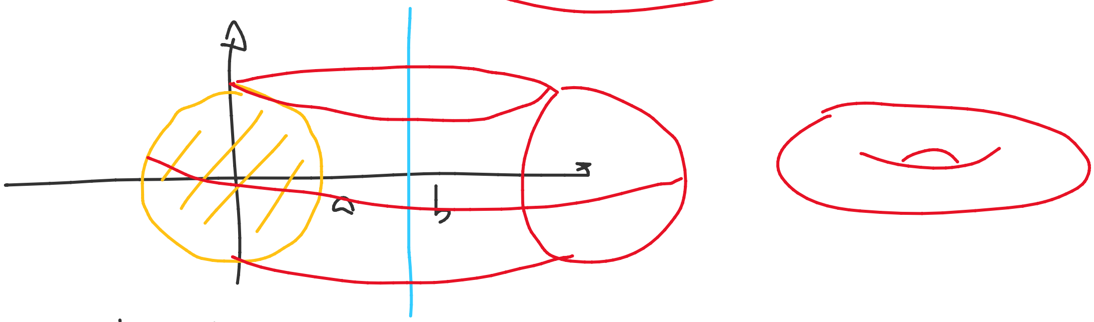
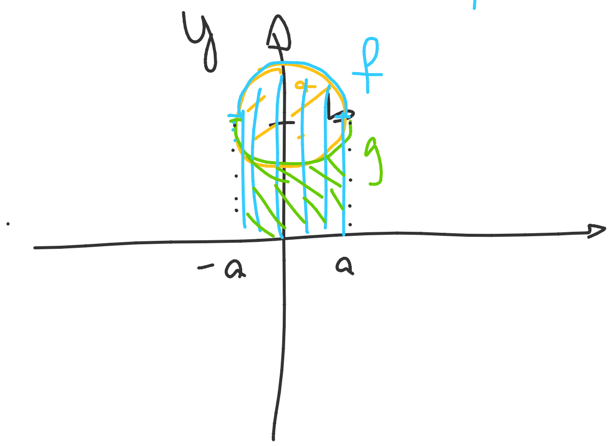
Exercícios 1

Lista 6

9. O disco $x^2 + y^2 \leq a^2$ é girado em torno da reta $x = b$, com $b > a$, para gerar um sólido com a forma de um pneu. Esse sólido é chamado **toro**. Calcule seu volume. (Sugestão: Note que $\int_{-a}^{a} \sqrt{a^2 - y^2}dy = \frac{\pi a^2}{2}$.)



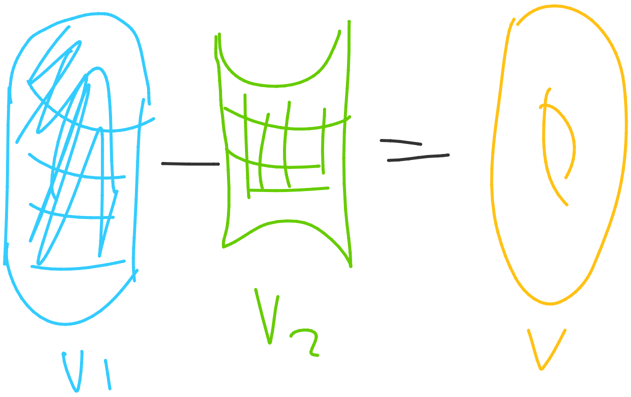


$$(x-0)^2 + (y-b)^2 = a^2$$

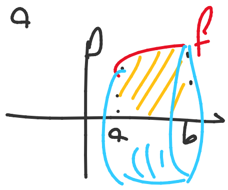
$$(y-b)^2 = a^2 - x^2$$

$$y = b \pm \sqrt{a^2 - x^2}$$

$$\Rightarrow \begin{cases} f(x) = b + \sqrt{a^2 - x^2} \\ g(x) = b - \sqrt{a^2 - x^2} \end{cases}$$



$$V = \pi \int_a^b h^2(x)\, dx$$



$$V = \pi \int_{-a}^{a} f^2(x)\, dx - \pi \int_{-a}^{a} g^2(x)\, dx$$

$$V = 2\pi \int^{a} \left(f^2(x) - g^2(x) \right) dx$$

$$V = 2\pi \int_0^a \left[\left(b+\sqrt{a^2-x^2}\right)^2 - \left(b-\sqrt{a^2-x^2}\right)^2 \right] dx$$

$$V = 2\pi \int_0^a \left(\cancel{b}+\sqrt{a^2-x^2}\cancel{-b}+\sqrt{a^2-x^2}\right)\left(b+\cancel{\sqrt{a^2-x^2}}+b-\cancel{\sqrt{a^2-x^2}}\right) dx$$

$$V = 2\pi \int_0^a 2\sqrt{a^2-x^2}\, 2b\, dx$$

$$V = 2\pi \cancel{2} \cdot 2b \underbrace{\int_{-a}^{a} \sqrt{a^2-x^2}\, dx}_{\frac{\pi a^2}{2}} = 4b\pi \cdot \frac{\pi a^2}{2}$$

h par $\Rightarrow \int_{-\pi}^{\pi} h(x)\, dx = 2 \int_0^{\pi} h(x)\, dx$

Lista 4

7. Seja $f(x) = x^7 + \pi x^3 - 8x^2 + ex + 1$. Quantas soluções distintas tem a equação $f''(x) = 0$? Mostre que a equação $f(x) = 0$ tem exatamente três soluções reais distintas.

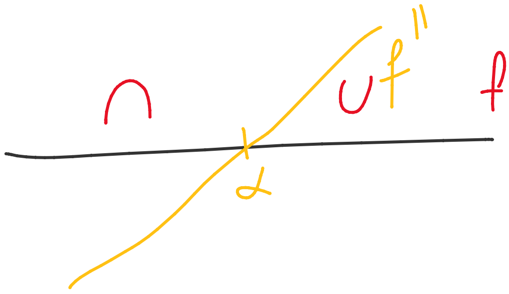
$f'(x) = 7x^6 + 3\pi x^2 - 16x + e$

$f''(x) = 42x^5 + 6\pi x - 16$

" exatamente

$f''(x) = 42x^5 + 6\pi x - 16$

$f'''(x) = 210x^4 + 6\pi > 0, \ \forall x \Rightarrow f''$ é estritamente crescente



$\Rightarrow f''$ tem apenas uma raiz real

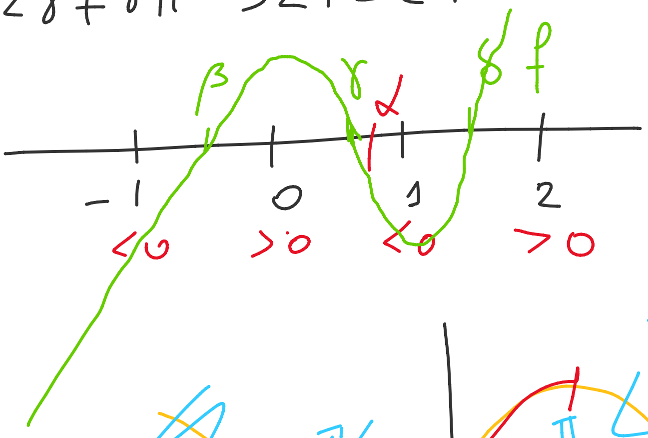
$f(x) = x^7 + \pi x^3 - 8x^2 + ex + 1$

$f(-1) = -1 - \pi - 8 - e + 1 < 0$

$f(0) = 1 > 0$

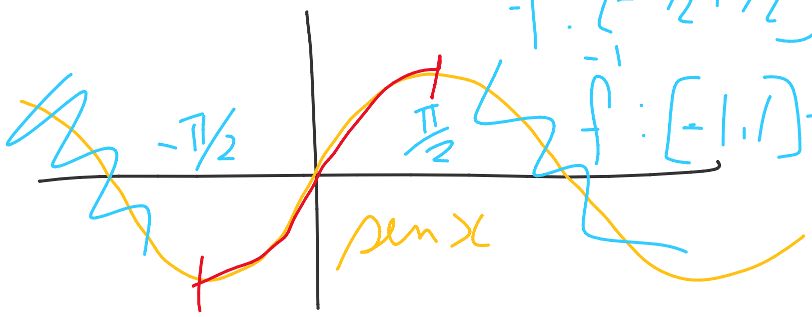
$f(1) = 1 + \pi - 8 + e + 1 < 0$

$f(2) = 128 + 8\pi - 32 + 2e + 1 > 0$



$f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to [-1, 1]$

$f^{-1} : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



Lista 5

40. $\displaystyle\int \frac{dx}{(1+x^2)\sqrt{1-x^2}} = \int \frac{\cancel{\cos u}}{(1+\operatorname{sen}^2 u)\cancel{\cos u}}\,du = \int \frac{1}{1+\operatorname{sen}^2 u}\,du =$

$x = \operatorname{sen} u \Rightarrow u = \operatorname{arcsen} x$

$dx = \cos u \, du$

$$= \int \frac{\frac{1}{\cos^2 u}}{\frac{1+\sin^2 u}{\cos^2 u}}\, du = \int \frac{\sec^2 u\, du}{\sec^2 u + tg^2 u} = \int \frac{\sec^2 u\, du}{1+tg^2 u + tg^2 u}$$

$$= \int \frac{\sec^2 u\, du}{1+2tg^2 u} = \int \frac{dv}{1+2v^2} = \int \frac{1}{1+(\sqrt{2}v)^2}\, dv =$$

$v = tg\, u$
$dv = \sec^2 u\, du$

$\sqrt{2}v = w$
$dv = \frac{1}{\sqrt{2}}\, dw$

$$= \int \frac{1}{1+w^2} \frac{1}{\sqrt{2}}\, dw = \frac{1}{\sqrt{2}} \operatorname{arctg} w + k$$

$$= \frac{1}{\sqrt{2}} \operatorname{arctg}(\sqrt{2}\, v) + k = \frac{1}{\sqrt{2}} \operatorname{arctg}(\sqrt{2}\, tg\, u) + k$$

$$= \frac{1}{\sqrt{2}} \operatorname{arctg}\left(\sqrt{2}\, tg(\operatorname{arcsen} x)\right) + k$$

$$\int \frac{1}{1+ax^2}\, dx = \int \frac{1}{1+(\sqrt{a}x)^2}\, dx =$$

$$\boxed{\int \frac{1}{1+y^2}\, dy = \operatorname{arctg}(y) + k}$$

Lista 4

Regras de L'Hospital

Lista 4

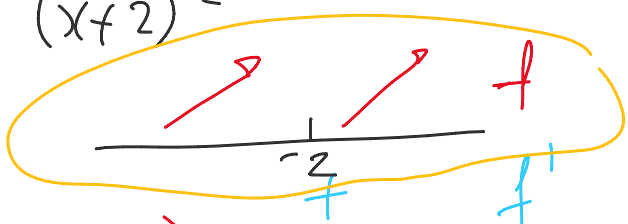
13. Esboce o gráfico das funções abaixo e dê as equações das assíntotas, quando existirem.

(c) $f(x) = \dfrac{2x^2+3x-8}{x+2}$

$$D_f = \mathbb{R} \setminus \{-2\}$$

$$f'(x) = \frac{(4x+3)(x+2)-(2x^2+3x-8)}{(x+2)^2}$$

$$f'(x) = \frac{4x^2+\cancel{3x}+8x+16-2x^2-\cancel{3x}+8}{(x+2)^2} = \frac{2x^2+8x+14}{(x+2)^2}$$



$$f''(x) = \frac{(4x+8)(x+2)^{\cancel{2}} - (2x^2+8x+14)\,2\,\cancel{(x+2)}}{(x+2)^{\cancel{4}\,3}}$$

$$f'(x) = \frac{\ldots}{(x+2)^{\cancel{4}\,3}}$$

$$f''(x) = \frac{\cancel{4x^2} + \cancel{8x} + \cancel{8x} + 16 - \cancel{4x^2} - 16x - 28}{(x+2)^3} = \frac{-12}{(x+2)^3}$$

$$\begin{array}{ccccc} & \cup & & \cap & f \\ \hline & + & -2 & - & f'' \end{array}$$

$$\lim_{x \to -2^-} f(x) =$$

$$\lim_{x \to -\infty} \frac{f(x)}{x} = 2 = m$$

$$\lim_{x \to -\infty} \left[f(x) - mx\right] = \lim_{x \to -\infty} \left[\frac{2x^2 + 3x + 8}{x+2} - 2x\right] =$$

$$= \lim_{x \to -\infty} \left[\frac{\cancel{2x^2} + 3x + 8 - \cancel{2x^2} - 4x}{x+2}\right] = -1 = n$$

$$y = 2x - 1$$

0 $\quad \ln\left((x^2)^2\right) 2x$

5. Calcule $\lim\limits_{x\to 0} \dfrac{\int_0^{x^2} \cos(t^2)\,dt}{\int_0^x e^{-t^2}\,dt}$. $\underset{L'H}{\overset{0/0}{=}} \lim\limits_{x\to 0} \dfrac{\cos((x^2)^2)\cdot 2x}{e^{-x^2}}$

$$= \lim_{x\to 0} \frac{2x \cancel{\cos x^4}^{1}}{\cancel{e^{-x^2}}_{1}} = 0$$

$$F(x) = \int_0^{\sqrt{x}} \operatorname{tg} t^3\,dt$$

$$G(y) = \int_0^y \operatorname{tg}(t^3)\,dt \Rightarrow G'(y) = \operatorname{tg}(y^3)$$

$$F(x) = G(\sqrt{x}) \Rightarrow F'(x) = G'(\sqrt{x})(\sqrt{x})'$$

$$= G'(\sqrt{x})\frac{1}{2\sqrt{x}}$$

$$F'(x) = \operatorname{tg}((\sqrt{x})^3)\cdot\frac{1}{2\sqrt{x}}$$

$$F(x) = \int_0^{x^2} \cos(t^2)\,dt$$

$$G(y) = \int_0^y \cos(t^2)\,dt \Rightarrow G'(y) = \cos(y^2)$$

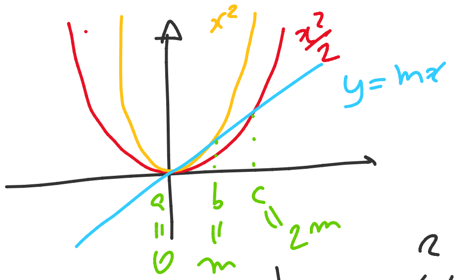
$$F(x) = G(x^2) \Rightarrow F'(x) = G'(x^2)\,2x$$

$$F(x) = G(x^2) \Rightarrow F'(x) = G'(x^2)\,2x$$

$$= \cos((x^2)^2)\,.\,2x$$

Lista 5

3. Determine $m > 0$ para que a área delimitada por $y = x^2$, $y = \frac{x^2}{2}$ e a reta $y = mx$ seja igual a 4.



$$\begin{cases} y = x^2 \\ y = mx \end{cases} \Rightarrow x^2 = mx$$

$$x(x-m) = 0 \quad \begin{cases} x = 0 \\ \text{ou} \\ x = m \end{cases}$$

$$\begin{cases} y = x^2/2 \\ y = mx \end{cases} \Rightarrow \frac{x^2}{2} = mx$$

$$x(x - m2) = 0 \quad \begin{cases} x = 0 \\ \text{ou} \\ x = 2m \end{cases}$$

$$A = \int_0^m \left(x^2 - \frac{x^2}{2}\right) dx + \int_m^{2m} \left(mx - \frac{x^2}{2}\right) dx = 4$$

$$A = \left[\frac{x^3}{6}\right]_0^m + \left[m\frac{x^2}{2} - \frac{x^3}{6}\right]_m^{2m} = 4$$

Lista 6

6. Mostre que $f(x) = \int_0^{1/x} \frac{1}{t^2+1}\,dt + \int_0^{x} \frac{1}{t^2+1}\,dt$ é constante em $(0,\infty)$. Qual o valor dessa constante?

$$f'(x) = \frac{1}{\left(\frac{1}{x}\right)^2+1}\cdot\left(-\frac{1}{x^2}\right) + \frac{1}{x^2+1} = \frac{1}{\frac{1}{x^2}+1}\left(-\frac{1}{x^2}\right) + \frac{1}{x^2+1} =$$

$$= \frac{-1}{\frac{1+x^2}{\cancel{x^2}}}\cdot\frac{1}{\cancel{x^2}} + \frac{1}{x^2+1} = -\frac{1}{1+x^2} + \frac{1}{x^2+1} = 0,\ \forall x>0$$

$\Rightarrow f$ é cte.

$$f(1) = \int_0^1 \frac{1}{t^2+1}\,dt + \int_0^1 \frac{1}{t^2+1}\,dt = 2\int_0^1 \frac{1}{1+t^2}\,dt =$$

$$= \operatorname{arctg}(t)\Big|_0^1 = \operatorname{arctg}(1) - \operatorname{arctg}(0)$$

$$= \pi/4 - 0 = \pi/4$$

$$\Rightarrow f(x) = \pi/4,\ \forall x>0$$

Lista 5

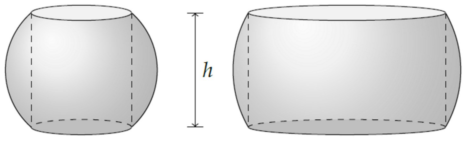
impar

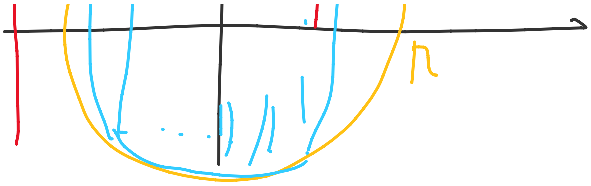
Questão 1. O valor de $\int_{-\pi/4}^{\pi/4} \left(\cos t + \underbrace{\sqrt{1+t^2}}_{\text{par}}\ \underbrace{\sin^3 t}_{\text{impar}}\ \underbrace{\cos^3 t}_{\text{par}}\right) dt$ é:

(a) $\sqrt{2}$; (b) $\frac{\sqrt{2}}{2}$; (c) $-\sqrt{2}$; (d) $-\frac{\sqrt{2}}{2}$; (e) 0.

Lista 6

12. Um anel esférico é o sólido que permanece após a perfuração de um buraco cilíndrico através do centro de uma esfera sólida. Se a esfera tem raio R e o anel esférico tem altura h, prove o fato notável de que o volume do anel depende de h, mas não de R.

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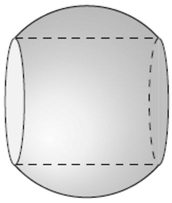


y
x