

## Resumos

### Abstracts

Sessão: Álgebra

*Session: Algebra*

Organizadores

*Organizers*

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# Basic superranks for varieties of algebras

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## Resumo

In the present work, all algebras are considered over a field of characteristic 0. Let  $\mathcal{V}$  be a variety of algebras and  $\mathcal{V}_r$  be a subvariety of  $\mathcal{V}$  generated by the free  $\mathcal{V}$ -algebra of rank  $r$ . Then one can consider the chain  $\mathcal{V}_1 \subseteq \mathcal{V}_2 \subseteq \cdots \subseteq \mathcal{V}_r \subseteq \cdots \subseteq \mathcal{V}$ , where  $\mathcal{V} = \bigcup_r \mathcal{V}_r$ . If this chain stabilizes, then the minimal number  $r$  with the property  $\mathcal{V}_r = \mathcal{V}$  is called the *basic rank* of the variety  $\mathcal{V}$  and is denoted by  $r_b(\mathcal{V})$ . Otherwise, we say that  $\mathcal{V}$  has the *infinite basic rank*  $r_b(\mathcal{V}) = \aleph_0$ .

Recall the main results on the basic ranks of the varieties of associative (Assoc), Lie (Lie), alternative (Alt), Malcev (Malc), and some other algebras. It was first shown by A. I. Mal'cev [1] that  $r_b(\text{Assoc}) = 2$ . A. I. Shirshov [2] proved that  $r_b(\text{Lie}) = 2$  and  $r_b(\text{SJord}) = 2$ , where SJord is the variety generated by all special Jordan algebras. In 1958, A. I. Shirshov posed a problem on finding basic ranks for alternative and some other varieties of nearly associative algebras [3, Problem 1.159]. In 1977, I. P. Shestakov proved that  $r_b(\text{Alt}) = r_b(\text{Malc}) = \aleph_0$  [2, 4]. The similar fact for the variety of algebras of type  $(-1, 1)$  was established by S. V. Pchelintsev [5]. Note that the basic ranks of the varieties of Jordan and right alternative algebras are still unknown.

A proper subvariety of associative algebras can be of infinite basic rank as well. For instance, so is the variety  $\text{Var } G$  generated by the Grassmann algebra  $G$  on infinite number of generators, or the variety defined by the identity  $[x, y]^n = 0$ ,  $n > 1$ .

It follows from the Kemer's Theorem [6] that the ideal of identities of arbitrary associative algebra coincides with the ideal of identities of the Grassmann envelope [7] of some finite dimensional superalgebra. This result suggests a generalization of the notion of basic rank that we call basic superrank.

First we consider a number of varieties of nearly associative algebras that have infinite basic ranks and calculate their basic superranks which turns out to be finite. Namely we prove that the variety of alternative metabelian (solvable of index 2) algebras has the two

basic superranks  $(1, 1)$  and  $(0, 3)$ ; the varieties of Jordan and Malcev metabelian algebras have the unique basic superranks  $(0, 2)$  and  $(1, 1)$ , respectively. Furthermore, for arbitrary pair  $(r, s) \neq (0, 0)$  of nonnegative integers we provide a variety that has the unique basic superrank  $(r, s)$ . Finally, we construct some examples of nearly associative varieties that do not possess finite basic superranks.

This is a joint work with Ivan Shestakov.

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# Identidades Polinomiais Graduadas em Álgebras T-primas

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## Resumo

A descrição das graduações por um grupo  $G$  em uma álgebra  $A$  e a descrição das identidades polinomiais graduadas correspondentes são problemas importantes. Neste contexto as álgebras T-primas, introduzidas por Kemer em sua solução para o problema de Specht, admitem graduações naturais. Apresentaremos aqui alguns resultados recentes nesta linha de pesquisa para algumas álgebras T-primas.

# WEAKLY-SKEW IDENTITIES OF THE CAYLEY-DICKSON ALGEBRA

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## **Resumo**

In this work we find all skew identities of the simple Malcev algebra  $\mathfrak{sl}(\mathbb{O})$  and the simultaneously skew and weak identities of the octonion algebra  $\mathbb{O}$ . For those special cases, we show positively the Shestakov-Zhukavets Conjecture. Finally, we show the Shestakov-Zhukavets Conjecture would be false with a slight relaxation of the hypothesis.

# PBW degenerated Demazure modules and Schubert varieties: posets and polytopes

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## Resumo

I will introduce the PBW filtration on cyclic modules for nilpotent complex Lie algebras. The associated graded modules are modules for abelianized versions of the Lie algebras and of some polynomial rings. The first natural question is to ask for monomial bases of these graded modules and I will report here on the state of art, for example on polytopes parametrizing bases. By considering the corresponding flag varieties, one obtains PBW-degenerations and further toric degenerations. I will explain how these degenerations are related to the well-known degenerations in the Gelfand-Tsetlin theory via poset combinatorics. Several open questions will be presented, especially in relations to local Weyl modules of current algebras, fusion product, Schur positivity conjectures.

# Pro- $p$ Completions of Poincaré Duality Groups

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## Resumo

This is a joint work with D.H. Kochloukova (University of Campinas, Brazil) and J.A. Hillman (University of Sydney, Australia) accepted for publication in the Israel Journal of Mathematics (2014). We consider some sufficient conditions for the pro- $p$  completion of an orientable Poincaré duality group of dimension  $n \leq 3$  to be a virtually pro- $p$  Poincaré duality group of dimension at most  $n \leq 2$ .

# Block designs from algebraic point of view

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## Resumo

Steiner triple systems play a major part in combinatorics; many interesting connections have been developed between their combinatorial and algebraic aspects. From this point of view, the study of their algebraic background can be useful.

This generates an interest towards Steiner quasigroups and loops. In this presentation we analyse their multiplication and automorphism groups. Specifically, we discuss which groups can be multiplication groups of Steiner loops (this concept is important for non-associative structures). This question has been solved for several classes of Steiner quasigroups and loops. For example, we prove that all automorphisms of a free Steiner loop (FSL) are tame, and the automorphism group cannot be finitely generated when the loop has more than 3 generators.

The automorphism group of the 3-generated FSL is generated by the symmetric group  $S_3$  and by the elementary automorphism  $\varphi = e_1(x_2)$ . We also conjecture that  $\text{Aut}(S(x_1, x_2, x_3))$  is the Coxeter group  $\langle (12), (13), \varphi | (\varphi(12))^3 = (\varphi(13))^4 = ((12)(13))^3 = 1 \rangle$ . These conjecture fits the context of the work by U. Umirbaev on linear Nielsen-Schreier varieties of algebras.

Furthermore, we discuss their growth and Cayley graphs.



# Profinite permutation $\mathbb{Z}_p$ -lattices for finite $p$ -groups

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## Resumo

For a finite group  $G$ , the representation theory of the integral group ring  $\mathbb{Z}_p G$  is in a formal sense frequently impossible to understand. One therefore tries to deal with a general module  $U$  by approximating it with modules that are “easier”. One such class of modules consists of those  $\mathbb{Z}_p$ -free modules having a  $\mathbb{Z}_p$ -basis that is preserved by the action of  $G$ , the  $\mathbb{Z}_p G$  “permutation lattices”. Already these modules are not well-understood. A theorem of Alfred Weiss from 1988 gives a detection theorem for finitely generated permutation  $\mathbb{Z}_p G$ -lattices when  $G$  is a finite  $p$ -group. We show that the same detection theorem applies for arbitrary profinite  $\mathbb{Z}_p G$ -lattices for finite  $p$ -groups  $G$ .

# Sobre Uma Condição de Finitude para Subgrupo Verbal com Respeito à Palavra de Engel

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## Resumo

Seja  $F$  um grupo livre sobre o conjunto  $\{x_1, x_2, \dots, x_k\}$ , cujos elementos chamamos de variáveis, uma palavra é um expressão da forma

$$w(x_1, x_2, \dots, x_k) = x_{i_1}^{\epsilon_1} \cdot x_{i_2}^{\epsilon_2} \dots x_{i_s}^{\epsilon_s},$$

onde  $i_1, i_2, \dots, i_s \in \{1, \dots, k\}$  e cada  $\epsilon_j$  é  $\pm 1$ . Em um grupo  $G$ , a palavra pode ser vista como uma aplicação de  $G \times \dots \times G$  com  $k$  fatores, onde substituímos as variáveis pelos elementos do grupo. Ao elemento  $w(g_1, \dots, g_k)$ , onde  $g_1, \dots, g_k$  são elementos de  $G$ , chamamos de  $w$ -valor de  $G$ . O subgrupo verbal  $w(G)$  é o subgrupo gerado pelo conjunto  $G_w$  consistindo de todos os  $w$ -valores de  $G$ . Se o subgrupo verbal  $w(G)$  for finito sempre que o conjunto gerador  $G_w$  for finito dizemos que a palavra  $w$  é concisa. Para  $x, y$  em  $G$  definimos  $[x, y] = x^{-1}y^{-1}xy = x^{-1}x^y$ , o comutador de  $x$  e  $y$ . Em [4] foi mencionada a conjectura de P. Hall sobre toda palavra ser concisa e sua prova para uma palavra não comutador, que é a palavra cuja soma dos expoentes resulta em um valor diferente de zero, as palavras derivadas  $\delta_k$  e a central inferior  $\gamma_k$  definidas por  $\gamma_1 = \delta_0 = x$ ,  $\delta_{k+1} = [\delta_k, \gamma_1]$  e  $\delta_{k+1} = [\delta_k, \delta_k]$  também foram provadas serem concisas e, mais tarde, Jeremy Wilson [5] estendeu este resultado para todas as palavras comutadores multilineares (que são as palavras construídas por agrupar os comutadores sempre usando variáveis diferentes).

O problema de P.Hall tem solução positiva para grupos periódicos, pois se  $w$  é uma aplicação finita em  $G$  o subgrupo  $w(G)$  é finito se, e só se, todos os valores de  $w$  em  $G$  são de ordem finita. Enquanto que para grupos livres de torção o problema resume-se a

provar se  $w \equiv 1$  quando  $w$  é aplicação finita. Nesta direção, Ivanov [2] prova que existe um grupo livre de torsão  $G$  com centro cíclico cujo grupo-fator central é um grupo infinito, tal que a palavra  $v(x, y) = [[x^{p^n}, y^{p^n}]^n, y^{p^n}]^n$  possui somente dois valores em  $G$  e o valor não trivial é gerador do centro.

A palavra  $n$ -ésima de Engel  $[x, {}_n y]$  pode ser identificada com os elementos de  $F$  e definida indutivamente por  $[x, {}_0 y] = x$ ;  $[x, {}_n y] = [[x, {}_{n-1} y], y]$ , para todos inteiros positivos  $n$ ,  $n$  é dito comprimento da palavra, e denotamos por  $e_n(G) = \{[g, {}_n h] \mid g, h \in G\}$  o conjunto de todos  $e_k$ -valores de Engel de  $G$ . O subgrupo verbal gerado por  $e_n(G)$  é chamado de  $n$ -ésimo subgrupo verbal de Engel de  $G$  e denotamos aqui por  $E_n(G)$ . Note que, a palavra de Engel não é comutador multilinear se  $n > 1$ , pois as variáveis  $y$  ocorrem mais que uma vez, e o problema de determinar sua concisão está ainda aberto.

Nesta direção de impor certas condições ao conjunto  $G_w$  e verificar em que isso influencia a estrutura do subgrupo  $w(G)$ , em [1] Rogério e Shumyatsky apresentaram o seguinte resultado:

**Teorema 1** *Seja  $k$  um inteiro positivo e  $G$  um grupo em que todos os  $\delta_k$ -comutadores estão contidos em uma união de número finito de subgrupos de Chernikov. Então  $G^{(k)}$  é Chernikov.*

E de modo similiar, estabilizaram o seguinte teorema:

**Teorema 2** *Seja  $k$  um inteiro positivo e  $G$  um grupo em que todos os  $\gamma_k$ -comutadores estão contidos em uma união de número finito de subgrupos de Chernikov. Então  $\gamma_k(G)$  é Chernikov.*

Neste sentido, onde  $w$  é a palavra de Engel, lidaremos com o seguinte resultado:

**Teorema 3** *Seja  $k$  um inteiro positivo e  $G$  um grupo em que todos os  $e_k$ -valores de  $G$  estão contidos em uma união finita de subgrupos de Chernikov. Então  $E_k(G)$  é Chernikov.*

Este Teorema foi resolvido para os casos em que  $G$  é um grupo nilpotente, solúvel e de  $m$ -Engel. Serão apresentados os Lemas que foram pensados para a prova do Teorema no caso geral, onde  $G$  é um grupo arbitrário qualquer.

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# Twisted Hall algebra of bound quiver with small homological dimension

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## Resumo

Let  $Q$  be a quiver,  $\mathfrak{g}$  be a symmetric Kac-Moody algebra associated with  $Q$ , and  $\text{Rep}_Q(\mathbb{F}_q)$  be the category of finite-dimensional representations of  $Q$  over a field  $\mathbb{F}_q$ . In his remarkable papers [1, 2] C. Ringel proved that if  $Q$  is a Dynkin quiver then there exists an isomorphism between the Hall algebra associated with  $\text{Rep}_Q(\mathbb{F}_q)$  and the positive part of quantized universal enveloping algebra  $U_t(\mathfrak{g})$  with  $t^2 = q$ .

We consider the case of a bound quiver  $Q$  assuming that the global dimension of  $Q$  is at most 2. To each such quiver we associate an associative algebra  $U_q(Q)$  given by relations and generators. In the case when  $Q$  is a representation-directed we show that there exists an isomorphism between  $U_q(Q)$  and the corresponding twisted Hall algebra  $\mathbf{H}_{\text{Rep}_Q(\mathbb{F}_q)}^{tv}$ .

As the limiting case of this construction we also study representations of commutative quivers over the so-called field with one element:  $\mathbb{F}_1$ . Such a field is not defined per se, but there is agreement on what should be the definition and basic properties of the category of vector spaces over  $\mathbb{F}_1$  as a limiting case of the categories of vector spaces over  $\mathbb{F}_q$  (see for example [3]). Following the ideas of M. Szczesny we show that the category  $\text{Rep}_Q(\mathbb{F}_1)$  has enough structure to define its Hall algebra and prove that there exists an epimorphism  $\rho : U_1(Q) \rightarrow \mathbf{H}_{\text{Rep}_Q(\mathbb{F}_1)}$ .

Based on joint work [4] with Evan Wilson.

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# Propriedade de Specht das identidades graduadas de álgebras não associativas

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## Resumo

Uma variedade  $\mathcal{V}$  em uma classe de álgebras (associativas, de Lie, de Jordan, etc) tem a propriedade de Specht, se todas as subvariedades de  $\mathcal{V}$  podem ser definidas por um sistema finito de identidades polinomiais. Quando isso ocorre, dizemos também que o ideal das identidades de  $\mathcal{V}$  satisfaz a propriedade de Specht. Em 1987, Kemer provou que toda variedade de álgebras associativas sobre um corpo de característica 0 tem essa propriedade. Se o corpo é infinito e de característica positiva, foram construídos contraexemplos. Para álgebras de Lie e de Jordan, pouco se sabe a respeito em característica 0.

Nesta palestra falaremos da validade da propriedade de Specht em certas variedades de álgebras não associativas graduadas, em característica zero.

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# On locally nilpotent derivations of Fermat Rings

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## Resumo

Let  $\mathbb{C}[X_1, \dots, X_n]$  be the polynomial ring in  $n$  variables over complex numbers  $\mathbb{C}$ . Define

$$B_n^m = \frac{\mathbb{C}[X_1, \dots, X_n]}{(X_1^m + \dots + X_n^m)},$$

where  $m \geq 2$  and  $n \geq 3$ . This ring is known as Fermat ring.

In a recent paper [4] D. Fiston and S. Maubach show that for  $m \geq n^2 - 2n$  the unique locally nilpotent derivation of  $B_n^m$  is the zero derivation. Consequently the following question naturally arises: is the unique locally nilpotent derivation of the Fermat ring  $B_n^m$  for  $m \geq 2$  and  $n \geq 3$  the zero derivation?

In the paper [1] we show that the answer to this question is negative for  $m = 2$  and  $n \geq 3$ . In other words, there exist locally nilpotent derivations over  $B_n^2$  nontrivial. Furthermore, we show that these derivations are irreducible. In the general case, we prove that for certain classes of derivations of  $B_n^m$  the unique locally nilpotent derivation is the zero derivation.

The question remains open for the case  $m \geq 3$ .

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# Conjugacy Classes of Torsion Elements in the Crystallographic Group $B_n/[P_n, P_n]$

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## Resumo

Let  $B_n$  (resp.  $P_n$ ) denote the Artin braid group (resp. the Artin pure braid group) with  $n$  strings and let  $n \geq 3$ . We show that the quotient  $\frac{B_n}{[P_n, P_n]}$  is a crystallographic group, where  $[P_n, P_n]$  means the commutator subgroup of  $P_n$ . This quotient has torsion elements in contrast to the (pure) braid groups  $P_n$  and  $B_n$ . We classify the torsion elements and its conjugacy classes in the crystallographic group  $\frac{B_n}{[P_n, P_n]}$ . Finally, for  $n \leq 7$  we show that  $\frac{B_n}{[P_n, P_n]}$  does not have non-abelian finite subgroups. The case  $n > 7$  seems to be an open question, or possibly the classification of all non-abelian subgroups of  $B_n/[P_n, P_n]$  for  $n > 7$  can be more general.

# On profinite groups with Engel-like conditions

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## Resumo

The positive solution of the restricted Burnside problem had important consequences in the study of profinite groups. For example, using Wilson's reduction theorem [W], Zelmanov has been able to prove local finiteness of profinite periodic groups [Z]. Later Wilson and Zelmanov [WZ] used the result to prove that every Engel profinite group is locally nilpotent. In this work we will show certain situations where we can guarantee the local virtuality of profinite groups.

Our work is motivated by a result of Shumyatsky [S], which shows that if  $G$  is a finitely generated profinite group in which  $[x_1, \dots, x_k]$  are Engel for every  $x_1, \dots, x_k \in G$ , then the  $k$ th term of the lower central series  $\gamma_k(G)$  is locally nilpotent.

In this work we will discuss the following results:

**Theorem A.** ([BS]). *Let  $G$  be a profinite group in which for every element  $x \in G$  there exists a natural number  $q = q(x)$  such that  $x^q$  is Engel. Then  $G$  is locally virtually nilpotent.*

**Theorem B.** ([BS]). *Let  $p$  be a prime and  $G$  a finitely generated profinite group in which for every  $\gamma_k$ -value  $x \in G$  there exists a natural  $p$ -power  $q = q(x)$  such that  $x^q$  is Engel. Then  $\gamma_k(G)$  is locally virtually nilpotent.*

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# Determination of the 2- cocycles for the three point Witt algebra

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## Resumo

We provide formulas for computing the cocycles on a 3-point Witt algebra  $Der(R)$ , using an isomorphism between two 3-point algebras  $Der(R)$  and  $Der(S)$ , where the cocycle is already defined. These cocycles can be used to construct universal central extensions and the 3-point Virasoro, which are useful for the representation theory of a 3-point current algebra. The computations determining the cocycles on  $Der(R)$  involve elegant applications of the Chu-Vandermonde convolution and other identities for sums of binomial coefficients.

# The cohomology ring of the sapphires that admit the Sol geometry

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## Resumo

Let  $G$  be the fundamental group of a sapphire that admits the *Sol* geometry and is not a torus bundle. We determine a finite free resolution of  $\mathbb{Z}$  over  $\mathbb{Z}G$  and calculate a partial diagonal approximation for this resolution. We also compute the cohomology rings  $H^*(G; A)$  for  $A = \mathbb{Z}$  and  $A = \mathbb{Z}_p$  for an odd prime  $p$ , and indicate how to compute the groups  $H^*(G; A)$  and the multiplicative structure given by the cup product for any system of coefficients  $A$ .

# Fibrados $p$ -Buchsbaum de posto 2 sobre o espaço projetivo

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## Resumo

Foi demonstrado várias vezes que um fibrado normalizado, 1-Buchsbaum e de posto 2 sobre  $P^3$  é o fibrado de correlação nula, e um fibrado normalizado, 2-Buchsbaum e de posto 2 sobre  $P^3$  é um instanton de carga 2. Mostraremos que a mesma relação não é verdadeira para  $p$ -Buchsbaum, com  $p$  maior o igual do que 3. Proporemos uma conjectura sobre a classificação dos fibrados 3-Buchsbaum.

# Characters of certain finite-dimensional modules for hyper current algebras

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## Resumo

This talk is concerned with the study of certain classes of modules for hyper algebras of current algebras. A hyper algebra is a Hopf algebra associated to a Lie algebra, similar to its universal enveloping algebra, and obtained from it by first choosing a certain integral form and then changing scalars. They provide a way to pass from a category of modules for a Lie algebra over an algebraically closed field of characteristic zero to its analog in positive characteristic. If the underlying Lie algebra is simply laced, we show that local Weyl modules are isomorphic to certain Demazure modules, extending to positive characteristic a result due to Fourier-Littelmann. More generally, we extend a result of Naoi by proving that local Weyl modules admit a Demazure flag, i.e., a filtration with factors isomorphic to Demazure modules. Using this, we prove a conjecture of Jakelić-Moura stating that the character of local Weyl modules for hyper loop algebras are independent of the (algebraically closed) ground field. This is a joint work with A. Bianchi and A. Moura.



# A characterization of non-matrix varieties for Jordan and alternative algebras

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## Resumo

A variety  $\mathcal{M}$  of associative algebras (over a field  $F$ ) is called “non-matrix” if  $F_2 \notin \mathcal{M}$ , where  $F_2$  is the usual matrix algebra of second order. Concerning this definition, other equivalent characterizations for a non-matrix variety were obtained, for instance, by considering algebraic (Cekanu, 1979) and nilpotent (Mishchenko et al., 2012) elements.

However, the theory of varieties of algebras is not restricted to the class of associative algebras. In addition to the Lie algebras, among many classes of non associative algebras, we highlight the alternative and Jordan algebras. These classes of algebras have many connexions and applications to several areas of Mathematics and Physics and have a well-developed structural theory, as in the class of associative algebras.

The concept “non-matrix variety” can be reformulated for alternative or Jordan algebras and our work is to adapt, extend or generalize some results, as mentioned above, for non-matrix varieties in these classes of algebras.

## ON $\mathbb{Z}_2$ -GRADED IDENTITIES OF $UT_2(E)$

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### Resumo

Let  $F$  be an infinite field of characteristic different from two and  $E$  be the infinite dimensional Grassmann algebra over  $F$ . Consider the upper triangular matrix algebra  $UT_2(E)$  with entries in  $E$  endowed with the  $\mathbb{Z}_2$ -grading inherited by the natural  $\mathbb{Z}_2$ -grading over  $E$ . In this talk we will show some recent results about the ideal of  $\mathbb{Z}_2$ -graded polynomial identities ( $T_{\mathbb{Z}_2}$ -ideal) of  $UT_2(E)$  and its relatively free algebra. In particular we show that the set of  $\mathbb{Z}_2$ -graded polynomial identities of  $UT_2(E)$  does not depend on the characteristic of the field. This is a joint work with Prof. Lucio Centrone (UNICAMP).

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