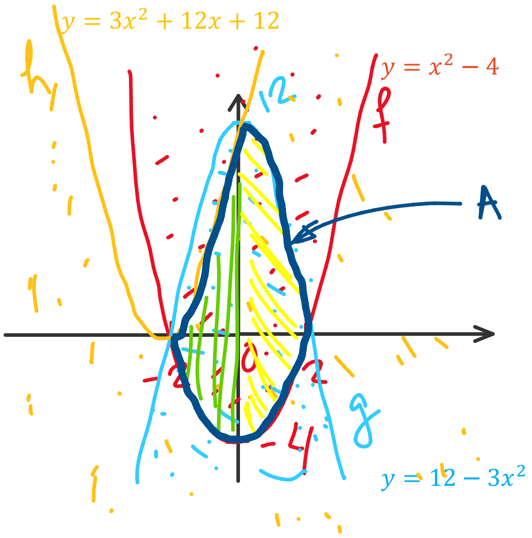
Integral de Riemann

<u>Exercício</u>: Desenhe a região $A$ e calcule a sua área:

a) $A = \{(x,y) \in \mathbb{R}^2 : y \geq x^2 - 4, y \leq 12 - 3x^2 \text{ e } y \leq 3x^2 + 12x + 12\}$




$$y = x^2 - 4 = (x-2)(x+2)$$

$$y = 12 - 3x^2 = 3(2-x)(2+x)$$

$$y = 3x^2 + 12x + 12 = 3(x^2+4x+4) = 3(x+2)^2$$

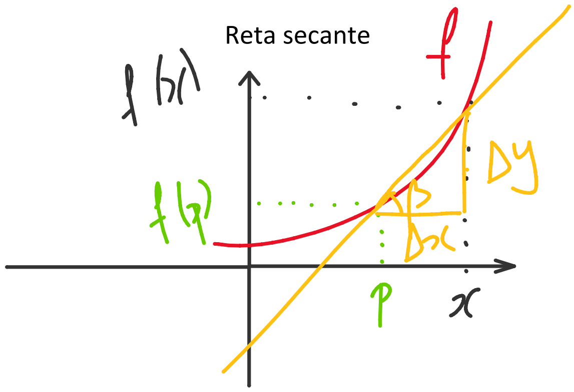
$$A = \int_{-2}^{0} (h(x) - f(x))\,dx + \int_{0}^{2} (g(x) - f(x))\,dx$$

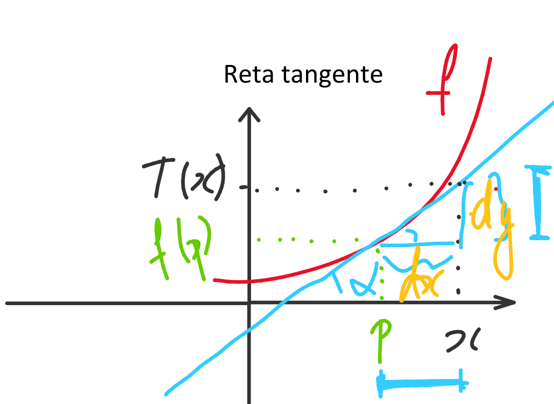
$$= \int_{-2}^{0} \left[(3x^2+12x+12) - (x^2-4)\right] dx + \int_{0}^{2} \left[(12-3x^2) - (x^2-4)\right] dx = \int_{-2}^{0} \left[2x^2 + 12x + 16\right] dx + \int_{0}^{2} \left[-4x^2 + 16\right] dx$$

$$= \left[\frac{2x^3}{3} + \frac{12x^2}{2} + 16x\right]_{-2}^{0} + \left[-\frac{4x^3}{3} + 16x\right]_{0}^{2} = \left[(0) - \left(\frac{2(-2)^3}{3} + \frac{12(-2)^2}{2} + 16(-2)\right)\right] + \left[\left(-\frac{4 \cdot 2^3}{3} + 16(2)\right) - (0)\right]$$

$$= \frac{16}{3} - 24 + 32 - \frac{32}{3} + 32 = -\frac{16}{3} + 40 = \frac{104}{3}$$

# Diferencial




$$tg\,\alpha = \frac{dy}{dx}$$

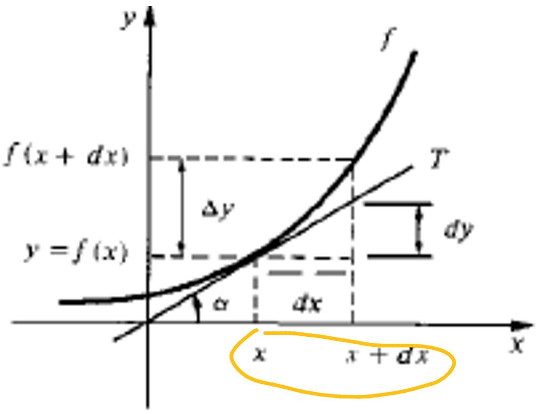
$$tg\,\beta = \frac{\Delta y}{\Delta x} = \frac{f(x)-f(p)}{x-p}$$

$$f'(p) = \lim_{x \to p} \frac{f(x)-f(p)}{x-p}$$

$$f'(p) = tg\,\alpha, \qquad \lim_{x \to p} \frac{f(x)-f(p)}{x-p} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

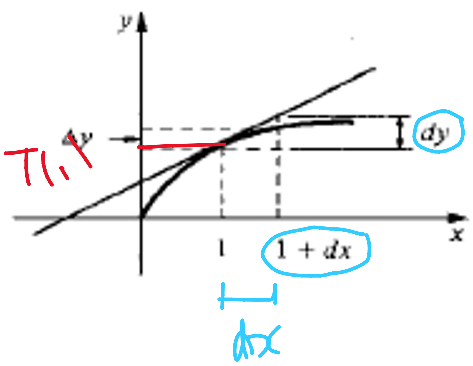
$$tg\,\alpha = f'(p) = \frac{dy}{dx}$$

$$dy = f'(p)\,dx$$




Ou seja $y = f(x) \Rightarrow dy = f'(x)\,dx$

Exercício: Utilizando diferencial, calcule um valor aproximado de $\sqrt{1{,}01}$.




$y = f(x) = \sqrt{x}$

$x_0 = 1$

$dx = 0{,}01$

$dy = f'(x_0).dx$

$dy = \frac{1}{2} 0{,}01 = 0{,}005$

$\sqrt{1{,}01} \simeq T(1) + dy = 1 + 0{,}005 = 1{,}005$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(x_0) = f'(1) = \frac{1}{2\sqrt{1}} = \frac{1}{2}$

Na calculadora temos que $\sqrt{1{,}01} \simeq 1{,}0049875621$

Observação: Considere a equação $x^2 + x = \text{sen}(u)$, calcular $\frac{d}{dx}(x^2 + x) = \frac{d}{du}(\text{sen}(u))$ é equivalente a calcular $\frac{du}{dx}$, onde $u = \text{arcsen}(x^2 + x)$

De fato, por um lado, temos: $\frac{d}{dx}(x^2+x) = \frac{d}{du}(\text{sen}\, u) \Rightarrow (2x+1)\,dx = \cos u\, du$

$$\Rightarrow \frac{du}{dx} = \frac{2x+1}{\cos u}, \text{ onde } u = \text{arc sen}(x^2+x)$$

$$\Rightarrow \frac{du}{dx} = \frac{2x+1}{\cos(\text{arcsen}(x^2+x))}$$

$$y = \cos(\text{arc sen}(x^2+x)) \geq 0$$

$$y^2 = \cos^2(\text{arc sen}(x^2+x)) = 1 - \text{sen}^2(\text{arc sen}(x^2+x))$$

$$y^2 = 1-(x^2+x)^2 \Rightarrow y = +\sqrt{1-(x^2+x)^2} \Rightarrow \frac{dx}{du} = \frac{2x+1}{\sqrt{1+(x^2+x)^2}} = (*)$$

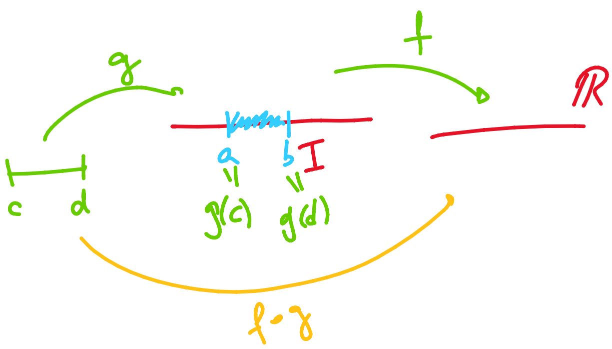
Por outro lado, temos $u = \text{arcsen}(x^2+x) \Rightarrow \frac{du}{dx} = \frac{1}{\sqrt{1-(x^2+x)^2}}(2x+1) = (**)$

Ou seja, $(*) = (**)$

# Mudança de variável na integral definida

Teorema: Seja $f$ contínua num intervalo $I$ e sejam $a$ e $b$ dois reais quaisquer em $I$. Seja $g: [c, d] \longrightarrow I$, com $g'$ contínua em $[c, d]$, tal que $g(c) = a$ e $g(d) = b$. Nestas condições

$$\int_a^b f(x)\,dx = \int_c^d f(g(u))g'(u)du.$$



Exercício: Calcule

a) $\int_1^2 (x-2)^5\,dx = \int_{-1}^{0} u^5\,du = \left.\frac{u^6}{6}\right|_{-1}^{0} = \left(\frac{0^6}{6}\right) - \left(\frac{(-1)^6}{6}\right) = 0 - \frac{1}{6} = -\frac{1}{6}$

$x - 2 = u$

$g(u) = x = u + 2$

$1\,dx = 1\,du$

$dx = du$

$x = g(u)$

$dx = g'(u)\,du$