Funções de $\mathbb{R}$ em $\mathbb{R}$ - parte 1 (T4)

<u>Definição</u>: Sejam A e B dois conjuntos. O produto cartesiano entre A e B é o conjunto definido da seguinte forma:

$$A \times B = \{(a, b): a \in A \ \text{ e } \ b \in B\}.$$

<u>Definição</u>: $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(a, b): a, b \in \mathbb{R}\}.$

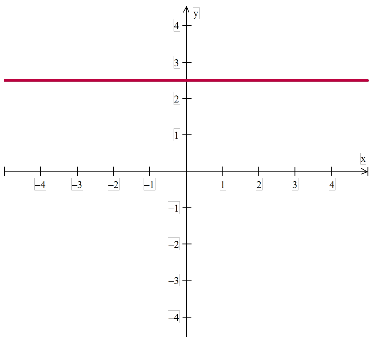
<u>Definição</u>: Dados dois conjuntos A e B, uma função de A em B é uma regra que associa a cada elemento de A um único elemento de B. Denotamos por (A, B, f) ou $f: A \longrightarrow B$.

$A = \mathrm{Dom}_f = \mathrm{D}_f =$ domínio de f
$B =$ contradomínio de f
$\mathrm{Im}_f =$ imagem de $f = \{y \in B: \exists\, x \in A$ tal que $f(x) = y\}$
$\mathrm{G}_f =$ gráfico de $f = \{(x, y) \in \mathbb{R}^2: y = f(x)$, para algum $x \in \mathrm{D}_f\}$
$\quad = \{(x, f(x)) \in \mathbb{R}^2: x \in \mathrm{D}_f\}$

<u>Exemplos</u>:

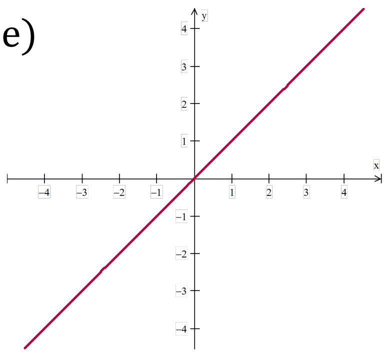
$a)\ f(x) = k, \text{com } k \in \mathbb{R}\}$ (função constante)

$\mathrm{D}_f = \mathbb{R}$
$\mathrm{Im}_f = \{k\}$

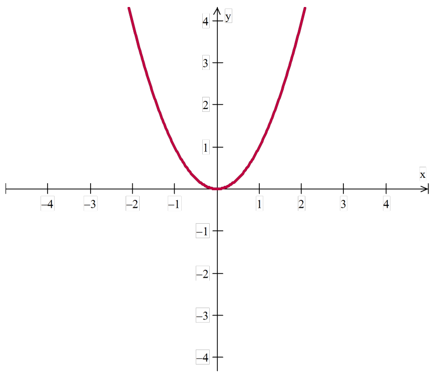


$b)\ f(x) = x$ (função identidade)

$\mathrm{D}_f = \mathbb{R}$
$\mathrm{Im}_f = \mathbb{R}$

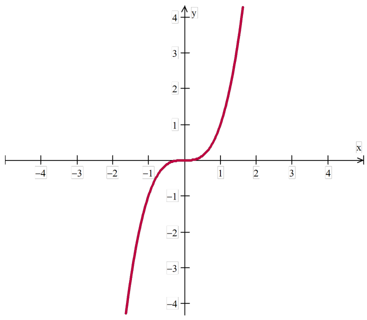


$c)\ f(x) = x^2$



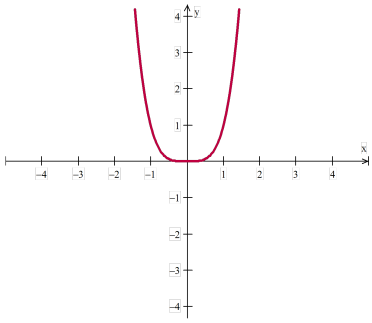
$\mathrm{D}_f = \mathbb{R}$
$\mathrm{Im}_f = \mathbb{R}_+ = \{x \in \mathbb{R}: \mathrm{x} \geq 0\}$

$d)\ f(x) = x^3$



$\mathrm{D}_f = \mathbb{R}$
$\mathrm{Im}_f = \mathbb{R}$

$e)\ f(x) = x^4$

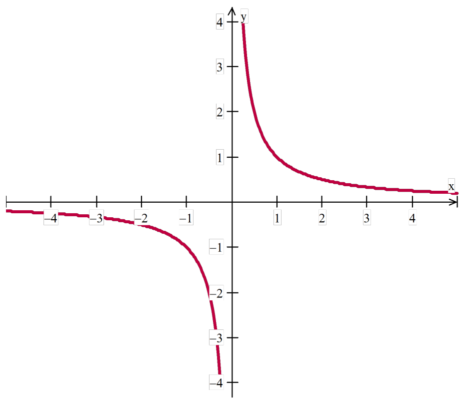


$\mathrm{D}_f = \mathbb{R}$
$\mathrm{Im}_f = \mathbb{R}_+$

$f)\ f(x) = \sum_{i=0}^{n} a_i x^i, a_i \in \mathbb{R}, a_i \neq 0$ (função polinomial)

$\mathrm{D}_f = \mathbb{R}$

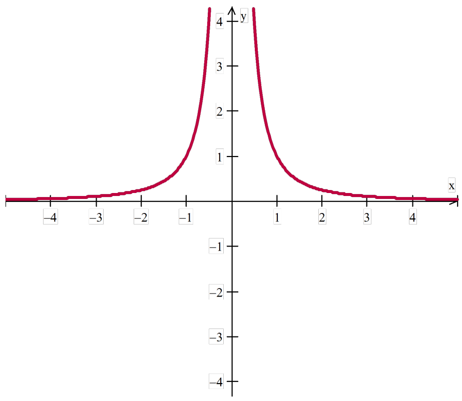
$g)\ f(x) = \frac{1}{x}$



$\mathrm{D}_f = \mathbb{R}^* = \{x \in \mathbb{R}: x \neq 0\}$
$\mathrm{Im}_f = \mathbb{R}^*$

$h)\ f(x) = \dfrac{1}{x^2}$

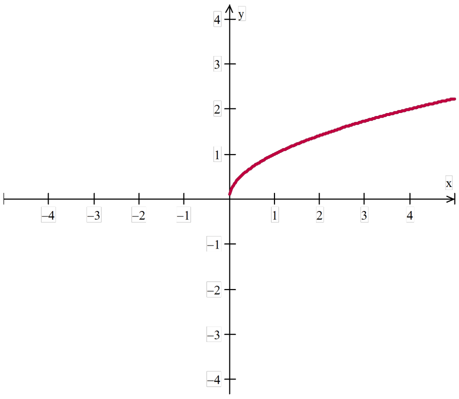
$D_f = \mathbb{R}^*$
$Im_f = {\mathbb{R}_+}^* = \{x \in \mathbb{R}: x > 0\}$



$i)\ f(x) = \frac{p(x)}{q(x)}$, p, q funções polinomiais com $q(x) \neq 0$. (funções racionais)

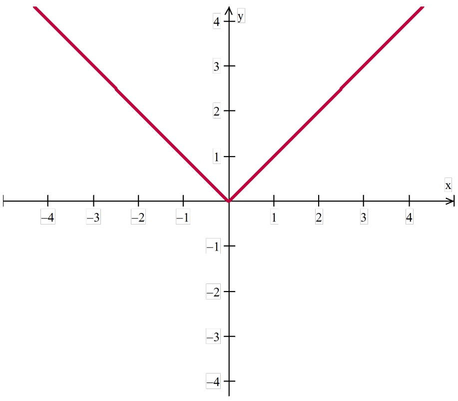
$j)\ f(x) = \sqrt{x}$

$D_f = \mathbb{R}_+$
$Im_f = \mathbb{R}_+$



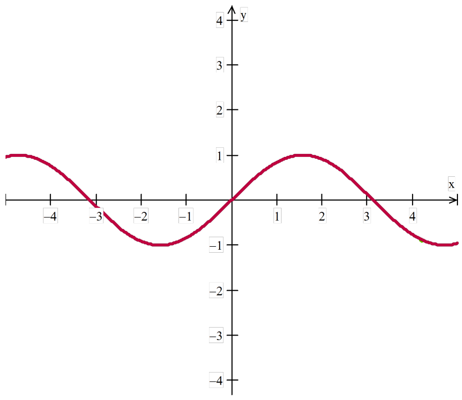
$k)\ f(x) = |x|$

$D_f = \mathbb{R}$
$Im_f = \mathbb{R}_+$



$l)\ f(x) = sen\ (x)$

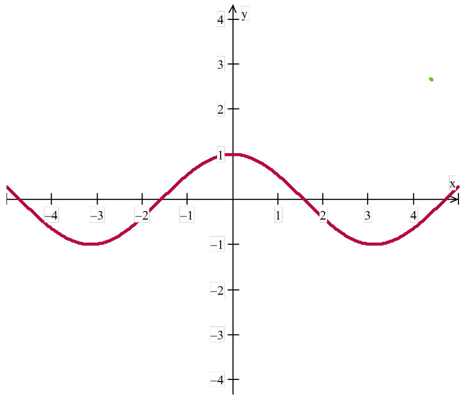
$D_f = \mathbb{R}$
$Im_f = [-1,1] = \{x \in \mathbb{R}: -1 \leq x \leq 1\}$



$m)\ f(x) = \cos(x)$

$D_f = \mathbb{R}$

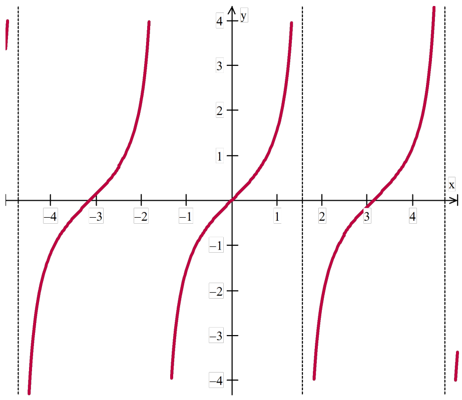
$Im_f = [-1,1] = \{x \in \mathbb{R}: -1 \leq x \leq 1\}$



$n)\ f(x) = \text{tg}(x) = \dfrac{\text{sen}(x)}{\cos(x)}$

$D_f = \{x \in \mathbb{R}: x \neq \dfrac{\pi}{2} + \text{k}\pi, k \in \mathbb{Z}\}$

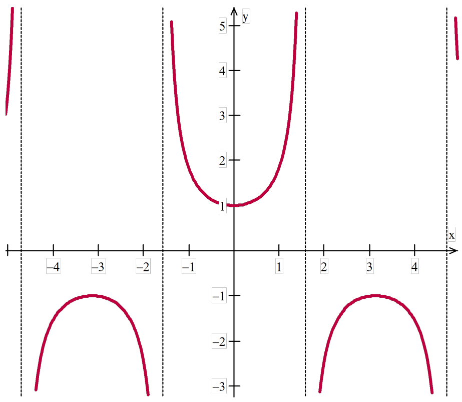
$Im_f = \mathbb{R}$



$o)\ f(x) = \sec(x) = \dfrac{1}{\cos(x)}$

$D_f = \{x \in \mathbb{R}: x \neq \dfrac{\pi}{2} + \text{k}\pi, k \in \mathbb{Z}\}$

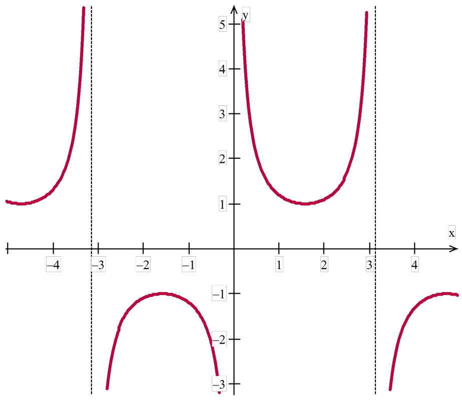
$Im_f =]-\infty, -1] \cup [1, +\infty[= \{x \in \mathbb{R}: x \leq -1 \text{ ou } x \geq 1\}$



$p)\ f(x) = \text{cossec}\,(x) = \dfrac{1}{\text{sen}(x)}$

$\text{D}_f = \{x \in \mathbb{R}: x \neq \text{k}\pi, k \in \mathbb{Z}\}$

$\text{Im}_f =]-\infty, -1] \cup [1, +\infty[= \{x \in \mathbb{R}: x \leq -1 \text{ ou } x \geq 1\}$



$q)\ f(x) = \text{cotg}\,(x) = \dfrac{\cos(x)}{\text{sen}(x)}$

$\text{D}_f = \{x \in \mathbb{R}: x \neq \text{k}\pi, k \in \mathbb{Z}\}$

$\text{Im}_f = \mathbb{R}$