions to Maharam's problem is still a valid research objective. In this talk we present one such attempt. We will show that via the theory of forcing one can add a function very close to the one constructed by Talagrand. We hope to elaborate on this naive approach to provide a new proof of Maharam's problem and hopefully one that is easier to understand. This is (of course) work in progress.

On the extent of separable, locally compact, selectively (a)-spaces

Samuel G. da Silva

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Resumo

The author has recently shown that separable, selectively (a)-spaces cannot include closed discrete subsets of size $\mathfrak{c}$. It follows that, assuming **CH**, separable selectively (a)-spaces have, necessarily, countable extent. However, it was also shown by the author that the weaker hypothesis "$2^{\aleph_0} < 2^{\aleph_1}$" is not enough to ensure countability of the closed discrete subsets of such spaces. In this note we show that, if one adds the hypothesis of local compactness, then a specific effective (meaning, Borel) parametrized weak diamond principle implies countable extent in this context.

Selection principles and chain conditions

L. F. Aurichi, A. Bella, Santi Spadaro* and L. Zdomskyy

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Resumo

We present joint work with Aurichi, Bella and Zdomskyy about chain conditions and their selective versions. For example, we provide:

1. Several partial answers to an old question of Bell, Ginsburg and Woods about the cardinality of weakly Lindelof first-countable regular spaces.
2. Characterizations of certain selective versions of separability and the ccc on spaces of continuous functions and hyperspaces of finite sets.
3. Topological characterizations of a few cardinal invariants of the continuum.

Generalized side conditions

Giorgio Venturi

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Resumo

In this talk I would like to present the method of generalized side conditions, first proposed by Neeman in 2011: a method that allows to give uniform consistency proofs for the existence of objects of size $\aleph_2$. Generally speaking a poset that uses models as side conditions is a notion of forcing whose elements are pairs, consisting of a working part which is some partial information about the object we wish to add and a finite $\in$-chain of elementary substructures of $H(\theta)$ (for some regular cardinal θ) whose main function is to preserve cardinals. I will present in details the pure generalized side conditions poset and I will briefly show how to force, with ?nite conditions, the forcing axiom PFA(T), a relativization of PFA to proper forcing notions preserving a given Souslin tree T. If I have time I will also discuss the possibility to generalize this method and its link with the problem of generalizing Forcing Axioms.

Referências

[1] I. Neeman, *Forcing with sequences of models of two types*, to appear in the Notre Dame Journal of Formal Logic.

[2] B. Veličković and G. Venturi, *Proper forcing remastered*. In *Appalachian Set Theory* (Cummings, Schimmerling, eds.), LMS lecture notes series, 406, 331–361, 2012.

[3] G. Venturi, *Side conditions and Souslin trees*, submitted.