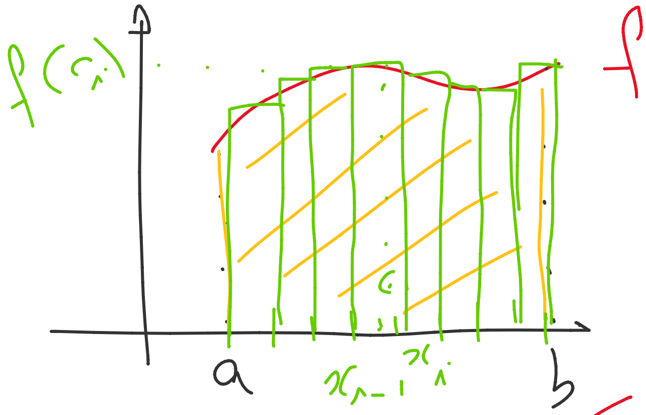
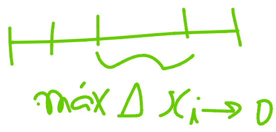
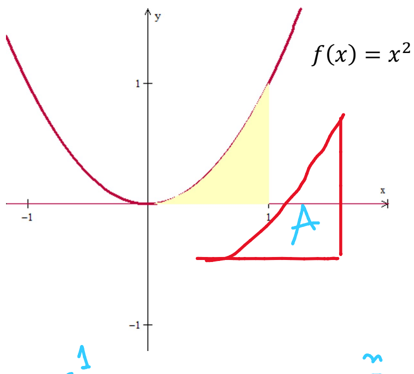
Teorema Fundamental do Cálculo (T4 e T5)

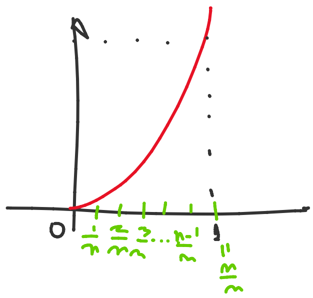




$$\boxed{\int_a^b f(x)dx} = \lim_{\text{máx}\Delta x_i \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i \in \mathbb{R}$$

Exercício: Calcule a área da figura abaixo





$$A_i = f(c_i)\cdot\Delta x_i$$

$$A_i = c_i^2 \frac{1}{n} = \left(\frac{i}{n}\right)^2 \cdot \frac{1}{n} = \frac{i^2}{n^3}$$

$$A = \int_0^1 x^2 dx = \lim_{\text{máx}\Delta x_i \to 0} \sum_{i=1}^{n} A_i = \lim_{\text{máx}\Delta x_i \to 0} \sum_{i=1}^{n} \frac{i^2}{n^3} = \lim_{n\to+\infty} \frac{1}{n^3} \sum_{i=1}^{n} i^2 =$$

$$= \lim_{n\to+\infty} \frac{1}{n^3} \frac{n(n+1)(2n+1)}{6} = \lim_{n\to+\infty} \frac{2n^3+3n^2+n}{6n^3} = \frac{2}{6} = \frac{1}{3}$$

$$\text{Obs: } \boxed{\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}}$$

$x_{i-1} = \frac{i-1}{n}$

$x_i = c_i = \frac{i}{n}$

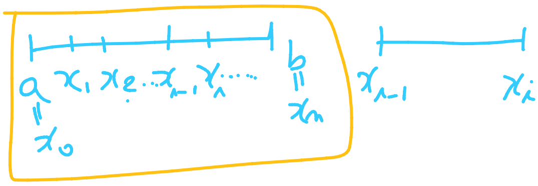
$$\Delta x_i = \frac{i}{n} - \left(\frac{i-1}{n}\right) = \frac{1}{n}$$

$\Delta x_i \to 0$, $n \to +\infty$

Teorema Fundamental do Cálculo: Se f for integrável em $[a,b]$ e se F for uma primitiva de f em $[a,b]$, então

$$\int_a^b f(x)dx = F(b) - F(a)$$

Demonstração:



Considere $F_i = F|_{[x_{i-1},x_i]}$

F_i é contínua em $[x_{i-1},x_i]$ e derivável em $]x_{i-1},x_i[$

$(T.V.M) \Rightarrow \exists\, c_i \in]x_{i-1},x_i[$ tq. $F(x_i)-F(x_{i-1}) = F'(c_i)(x_i - x_{i-1}), \ \forall i=1,\dots,n$

$$\Rightarrow \boxed{F(x_i)-F(x_{i-1}) = f(c_i)\Delta x_i}, \ i=1,\dots,n$$

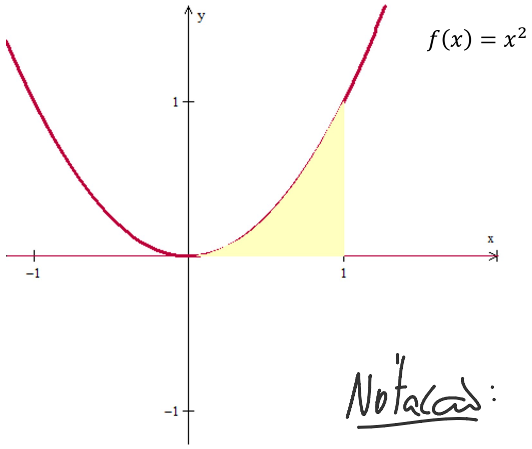
$$\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} f(c_i)\Delta x_i = \sum_{i=1}^{n}[F(x_i)-F(x_{i-1})] = \cancel{F(x_1)} - F(x_0) + \cancel{F(x_2)} - \cancel{F(x_1)}$$

$$+ \cancel{F(x_3)} - \cancel{F(x_2)} + \cancel{F(x_4)} - \cancel{F(x_3)} + \dots + \cancel{F(x_{n-2})} - \cancel{F(x_{n-3})} + \cancel{F(x_{n-1})} - \cancel{F(x_{n-2})}$$

$$+ F(x_n) - \cancel{F(x_{n-1})} = F(x_n) - F(x_0) = F(b) - F(a)$$

$$\int_a^b f(x)dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i = \lim_{\max \Delta x_i \to 0} [F(b) - F(a)] = F(b) - F(a)$$

Exercício: Calcule a área da figura abaixo



$$A = \int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}$$

Notação: $F(x)\Big|_a^b = \left(F(x)\right)_a^b = F(b) - F(a)$

Exercício: Calcule as seguintes integrais definidas

a) $\int_{-2}^{3} (x^3 - x)dx = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-2}^{3} = \underbrace{\left[\frac{3^4}{4} - \frac{3^2}{2}\right]}_{F(3)} - \underbrace{\left[\frac{(-2)^4}{4} - \frac{(-2)^2}{2}\right]}_{F(-2)}$

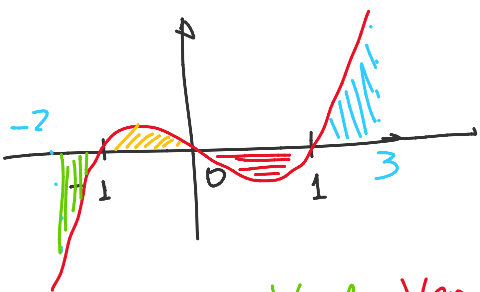
$$= \frac{81}{4} - \frac{18}{4} - \frac{16}{4} + \frac{8}{4} = \frac{55}{4}$$

b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \operatorname{sen}(x)dx = \left[-\cos x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \left(-\cos\left(\frac{\pi}{2}\right)\right) - \left(-\cos\left(-\frac{\pi}{2}\right)\right) = 0 - 0 = 0$

Pergunta: Qual é o significado geométrico dos números obtidos acima?

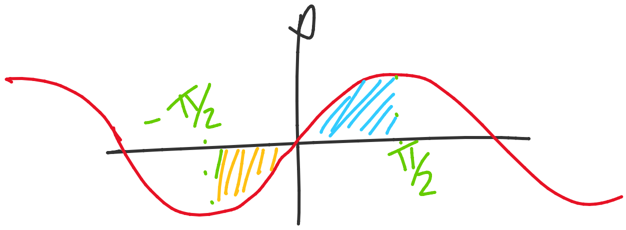
a) $\int_{-2}^{3} (x^3 - x)dx = \frac{55}{4}$

$x^3 - x = x(x^2 - 1) = x(x+1)(x-1)$



$A_{am} + A_{az} - V_{erde} - V_{erm} = \frac{55}{4}$

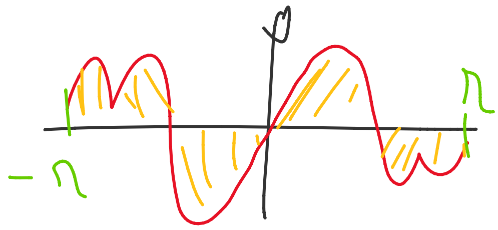
$$b)\ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \operatorname{sen}(x)dx = 0$$



A2 − LAR = 0

<u>Observação 1</u>: Se f é uma função ímpar e $r > 0$, então temos que

$$\int_{-r}^{r} f(x)dx = 0$$



<u>Observação2</u>: Se f é uma função par e $r > 0$, então temos que

$$\int_{-r}^{r} f(x)dx = 2\int_{0}^{r} f(x)dx$$