Construção de gráficos (T4 e T5)

Exercício: Esboce o gráfico de

L4)13)d) $f(x) = \dfrac{x^3}{x^2-1}$

i) $\boxed{D_f = \mathbb{R} \setminus \{-1, 1\}}$

ii) $f'(x) = \dfrac{3x^2(x^2-1) - x^3 \cdot 2x}{(x^2-1)^2} = \dfrac{x^4 - 3x^2}{(x^2-1)^2} = \dfrac{\overset{>0}{x^2}\,\overset{?}{(x^2-3)}}{\underset{>0}{(x^2-1)^2}} = 0$

		$-\sqrt{3}$		-1		0		1		$\sqrt{3}$	
f'	+		−		−		−		−		+
f	↗		↘		↘		↘		↘		↗

iii) $f''(x) = \dfrac{(4x^3-6x)(x^2-1)^{\cancel{2}\,1} - (x^4-3x^2)\,2\,\cancel{(x^2-1)}\,2x}{(x^2-1)^{\cancel{4}\,3}}$

$f''(x) = \dfrac{\cancel{4x^5} - \cancel{6x^3} - 4x^3 + 6x - \cancel{4x^5} + \cancel{12x^3}}{(x^2-1)^3}$

$f''(x) = \dfrac{2x^3 + 6x}{(x^2-1)^3} = \dfrac{\overset{?}{2x}\,\overset{>0}{(x^2+3)}}{(x^2-1)^3} = 0$

		-1		0		1	
f	∩		∪		∩		∪
$2x$	−		−		+		+
x^2-1	+		−		−		+
f''	−		+		−		+

pontos de inflexão: 0

iv)

$\boxed{\lim\limits_{x \to -1^+} \dfrac{x^3}{x^2-1} = +\infty \qquad \lim\limits_{x \to -1^-} \dfrac{x^3}{x^2-1} = -\infty}$ (I)

$\boxed{\lim\limits_{x \to 1^+} \dfrac{x^3}{x^2-1} = +\infty \qquad \lim\limits_{x \to 1^-} \dfrac{x^3}{x^2-1} = -\infty}$ (II)

$\boxed{\lim\limits_{x \to -\infty} \dfrac{x^3}{x^2-1} = -\infty \qquad \lim\limits_{x \to +\infty} \dfrac{x^3}{x^2-1} = +\infty}$ (III)

iv) $f(0)=0$ $\qquad f(\sqrt{3})=\dfrac{3\sqrt{3}}{2}$ $\qquad f(-\sqrt{3})=-\dfrac{3\sqrt{3}}{2}$

v) Assíntotas

(I) $\Rightarrow$ $x=-1$ é uma assíntota vertical

(II) $\Rightarrow$ $x=1$ é uma assíntota vertical

(III) $\Rightarrow$ não existem assíntotas horizontais

Verificação da existência das assíntotas oblíquas

$$\lim_{x\to+\infty}\frac{f(x)}{x}=\lim_{x\to+\infty}\frac{\frac{x^3}{x^2-1}}{x}=\lim_{x\to+\infty}\frac{x^3}{x^3-x}=\boxed{1=m}$$

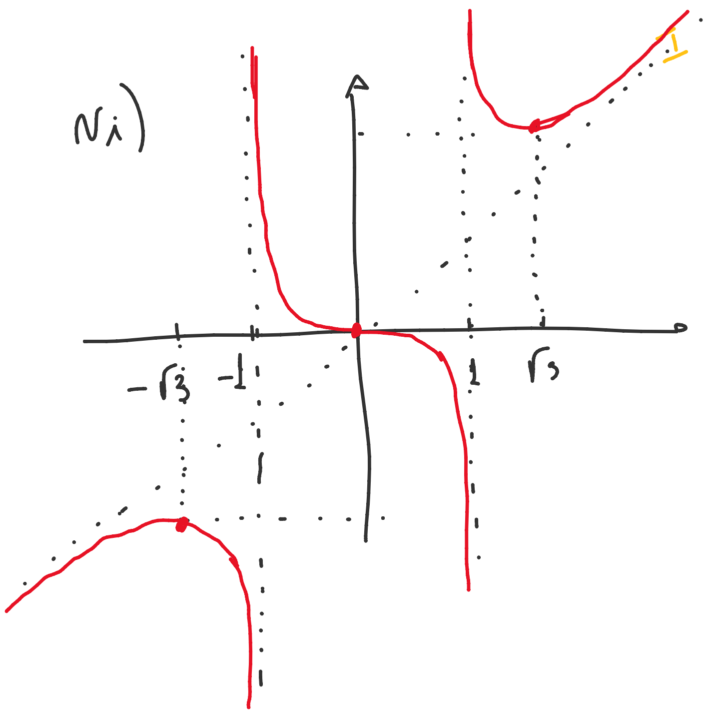
$$\lim_{x\to+\infty}\left[f(x)-mx\right]=\lim_{x\to+\infty}\left[\frac{x^3}{x^2-1}-1x\right]=\lim_{x\to+\infty}\left[\frac{\cancel{x^3}-\cancel{x^3}+x}{x^2-1}\right]=\boxed{0=n}$$

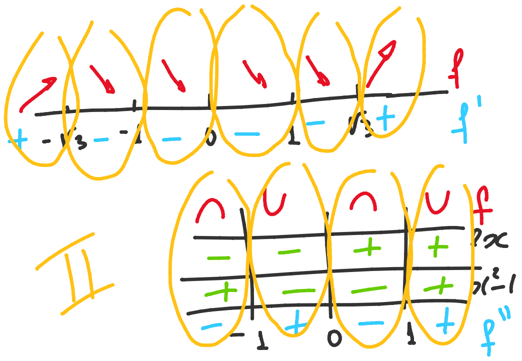
$\Rightarrow y=1x+0$ é uma assíntota oblíqua

$$\lim_{x\to-\infty}\frac{f(x)}{x}=1 \quad e \quad \lim_{x\to-\infty}\left[f(x)-mx\right]=0$$

$\Rightarrow y=1x+0$ é uma assíntota oblíqua

vi)





$$\boxed{\lim_{x\to-1^+}\frac{x^3}{x^2-1}=+\infty} \qquad \boxed{\lim_{x\to-1^-}\frac{x^3}{x^2-1}=-\infty}$$

$$\boxed{\lim_{x\to1^+}\frac{x^3}{x^2-1}=+\infty} \qquad \boxed{\lim_{x\to1^-}\frac{x^3}{x^2-1}=-\infty}$$

$$\boxed{\lim_{x\to-\infty}\frac{x^3}{x^2-1}=-\infty} \qquad \boxed{\lim_{x\to+\infty}\frac{x^3}{x^2-1}=+\infty}$$