Science – The Sharper Image:

The Quest for Knowledge and the Recursive Production of Objective Realities

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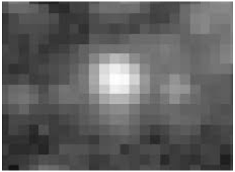
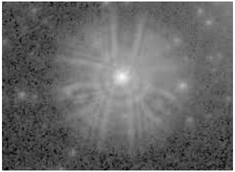
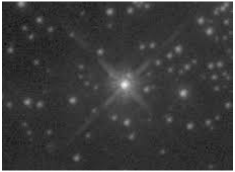




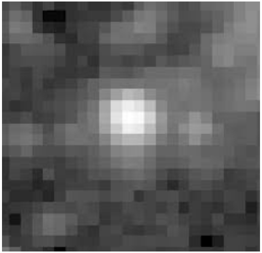
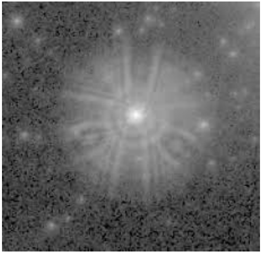


XIX EBL, 6-10/05/19, J. Pessoa; XI Principia, 19-22/08/19, Florianópolis;

Newton da Costa: $16 + 09 + 2019 = 90$, IME-USP, São Paulo

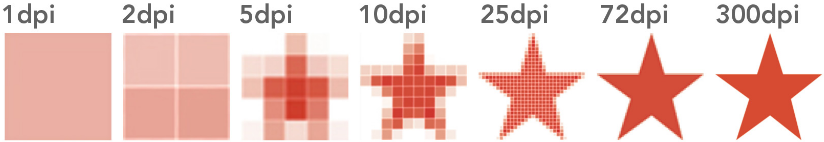
http://www.ime.usp.br/~jstern/miscellanea/jmsslide/galileos.pdf

Star Melnick-34 in the Tarantula Nebula
- European Souther Observatory ground based telescope;
- The Hubble's wide field planetary camera, 1990; and
- after space shuttle Endeavour corrective mission, 1993.

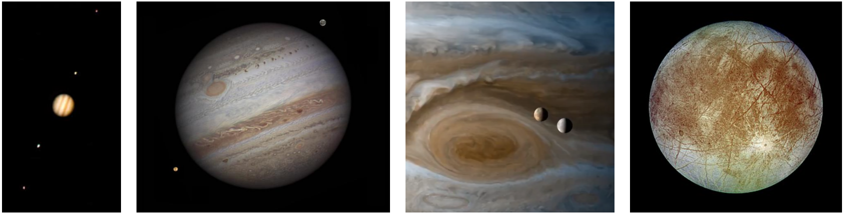




(←) Aberration & artifacts: Distortion & spurious effects like pixelation, point replication (dimmer copies arround it), halos, coronas, spikes, etc.

Sharper images: Resolution & Magnification



($\rightarrow$) Resolution: Specifies the observer's ability to distinguish apart (resolve) two nearby objects



($\rightarrow$) Magnification factor: How many times larger the observer sees an object (Jupiter*, Io, Europa*, Ganymede and Callisto)

This Presentation

I- Metaphor: Science provides the eye (of the mind) with *sharper* images of objects in the environment we live in;

II- What do we see through a magic glass?

- Galileo Galilei and his telescope face disbelief;

III- The Game of Science - Objective Cognitive Constructivism

- Knowing and playing with great scientists;

IV- Recursive production of *objective* realities

- Essential characteristics of scientific laws – – *Precision, Stability, Separability and Composability*;

V- Specialized tools of production

- Technological know-how: Fabrication, computing, etc;
- Testing and adjusting scientific equipment & hypotheses;
- Scientific ontologies and metaphysics;

VI- Final & additional remarks.

What do we see through a Magic Glass?

- Galileo (1610) reported observation of Jupiter's moon transit -

Cesare Cremonini (1550-1631) supposed refusal to look trough Galileo's telescope as reporter in letters from Galileo to Kepler and from Paolo Gualdo to Galileo (29/06/1611):

My dear Kepler, what would you say of the learned here, who, replete with the pertinacity of the asp, have steadfastly refused to cast a glance through the telescope?
What shall we make of this? Shall we laugh, or shall we cry?

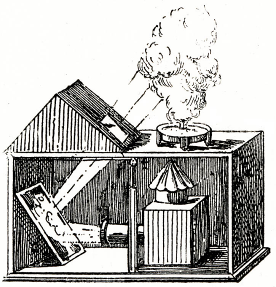
I do not wish to approve of claims about which I do not have any knowledge, and about things which I have not seen .. and then to observe through those glasses gives me a headache.

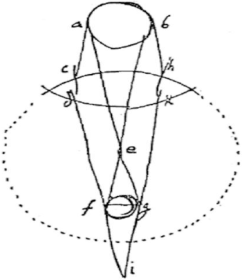
What do we see through a Magic Glass?

Christopher Clavius (1610), as quoted in GO (X, p.442):

Clavius said ...about the four stars [moons of Jupiter]: That is ridiculous, for one needs to manufacture a spyglass that creates them for next display them; ... Galileo can have his opinion and he [Clavius] shall have his own.

- Optics by Giambattista della Porta (1589,1610)



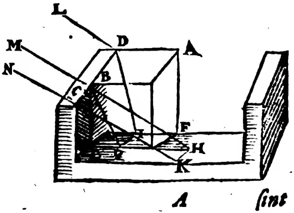


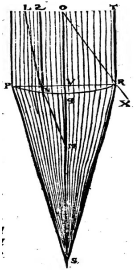
- Galileo’s trade secrets on telescope manufacturing

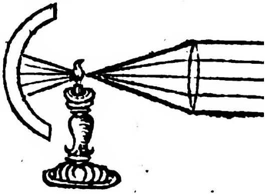
Kepler's Simple (linear) Law & Explanations

Kepler (1610, p.17-18), Dissertatio cum Nuncio Sidereo:

In the Optical part of astronomy, I gave and explained a lucid geometrical demonstration of what happens in simple lenses.





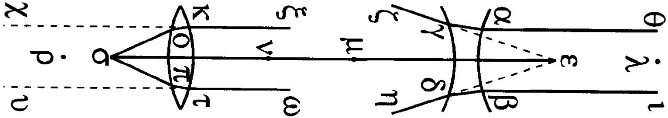


- Model relating refraction index of different media and angle to normal to interface: $\theta_2/\theta_1 = n_1/n_2$;
- Models for thin spherical lenses and spherical mirrors.

Kepler's Simple (linear) Composition Rules

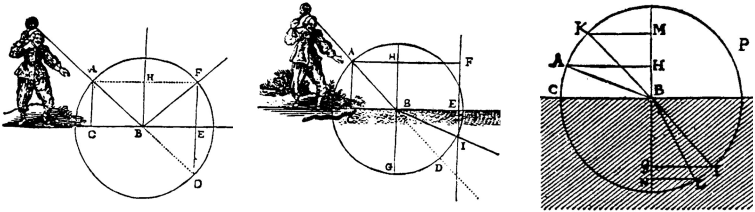
Kepler (1610, p.17-18), Dissertatio cum Nuncio Sidereo:

...concerning the circular tubes with two lenses, I am trying to convince the incredulous to have faith in these instruments...



- Models for compound optical instruments

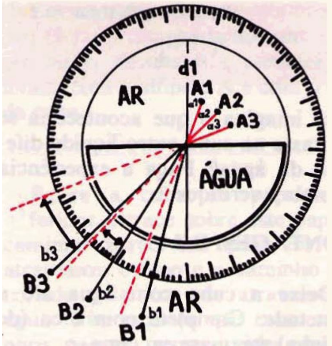
Snell-Descartes better (but non-linear) Explanations



René Descartes (1596-1650) tennis ball model:
Reflection: Tangential/ normal velocities are equal/ inverted.
Refraction: By analogy, Tangential/ normal velocities are unchaged/ increased or decreased by crossing the interface between media (with refractive indexes) n_1 and n_2.
This model immediately leads to the law relating angle to the interface's normal and light's velocity in different media:

- $\sin(\theta_2)/\sin(\theta_1) = n_1/n_2 = v_2/v_1$
- $\approx \theta_2/\theta_1$ for $\theta \approx 0$, paraxial approximation

Game of Science: Precise Laws



Os Cientistas (FUNBEC/ Abril, 1972) - Descartes - kit of experiments.

Willard v. O. Quine (1969) slogan: No entity without identity :-)

- Objective Cognitive Constructivism:
- Entities are defined by identity (invariance, equality) relations!
- An entity is as objective (truth, real) as verifiably sharp or precise are the identity relations that define it.

Game of Science: Compositionality Rules

Polyopticon (D.F.Vasconcellos, 1960) Optical construction kit

- Objective Cognitive Constructivism:
- We refer to entities by words (tokens) in a language;
- Good syntactical and semantical articulation rules for this language must correspond to the compositionaly properties of the objects in the corresponding context of reference;
- Identity relations and compositionality rules define, in turn, the meaning of words (and the objects they stand for).
- Playing optical games we learn optics (the only way!)

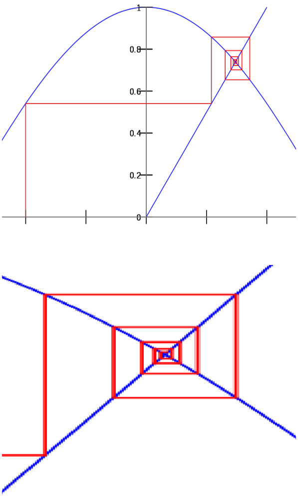
Theoretical	|	Metaphysical	|	Technological
Mathematical formalization	⇒	Causal explanation	⇒	Instrument specification
⇑				⇓
Speculative interpretation		Sharp images		Project design & calculation
⇑				⇓
Aberration analysis	⇐	Calibration/ observations	⇐	Fabrication /assembly
optics	|	Operational	|	implementation

Optical instrumentation production diagram

Theoretical	|	Metaphysical	|	Technological
Mathematical laws and models	$\Rightarrow$	Causal explanation	$\Rightarrow$	Hypotheses formulation
$\Uparrow$				$\Downarrow$
Speculative interpretation		Precise laws/rules		Empirical trial design
$\Uparrow$				$\Downarrow$
Statistical analysis	$\Leftarrow$	Data processing	$\Leftarrow$	Experiment execution
optics	|	Operational	|	implementation

Scientific Laws and Rules production diagram

v. Foerster: Objects are Tokens for Eigen-Solutions



$x_{n+1} = \cos(x_n), \ \ x_0 = -1$

x^* is a Fixed Point of $f(x)$ if $x^* = f(x^*)$;
i.e., x^* is Invariant by application of $f(\,)$

Under appropriante regularty conditions
(ex: $C^2 \wedge |f'(x)| < 1$ for $a < x^* < b$)
$z = f^n(x) = f(\ldots f(f(x))\ldots) \longrightarrow x^*$;
This is an attractive or stable fixed point.

Distinct stable fixed points define
separate attraction regions.

Generalized Fixed Points are known
as Eigen-Value, Eigen-Solution, etc.

Eigen-solutions are Precise & Stable
(and according to the appropriate
context, separable and composable).
Hence, Heinz von Foerster metaphor:
Objects are Tokens for Eigen-Solutions.

Essential characteristics of Scientific Laws

Characteristics of Objective (good) Scientific Laws:

- Invariance: They express invariant relations:

For any angle θ_1 in $[0, \pi/2]$: $sin(\theta_1)/\sin(\theta_2) = n_2/n_1$;

- Precision (shapness): This is the meaning of the symbol $=$;

- Stability: Even considering material imperfections, errors in measurement, etc., we can still observe these relations with good accuracy. Moreover, we are able to build real systems based on these laws that behave accordingly (fairly well).

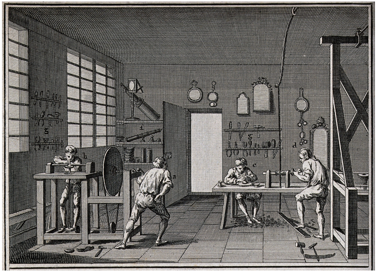
- Separability and Compositionality Rules, examples:

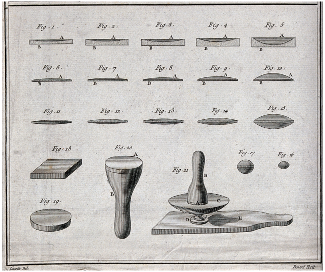
- Keplerian (paraxial) Compositionality Rules (A3)

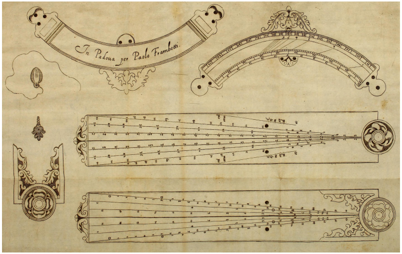
- Each propagation medium have its individual (separate) refractive index, that can be combined according to the rule:

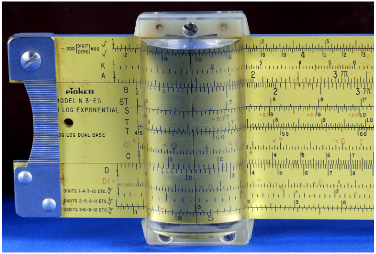
If $\sin(\theta_1)/\sin(\theta_2) = n_2/n_1$ and $\sin(\theta_1)/\sin(\theta_3) = n_3/n_1$
then $\sin(\theta_2)/\sin(\theta_3) = n_3/n_2$

Tools of Production: (a) Technological Know-How



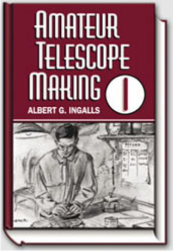


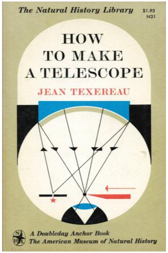


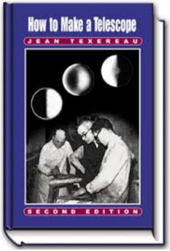


Lucotte (1770), Galileo sector (1606), Pickett N3 (1965)

Amateur Telescope Making



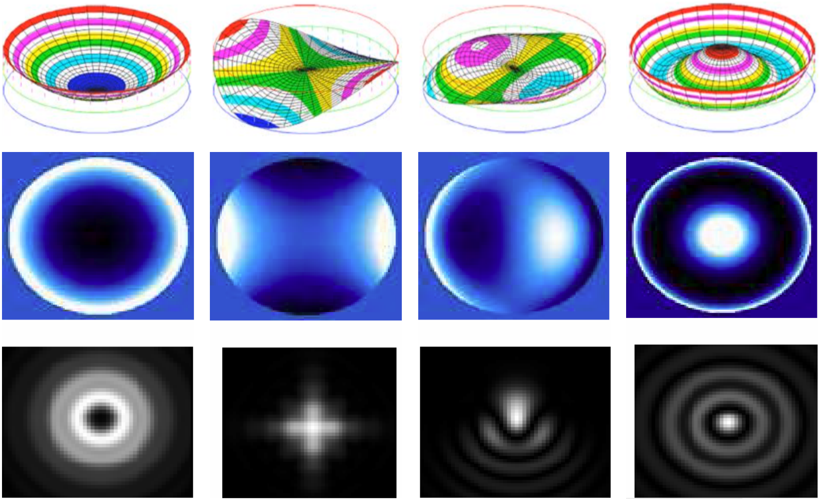




Grinding, testing & adjusting a telescope mirror precise shape

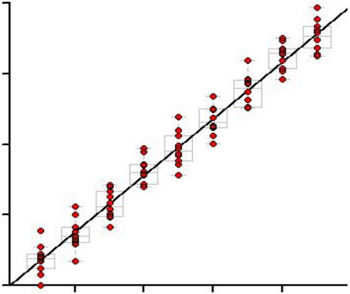
Mirror's surface distortions: Like waves & ripples in a coffee cup

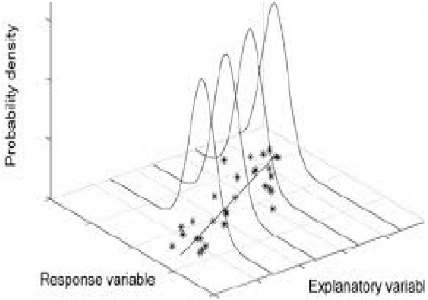
Testing/ Calibrating Scientific Equipment



Zernike Eigen-Functions, $Z(n, m)$, and corresponding objects in Optics (Foucault knife-edge test aberrations) and Ophthalmology: Defocus (2,0); Astigmatism (2,2); Coma (3,1); Spherical (4,0)
– Genberg & Michels (2004); Zou & Wattellier (2012).

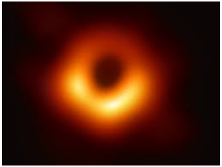
(b) Testing/ Verifying Scientific Hypotheses

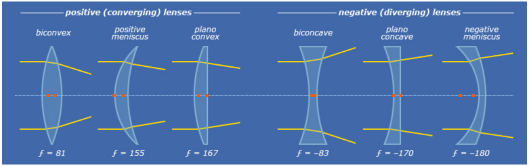


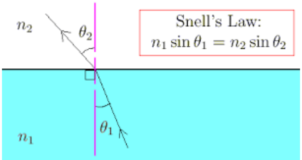


- Adjusting a polynomial line: $y = a + bx + cx^2 + \epsilon$, where $y = \sin(\theta_2)$, $x = \sin(\theta_1)$; and verifying or testing if the hypothesis $H : \{a{=}0,\ b{=}(n_1/n_2),\ c{=}0\}$ should be accepted.
- Repeated measurements, box-plots, visual inspection
- Statistics (linear regression) + Hypothesis Test theory

(c) Ontologies: Things we make, see, explain & name







- A Scientific ontology is a carefully controlled language with words (tokens) for objects emerging in the production cycle of a scientific discipline. Its grammar and valid articulation rules must correspond to underlying laws & compositionality rules.
- Jupiter's moons, Saturn's rings; Refractive indexes, focal distances, types of lenses, align, collimate; Arithmetic & trigonometric functions, commutative, associative, distributive...

(d) Metaphysics (entities & statistical parameters)

- Metaphysics concerns causal explanations telling why things are the way they do. These are the narratives, metaphors, and abstracts symbolic statements used to build our systemic understanding & intuition about the world and the way it works.
- Causal explanations are an essential part of the scientific production cycle. Scientific hypotheses are formulated using metaphysical concepts, i.e., non directly observable objects.
- In a statistical model, the *sampling distribution*, $p(x \mid \theta)$, "explains" relations betwen observed random variables, x , by the *parameters*, latent or non-observable random variables, θ .
- *Precise* or *Sharp* Statistical hypotheses are formulated as *equality* constraints in the parameter space; $H : h(\theta) = 0$
- In Objective Cognitive Constructivism, statistical theory and methods for testing sharp hypotheses is the anchor for ontology and metaphysics

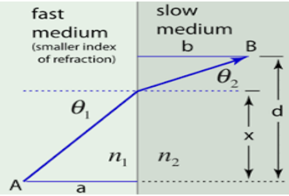
Fermat's Least Time (teleological) Principle

Lifeguard problem: Station at $[x, y] = [0, 0]$; shoreline at $x = a$;
drowning person at $[a + b, d]$; top speed limit $= c$;
running speed= c/n_1 ; swimming speed $= c/n_2$; $1 < n_1 < n_2$.
Best entry point $[a, y]$? (best transition from land to water?)

$$\min_y! \quad J(y) = \frac{n_1}{c}\sqrt{a^2 + y^2} + \frac{n_2}{c}\sqrt{b^2 + (d - y)^2}$$

$$\Rightarrow \quad \frac{dJ}{dy} = \frac{n_1}{c}\frac{-2y}{2\sqrt{a^2 + y^2}} + \frac{n_2}{c}\frac{-2(d - y)}{2\sqrt{b^2 + (d - y)^2}} = 0$$

$$\Rightarrow \quad n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$



Descartes' and Fermat's explanations predict higher / lower speeds for larger refractive indexes. Without measures for the seed of light, c , both are "good explanations" for scientific production!

Analytical (variational/ teleological) Mechanics

An insight for teaching Analytical* Mechanics:

- Lagrangian and Hamiltonian variational formalisms are (almost**) equivalent to standard Newtonian Mechanics, that is based solely on efficient causes (forces). However...

- The key to understand this discipline, master the art of setting up appropriate conceptual frameworks and writing down adequate systems of equations for solving exercises and real problems, is to fully embrace its teleological spirit.

- We must thing about where the particle, spin-top, etc. wants to go, what it wants to do and how it is going to do it, with the “goal” or “purpose” of minimize the action (or other pertinent quantity) associated with its trajectory.

* J. Marion (1970). Classical Dynamics of Particles and Systems;
** R. Abraham, J. Marsden (1980). Foundations of Mechanics.

Tokens for Commodities – Acknowledgments

(invariant) prices ∼ (strong) currencies ∼ (perfect) markets

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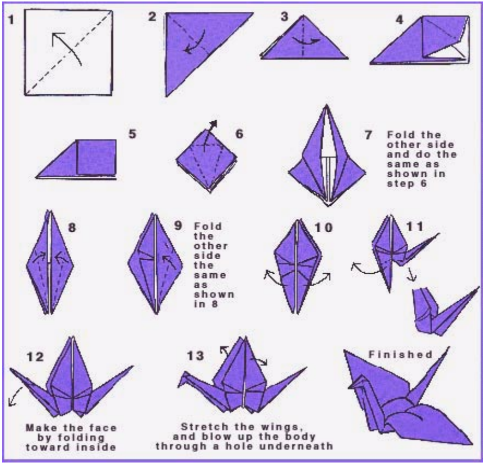
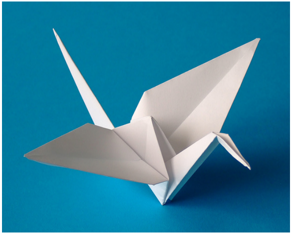
A1- Richard Dawkins' Origami Metaphor

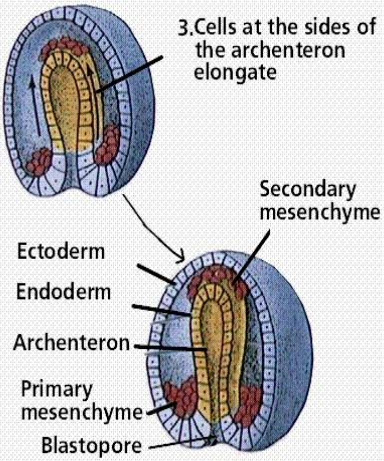


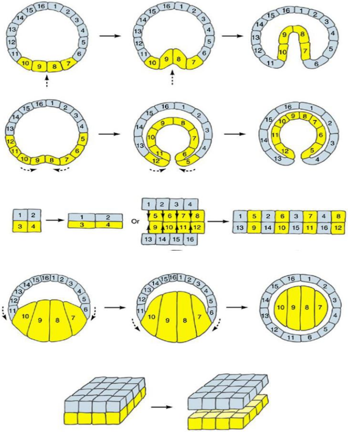


Origami folding instructions for a Crane (Tsuru).
What happens in Chinese Whispers game with both cranes?
Why? Folds are: Exact, Stable, Separable and Composable!

– Stern (2014), Jacob's Ladder & Scientific Ontologies.

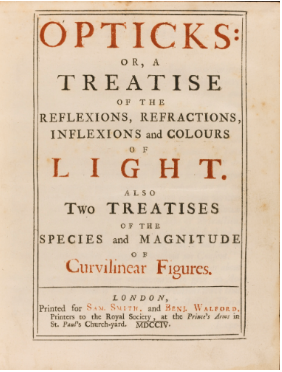
A1- Richard Dawkins' Origami Metaphor





Real cranes are also (self) assembled like an origami!
Tissue movements at Organic morphogenesis (gastrulation):
Invagination, involution, convergent extension, epiboly, delamination.

A2- Mathematics as a Natural Science



OPTICKS:
OR, A
TREATISE
OF THE
REFLEXIONS, REFRACTIONS,
INFLEXIONS and COLOURS
OF
LIGHT.
ALSO
Two TREATISES
OF THE
SPECIES and MAGNITUDE
OF
Curvilinear Figures.

LONDON,
Printed for SAM. SMITH, and BENJ. WALFORD,
Printers to the Royal Society, at the Prince's Arms in
St. Paul's Church-yard. MDCCIV.

As far as the statements of mathematics refer to actual truth, they are not certain; and as far as they are certain, they do not refer to actual truth. Albert Einstein.

(Kappa:) *If you want mathematics to be meaningful, you must resign of certainty. If you want certainty, get rid of meaning. You cannot have both.* Imre Lakatos.

The role of the alleged 'foundation' is rather comparable to the function discharged, in physical theory, by explanatory hypotheses ... the actual function of axioms is to explain the phenomena described by the theorems of this system rather than to provide a genuine 'foundation' for such theorems. Kurt Gödel.

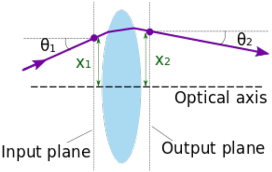
A2- Mathematics as a Natural Science

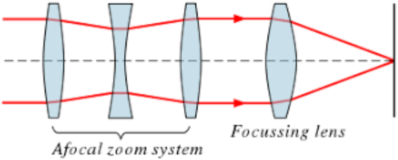
Eugen Wigner and Richard Hamming are astonished by "the unreasonable effectiveness of mathematics in natural sciences."

From the perspective of Cognitive Constructivism, nothing is more natural than the effectiveness of mathematics, for mathematics is nothing but the order of the natural world (including ourselves) expressed in language (as well as we currently can)

Of course, a deeper mystery remains untouched, namely, the existence of an orderly cosmos & not only chaos. Actually, not only the existence of any cosmos, but the existence of a "good" one, in which we can find the sharply defined, stable, separable and composable eigen-solutions we need to use as build blocks in the construction of knowledge.

- see Stern (2011a,b).





Transfer (or propagation) Matrix approach to optics:
x - locates the ray as it passes along the system's main axis;
y - describes the ray's height relative to main axis x;
α - describes the ray's angle (in radians) relative to the axis.

- The action of an optical element is modeled by a matrix A

$$\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = A \begin{bmatrix} y \\ \alpha \end{bmatrix} = \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix} = \begin{bmatrix} A_{1,1} y + A_{1,2}\alpha \\ A_{2,1} y + A_{2,1}\alpha \end{bmatrix}$$

1st / 2nd column of A describe the optical elemet's action on a light ray with zero inclination / hight.

A3- Keplerian (paraxial) Linear Composition

$D(d)$ – Distance d simple translation; $M(r)$ – Concave mirror; $F(n, n')$ – Flat (orthogonal to x) interface from medium n to n'; $S(n, n', r)$ – Spherical thin interface or radius r , where r is positive / negative for a convex / concave interface;

$$D(d) = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \; ; \; F(n, n') = \begin{bmatrix} 1 & 0 \\ 0 & n/n' \end{bmatrix} \; ;$$

$$S(n, n', r) = \begin{bmatrix} 1 & 0 \\ (n - n')/r\, n' & n/n' \end{bmatrix} \; ; \; M(r) = \begin{bmatrix} 1 & 0 \\ 2/r & 1 \end{bmatrix}$$

- System C composed by elements A and B (left to right):

$$C = BA = \begin{bmatrix} B_{1,1}A_{1,1} + B_{1,2}A_{2,1} & B_{1,1}A_{1,2} + B_{1,2}A_{2,2} \\ B_{2,1}A_{1,1} + B_{2,2}A_{2,1} & B_{2,1}A_{1,2} + B_{2,2}A_{2,2} \end{bmatrix}$$

Gerrard & Burch (1975, Ch.1 and 2). Introd. to Matrix Methods in Optics.
D. Kleinfeld, P. Tsai (13/04/2004). An introduction to basic optical design.

A3- Keplerian (paraxial) Compound Instruments

$L(n, n', r_1, r_2)$ – Lens made by a thin sequence of convex and concave interfaces ($r_1 > 0$ and $r_2 < 0$), with external/internal media n and n'. $(1/f) = (1/r_1 - 1/r_2)(n' - n)/n$

$$L(n, n', r_1, r_2) = S(n', n, r_2)S(n, n', r_1) = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} = L(f)$$

$T(f_1, f_2, d)$ – Telescope using two thin lenses with focal distances f_1 and f_2 separated by a distance d.

$$T(f_1, f_2, d) = L(f_2)D(d)L(f_1) = \begin{bmatrix} 1 - d/f_1 & d \\ (d - f_1 - f_2)/(f_1\, f_2) & 1 - d/f_2 \end{bmatrix}$$

Collimated telescope, $T(f_1, f_2, d)$, adjusting $d = f_1 + f_2$.

$$C(f_1, f_2) = \begin{bmatrix} -f_2/f_1 & f_1 + f_2 \\ 0 & -f_1/f_2 \end{bmatrix}$$

A4- Symmetry / Invariance / Conservation Laws

Emmy Noether's Theorem (1918): There is a correspondence between Symmetries of a theory, its Conserved quantities and its Invariant (ontological) objects, see Stern (2011b).
Analysis by Christiaan Huygens (1656) of Galilean relativity:

A4- Symmetry / Invariance / Conservation Laws

⓪ $v_0 \longrightarrow$ ① $v_1 \rightarrow$ $\Rightarrow$ ⓪ $w_0 \rightarrow$ ① $w_1 \longrightarrow$

Simple application: Collision between two bodies

- Galilean transformation (GT) as an Invariance Group:

• Consider Reference Frame $\boxed{F'}$ moving at $f' \longrightarrow$ uniform velocity relative to this slide (F); Galilean Relativity states that:
> Velocities in F' are: $v'_0 = v_0 - f'$, $v'_1 = v_1 - f'$, $w'_0 = w_0 - f'$, ...
> Laws of Physics should be the same in F or F'

- Velocities u, v are not invariant by GT; Nevertheless, $\Delta_0 = (w_0 - v_0)$ and $\Delta_1 = (w_1 - v_1)$ are invariants.
- $\exists\ m_1(u) \in R \mid 1\Delta_0 = -m_1(u)\ \Delta_1$; where u is the scale (unit of measure) for velocities
- Scale invariance (no privileged unit) $\Rightarrow m_1(u) = m_1$;
- Constant m_1 is the (collision) inertial <mass> of particle p_1, relative to the standard unit mass $m_0 = 1$ of particle p_0.

A4- Symmetry / Invariance / Conservation Laws

- **GT** relativity + **Scale** invar. $\Rightarrow\ 1(w_0 - v_0) = -m_1(w_1 - v_1)$ is valid for any initial condition v_0, v_1 ; Assuming additional + **Compositionality** rules for relative inertia $\propto$ masses $\Rightarrow$

① $v_1 \longrightarrow$ ② $v_2 \rightarrow$ $\Rightarrow$ ① $w_1 \rightarrow$ ② $w_2 \longrightarrow$

$\Rightarrow$ Momentum Conservation law:

<MC> $m_1 v_1 + m_2 v_2 = m_1 w_1 + m_2 w_2$, where

m_1 and m_2 are the (collision) inertial masses of particles p_1 and p_2, relative to p_0 with standard unitary mass $m_0 = 1$.

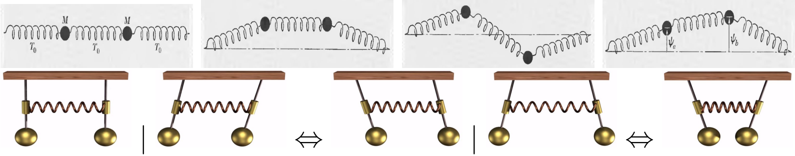
- Time Reversibility (TR) symmetry (elastic collision): $(v_1 - v_2) = -(w_1 - w_2)$;

- <MC> + **TR** $\Rightarrow$ Energy Conservation law:

<EC> $m_1 v_1^2 + m_2 v_2^2 = m_1 w_1^2 + m_2 w_2^2$.

– see A.P. French (1971), Newtonian Mechanics;
E. Mach (1960), The Science of Mechanics.

Bases for two coupled oscillators: Transverse and longitudinal

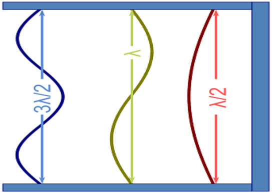


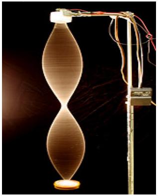
Left: Static invariant states (equilibrium) for the two systems.
Right: Dynamic invariant states for these systems:
Two *normal modes* of movement for the oscillating particles:
Symmetric mode – same amplitude and same phase,
Antisymmetric mode – same amplitude but opposite phases.

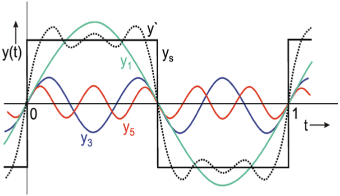
- (De)Composition: Any free movement of these systems is a linear superposition of their normal modes (eigen-solutions).
- Stability: Energy stored at each normal mode is constant.
- Precision: System's Symmetries impose strict invariant (eigen) forms and oscillating factor frequencies (eigen-values)

– Franklin (1968), French (1974), Sadun (2001)

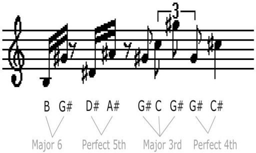
A5- Dynamic Invarnts & Eigen-Functions (continuum)

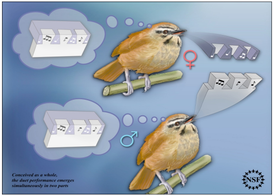






- Continuous string ($n \to \infty$ beads) $\Rightarrow$ trigonometric *basis*, $\lambda_n = 2L/n$, harmonic freq. $k_n = 2\pi/\lambda_n$; Hilbert space rules.





- Musical scales & harmonic chords: Like (a jigsaw?) puzzle!
- "Known" by men & wrens without mathematical formalisms;
- Perceived & used by essential properties of eigen-solutions;
- Eigen-Solutions (relations) can be named!

A5- Well-Adapted Eigen-Functions

Continuous string of density ρ under tension T, wave velocity $v = (T/\rho)^{1/2}$, length L and fixed extremes $y(0) = y(L) = 0$. Its vibrations are a superposition of Normal Modes

$$y(x,t) = \sum_{n=0}^{\infty} A_n \sin\left(\frac{n\pi v}{L} t + \varphi_n\right) \sin\left(\frac{n\pi}{L} x\right) \; ;$$

$$E_n = A_n^2 n^2 \left(T\pi^2/4L\right)$$

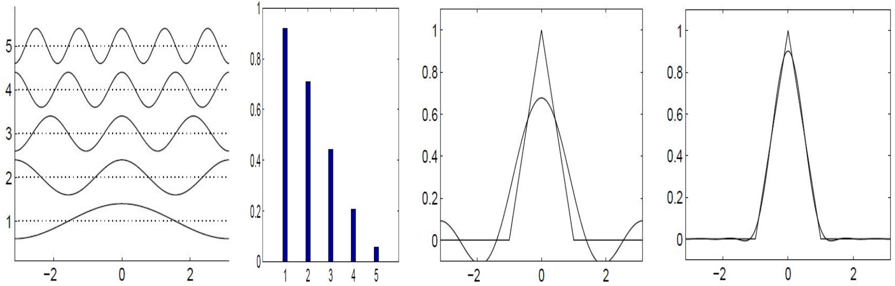
- Finite Energy: $E = \sum_{n=0}^{\infty} E_n < \infty$

$$\Rightarrow \sum_{n=0}^{\infty} A_n^2 n^2 < \infty \Rightarrow A_n = \mathcal{O}(\,1/n^{3/2+\delta}\,)$$

- This set of (basis of) eigen-functions is well-adapted:

Truncating the sums on a (relatively small) upper limit u , $\sum_{n=0}^{u}$, should provide a good description for a real string.

–see M. Jarvis (2016), Waves & Normal Modes



$$T_{2a}(x) = \begin{cases} a^{-2}(a - |x|), \text{ if } |x| < a, \\ 0, \text{ otherw. in } [-\pi, \pi] \end{cases} = \frac{1}{2\pi} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1 - \cos(na)}{n^2 a^2} \cos(nx)$$

$$R_{2a}(x) = \begin{cases} (2a)^{-1}, \text{ if } |x| < a, \\ 0, \text{ otherw. in } [-\pi, \pi] \end{cases} = \frac{1}{2\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{\sin(na)}{na} \cos(nx)$$

T_{2a} - Triangular spike Fourier syntheses w. 2 & 5 harmonics, $\mathcal{O}(1/n^2)$;
R_{2a} - Rectangular spike – this is a discontinuous function $\Rightarrow$
$\Rightarrow \infty$ energy $\Rightarrow$ Fourier series slow convergence, $\mathcal{O}(1/n)$.

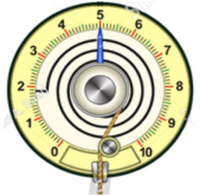
- Piskunov (1969), Folland (1992), Sadun (2001).

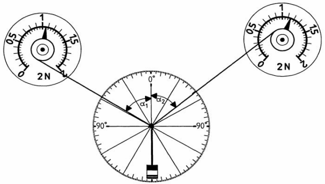
A5- Well-Adapted Eigen-Solutions & Ontologies

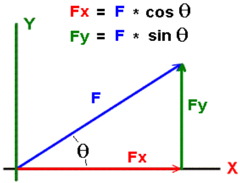
- Useful lexicons are short - or, at least, manageable;
- (Pocket) Maps are not the territory, but we need them!
- Any ontology can only hope to be well-adapted to its domain of reality, that is, in its pertinent scope of discourse;
- An ontology's domain of reality is a "complete" set of objects (tokens referring to objective eigen-solutions) that emerge in the production cycle of the pertinent scientific discipline(s);
- Cognitive Constructivism provides good anchors to the universe/ cosmos/ order subjacent to well-constructed realities;
- There *-is-* a territory to be mapped, i.e., in which a traveler can find and record his/ her/ the system's *-very own-* ways;
- Circularity in virtuous cycles, reinforcing well-anchored eigen-solutions (or finding new/ better ones); No need for skepticism.

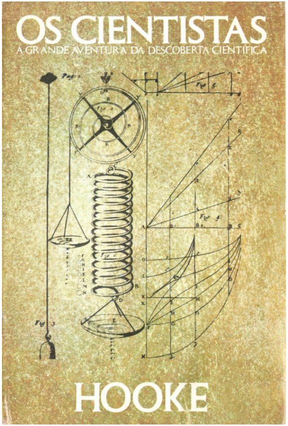
- Münchhausen trilemma (H.Albert, 1985): How to know something is true? • Circular argumentation; • Infinite regress; • Finite regress with somehow guaranteed terminal nodes.

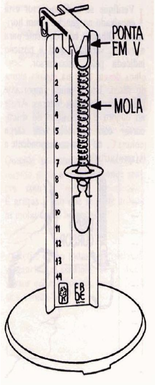
A6- Playing with force (de)composition & equilibria

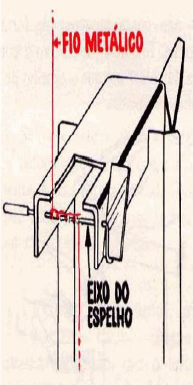












A6- Games for electronics & social construction