Técnicas de primitivação (T5)

I- <u>Substituição</u>: integral da forma $\int f(g(x))g'(x)dx$

$$F(g(x))+k=\int (F(g(x)))'\,dx=\int F'(g(x))\,g'(x)\,dx$$

<u>Exemplos</u>:

a) $$\int x\cos x^2\,dx=\int \cos x^2 \cdot x\,dx=\int (\cos u)\frac{1}{2}\,du=\frac{1}{2}\,\mathrm{sen}\,u+k=\frac{1}{2}\,\mathrm{sen}\,x^2+k$$

$x^2=u$
$2x\,dx=du$

b) $$\int \frac{x}{x^2+1}\,dx=\int \frac{\frac{1}{2}du}{u}=\frac{1}{2}\int \frac{1}{u}\,du=\frac{1}{2}\ln|u|+k=\frac{1}{2}\ln(x^2+1)+k$$

$x^2+1=u$
$2x\,dx=du$

c) $$\int \mathrm{sen}^4x\cos^3x\,dx=\int \mathrm{sen}^4x\cos^2x\cos x\,dx=\int \mathrm{sen}^4x(1-\mathrm{sen}^2x)\cos x\,dx$$

$u=\mathrm{sen}\,x$
$du=\cos x\,dx$

$$=\int u^4(1-u^2)\,du=\int (u^4-u^6)\,du=\frac{u^5}{5}-\frac{u^7}{7}+k=\frac{\mathrm{sen}^5x}{5}-\frac{\mathrm{sen}^7x}{7}+k$$

d) $$\int \mathrm{tg}\,x\,dx=\int \frac{\mathrm{sen}\,x}{\cos x}\,dx=\int \frac{-du}{u}=-\int \frac{1}{u}\,du=-\ln|u|+k=-\ln|\cos x|+k$$

$u=\cos x$
$du=-\mathrm{sen}\,x\,dx$

e) $$\int \sec x\,dx=\int \frac{\sec x}{1}\,\frac{(\sec x+\mathrm{tg}\,x)}{\sec x+\mathrm{tg}\,x}\,dx=\int \frac{\sec^2x+\sec x\,\mathrm{tg}\,x}{\sec x+\mathrm{tg}\,x}\,dx$$

$$=\int \frac{du}{u}=\ln|u|+k=\ln|\sec x+\mathrm{tg}\,x|+k$$

$u=\mathrm{tg}\,x+\sec x$
$du=(\sec^2x+\sec x\,\mathrm{tg}\,x)\,dx$

II- Por partes:

$$(f\cdot g)' = f'g + g'f$$
$$fg' = (fg)' - f'g$$
$$\int fg' = \int (fg)' - \int f'g$$

$u = f(x) \qquad v = g(x)$

$du = f'(x)\,dx \qquad dv = g'(x)\,dx$

$$\boxed{\int fg' = fg - \int gf'}$$
$$\boxed{\int f(x)g'(x)dx = f(x).g(x) - \int g(x)f'(x)dx}$$
$$\boxed{\int u\,dv = uv - \int v\,du}$$

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx \qquad \int u\,dv = uv - \int vdu$$

Exemplos:

a) $\int \underbrace{x}_{u} \underbrace{\cos x\ dx}_{dv} = x\,\mathrm{sen}\,x - \int \mathrm{sen}\,x\,dx = x\,\mathrm{sen}\,x + \cos x + k$

$$\boxed{\begin{matrix} u = x & \Rightarrow du = dx \\ dv = \cos x\,dx \Rightarrow & v = \mathrm{sen}\,x \end{matrix}}$$

b) $\int x^2\ \mathrm{sen}\,x\ dx = x^2(-\cos x) - \int (-\cos x)2x\,dx = -x^2\cos x + 2\int x\cos x\,dx$

$$\boxed{\begin{matrix} u = x^2 & du = 2x\,dx \\ dv = \mathrm{sen}\,x\,dx & v = -\cos x \end{matrix}} \qquad \begin{matrix} u = x & du = dx \\ dv = \cos x\,dx & v = \mathrm{sen}\,x \end{matrix}$$

$$= -x^2\cos x + 2\left[x\,\mathrm{sen}\,x + \cos x\right] + C$$

c) $I = \int e^x\ \mathrm{sen}\,x\ dx = -e^x\cos x - \int -\cos x . e^x dx = -e^x\cos x + \int e^x\cos x\,dx$

$$\boxed{\begin{matrix} u = e^x & du = e^x dx \\ dv = \mathrm{sen}\,x\,dx & v = -\cos x \end{matrix}}$$

$$= -e^x\cos x + e^x\,\mathrm{sen}\,x - \underbrace{\int \mathrm{sen}\,x\, e^x dx}_{I}$$

$$\begin{matrix} u = e^x & du = e^x dx \\ dv = \cos x\,dx & v = \mathrm{sen}\,x \end{matrix}$$

$$I = -e^x\cos x + e^x\,\mathrm{sen}\,x - I \Rightarrow 2I = e^x(\mathrm{sen}\,x - \cos x) + k$$

$$\int e^x\,\mathrm{sen}\,x\,dx = I = \frac{1}{2}e^x(\mathrm{sen}\,x - \cos x) + \frac{k}{2}$$

d) $\int x \ln x \; dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx$

$$\boxed{\begin{array}{ll} u = \ln x & du = \frac{1}{x}\,dx \\ dv = x\,dx & v = \frac{x^2}{2} \end{array}}$$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + k$

e) $\int \ln x \; dx = x\ln x - \int x\cdot\frac{1}{x}\,dx = x\ln x - \int 1\,dx = x\ln x - x + k$

$$\boxed{\begin{array}{ll} u = \ln x & du = \frac{1}{x}\,dx \\ dv = dx & v = x \end{array}}$$

$\int \sec x\,dx = \ln|\sec x - \operatorname{tg} x| + c$

f) $\underbrace{\int \sec^3 x \; dx}_{= I} = \int \sec x \sec^2 x\,dx = \sec x \operatorname{tg} x - \int \sec x \operatorname{tg} x \operatorname{tg} x\,dx$

$$\boxed{\begin{array}{ll} u = \sec x & du = \sec x \operatorname{tg} x\,dx \\ dv = \sec^2 x\,dx & v = \operatorname{tg} x \end{array}}$$

$= \sec x \operatorname{tg} x - \int(-\sec x + \sec^3 x)\,dx = \sec x \operatorname{tg} x + \int \sec x\,dx - \underbrace{\int \sec^3 x\,dx}_{I}$

$I = \frac{1}{2}\left[\sec x \operatorname{tg} x + \int \sec x\,dx\right] = \frac{1}{2}\left[\sec x \operatorname{tg} x + \ln|\sec x + \operatorname{tg} x|\right] + k$