Continuous versions of Haack's puzzles:

Equilibrium- steady- eigen-states, ontologies, and their essential properties: Precision, stability, (de)composition

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http://www.ime.usp.br/∼jstern/miscellanea/jmsslide/Princ15A30.pdf

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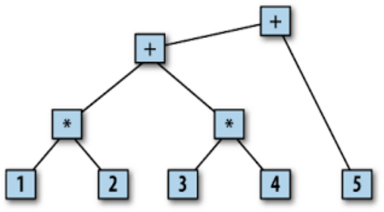
This Presentation

- II - Haack's foundherentism and crosswords:
  Essential properties of good puzzles and their solutions
- III - Invariants and equilibria in Linear Systems:
  Stability, Precision, Separation, Composition
- IV - *Chemical Affinity* tables as analytical puzzles:
  Historical evolution, from discrete inequalities to continuous equilibria, from static to dynamic invariants
- V - Dynamic invariants, eigen-solutions, etc.
- VI - Back to (cross) words:
  Production cycles, ontology and symbol grounding
- VII - Final remarks; References; FAQs

Epistemology & Susan Haack's Foundherentism



Finite regress | Infinite regress | Circular arguments

- How to prove that an hypothesis (in a theory) is true?

Trilemma* of Baron Münchhausen or Agrippa the Skeptic:

(1) Finite regress (deduction) – from *foundational* statements: Unquestionable ideals, axioms, empirical facts, observations...

(2) Infinite regress (I know it because by father does, because his father does, and so on... Hard to use in empirical science);

(3) Circular argumentation: Theory's general *coherence*.

(1+3): Foundherentism, like a crossword puzzle:

* see Stern (2015)

Crosswords Metaphor

How reasonable a crossword entry is depends on how well it is supported by its clue and any already-completed intersecting entries, how reasonable those other entries are, independent of the entry in question, and how much of the crossword has been completed. How warranted an empirical claim is depends, analogously, on how well it is supported by experience and background beliefs, how warranted those background beliefs are, independent of the claim in question, and how much of the relevant evidence the evidence includes.

[T]he natural sciences, at least, have come up with deep, broad and explanatory theories which are well anchored in experience and interlock surprisingly with each other, and, as plausibly filling in long, much-intersected entries in a crossword puzzle greatly improves one's prospects of completing more of the puzzle, these successes have enabled further successes.

Haack (1999, p.198-199).

The Crossword Metaphor and its Limits

(I): What is the positive role of circular argumentation in construing, proving or corroborating a scientific theory? Other than coherence as a trivial necessary condition?

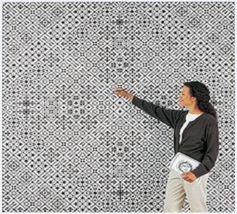
(II): What is the most appropriate form to present a scientific hypothesis H (in a statistical model and logical formalism) ?

(III): How to build and interpret $\text{ev}(H \mid X)$, a value of evidence in support of hypothesis H given the observational data X?

A common objection to coherentism is that it cannot account for truth... By stretching Susan Haacks crossword metaphor to its limits, we show that there are circumstances under which this objection is untenable. Atkinson and Peijnenburg (2010, abstr.)

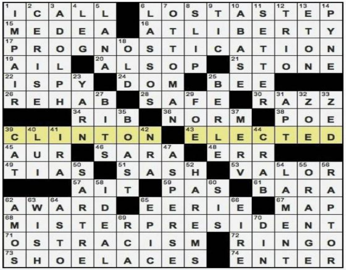
It would seem foundherentism is in need of some additional, "objective", virtuous criteria to explicate precisely what Haack means by the evaluation of C-evidence for p.
Lightbody (2006, p.19)

Theories of Everything and Huge Crosswords



As the complexity of the crossword increases, the ambiguity in general decreases: it becomes more and more difficult to come up with different solutions... The number of coherent ways of filling in a finite crossword, with a finite alphabet, irrespective of lexical constraint, is finite. In the end, if the crossword puzzle is sufficiently complicated, there might be only one solution.
Atkinson and Peijnenburg (2010, p.353-354).

Wanted: Operational notion of truth for theories limited in scope but capable of – very high *precision* – like Newtonian physics, Lavoisier chemistry, or Ohm + Kirchhoff circuit theory.



New York Times puzzle from Nov/05/1996 (election day -1)
Clues & Answers: ACROSS: (17) forcast, A: **prognostication**;
(68) title for 39th next year, A: **mister-president**;
(39+43) lead story in tomorrow newspaper,
Answer-1: **Clinton-elected**, Answer-2: **BobDole-elected**.
DOWN: (23) sewing shop purchase, A1: yarN, A2: yarD; (27) short writings, A1: biTs, A2: biOs; (35) trumpet, A1: bOast, A2: bLast; (39) black halloween animal, A1: Cat, A2: Bat; (40) french 101 word, A1: Lui, 2: Oui; (41) provider of support - for short, A1: Ira, A2: Bra; (42) much-debated political inits. A1: Nra, A2: Era.

Standard Features of the Most Amazing Crossword

- Standard size grid, 15 X 15.

- Game standard *composition* rules:

• Correct spelling and meaning of each *separate* word;
– Standard dictionary = Fixed, pre-determined *basis*;
• Letter coincidence where words intersect at the grid.

- Three *special* answer-words of maximum size ($S = 15$) running all the way across the entire length of the puzzle: (prognostication, Clinton/BobDole -elected, mister-president)

• Probability(misfit) $\lesssim F^S$, $F =$ highest letter frequency < 1;
Exponential decay on $S \Rightarrow$ Special words must fit *precisely*:
Accidental (incorrect / unintended / false) fits are very unlikely*

• Special words can be thought as long bolts that hold the entire puzzle together, making it *stable*;

* Rydberg constant relative uncertainty $\simeq$ 5.9 E–12 (CODATA),
$F \simeq 0.13$ (letter e), $S = 15$, $F^S \simeq$ 5.1 E–14

Special Features of the Most Amazing Crossword

- Additional layer of composition rules, namely, English language grammar and semantics, for the three special words: Prognostication: Clinton/BobDole elected mister-president!

- Several *genuine** solutions for the central special word:
- Each one fits in an acceptable / true solution for the puzzle;
- As initial input (guess), each one *regenerates* its own solution.

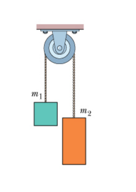
- Genuine (*eigen*) solution set spans all pertinent possibilities: they form a *basis* for the outcomes of the presidential election.

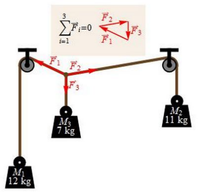
We will now study some simple continuous systems exhibiting *special (eigen*) solutions* (equilibria, functional invariants) with **essential properties** similar to those found in discrete puzzles, namely: **Precision, stability, composition, separation.**

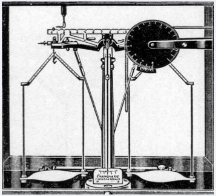
Next, we will show how to close the gap between discrete and continuous systems, using examples from history of chemistry.

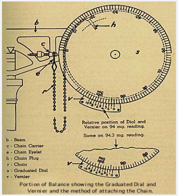
* *eigen* (German), *auto* (Latin), *self* (English)

Pulley Systems and Analytical Balances







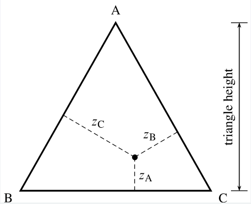


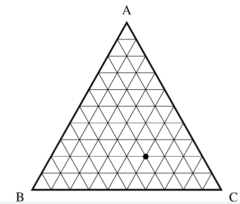
2 masses + 1 pulley – No stable equilibrium;
3 masses + 2 puleys – Stable solution:

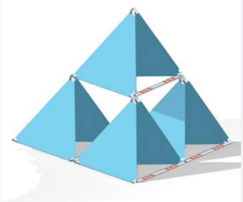
$$S\,\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \, , \quad \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} \, , \quad S = \begin{bmatrix} \sin(\theta_1) & \sin(\theta_2) & -1 \\ \cos(\theta_1) & \cos(\theta_2) & 0 \end{bmatrix} \, .$$

Decomposition / re-composition of μ-forces on the $[x, y]$ basis;
Hierarchical structure: Linear system w. non-linear coefficients;
Geometry $[\theta_1, \theta_2]$ solves $[\mu_1, \mu_2, \mu_3] \Rightarrow$ also solves $\alpha[\mu_1, \mu_2, \mu_3]$;
Variable geometry will adapt to restore system equilibrium in case of any disturbance on the fixed forces μ;
Precision instrument (5 parts per million).

Spring System and Compositional Diagrams







$$S\,\mu(z) = \mathbf{0} \quad \text{and} \quad \mathbf{1}^t z = h \ \text{(Viviani's theorem), where}$$

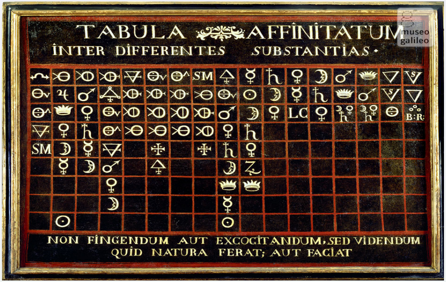
$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} , \ \mu(z) = \begin{bmatrix} k_1(z_1 - r_1) \\ k_2(z_2 - r_2) \\ k_3(z_3 - r_3) \end{bmatrix} , \ \mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} , \ \mathbf{1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} ,$$

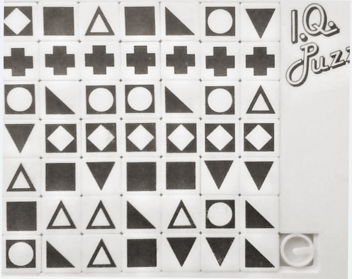
$$S = \begin{bmatrix} \sin(270^\circ) & \sin(30^\circ) & \sin(150^\circ) \\ \cos(270^\circ) & \cos(30^\circ) & \cos(150^\circ) \end{bmatrix} = \begin{bmatrix} -1 & 1/2 & 1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} .$$

μ - *Driving forces*: k, r, z - elastic const., rest & extended length;
S - Matrix of *geometric coefficients* (fixed, vs. variable forces);
$h = 1 \Rightarrow z =$ Fractional composition*, De Finetti diagram;
* (p.14): molar fractions in chemical system

Affinity Table by Étienne François Geoffroy (1718)

–Closing the gap from discrete puzzles & continuous systems–





- Chemical reactions conceived as one-directional:

They go all the way for greatest affinity (like the 1-pulley system)

- Affinity tables – "The Axioms of Chemistry":

Chemical substances ordered by reactivity (substitution)

- *Non fingendum aut excogitandum, sed videndum quid natura ferat; aut faciat.* – Not obtained by imagination or speculation, but by seeing what nature makes or actually does. (does it?)
- It looks like a (sliding block?) puzzle!

Affinity Table by Guyon de Morveau (1786)

Base/ Acid	Vitriolic	Nitric	Muriatic	Acetic	Mephitic
Barytes	65	62	36	28	14
Potash	62	58	32	26	9
Soda	58	50	31	25	8
Lime	54	44	20	19	12
Ammonia	46	38	14	20	4
Magnesia	50	40	16	17	6
Alumina	40	36	10	15	2

$$\begin{array}{c} \text{Muriate of} \\ \text{Barytes,} \\ BaCl_2 \end{array} \left\{ \begin{array}{ccc} \multicolumn{3}{c}{\overline{\underline{\text{Muriate of Potash, KCl}}}} \\ \text{Muriat.Ac.} & 32 & \text{Potash} \\ 36 & + & 9\ (=45) \\ \text{Barytes} & 14\ (=46) & \text{Meph.Ac.} \end{array} \right\} \begin{array}{c} \text{Mephite} \\ \text{of Potash,} \\ K_2CO_3 \end{array}$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{\text{Mephite of Barytes, } BaCO_3}$$

$$BaCl_2 + K_2CO_3 \rightarrow 2KCl + BaCO_3 \ ,$$

$$\text{Quiescent affinities} = 36 + 9 = 45 < \text{Divellent affinities} = 32 + 14 = 46 \ .$$

- Integer numbers tabulated – Only used to specify an order;
- $\Rightarrow$ Affinity numbers should have no other “real” meaning...
- It looks like a (inequality–sudoku?) puzzle!

– Nunc pro tunc notations... apologies.

Affinity Table in Thermodynamics / Statistical Physics

The following properties are listed at T = 298.15K:

$\Delta_f H^0$	Standard enthalpy of formation	$\Delta_f G^0$	Standard Gibbs energy of formation
S^0_m	Standard molar entropy	C_{mp}	Molar heat capacity at constant pressure

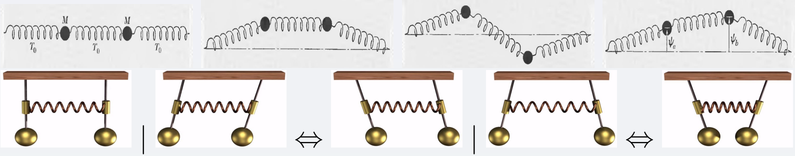
The standard state pressure is 100 kPa (1 bar). An entry of 0.0 for $\Delta_f H^0$ for an element indicates the reference state of that element. Blanks indicate no data available.

Molecular formula	Name	State	$\Delta_f H^0$ kJ mol^{-1}	$\Delta_f G^0$ kJ mol^{-1}	S^0_m J mol^{-1} K^{-1}	C_{mp} J mol^{-1} K^{-1}
Compounds not containing carbon						
Ac	Actinium	gas	406.0	366.0	188.1	20.8
Ag	Silver	cry	0.0	0.0	42.6	25.4
AgBr	Silver bromide	cry	−100.4	−96.9	107.1	52.4
$AgBrO_3$	Silver bromate	cry	−10.5	71.3	151.9	
AgCl	Silver chloride	cry	−127.0	−109.8	96.3	50.8
$AgClO_3$	Silver chlorate	cry	−30.3	64.5	142.0	
Al	Aluminum	cry	0.0	0.0	28.3	24.4
		gas	330.0	289.4	164.6	21.4

- Guldberg & Waage (1879): Reaction networks in equilibrium;
- J.C.Maxwell (1859), L.Boltzmann (1864), J.W.Gibbs (1884): Stat.Phys.; T.E.de Donder (1923): Affinity $= \mu(c, z, P, T, ...)$;
- $\mu(\;)$ – *Affinity*, function of thermodyn. consts. & state variabs, action is analogous to *driving forces* in spring system (p.11);
- z – network (regenerated) molar fractions, determined by a
- Linear system: S – fixed matrix of *stoichiometric coefficients*, c – vector of thermodynamic constants, similar to coeff.matrix & consts. (k, r) characterizing spring system

Dynamic Invariants and Eigen-Solutions (discrete)

Bases for two coupled oscillators: Transverse and longitudinal

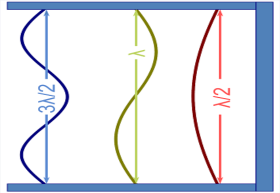


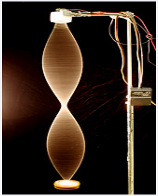
Left: Static invariant states (equilibrium) for the two systems.
Right: Dynamic invariant states for these systems:
Two *normal modes* of movement for the oscillating particles:
Symmetric mode – same amplitude and same phase,
Antisymmetric mode – same amplitude but opposite phases.

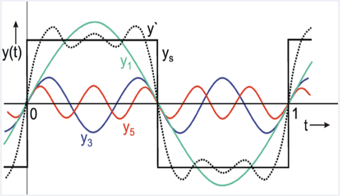
- (De)Composition: Any free movement of these systems is a linear superposition of their normal modes (eigen-solutions).
- Stability: Energy stored at each normal mode is constant.
- Precision: System's Symmetries impose strict invariant (eigen) forms and oscillating factor frequencies (eigen-values)

see Crawford (1968), Sadun (2001)

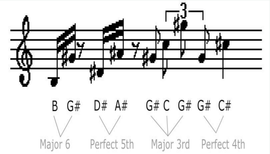
Dynamic Invariants and Eigen-Solutions (continuum)

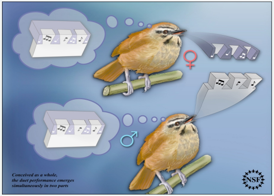






- Continuous string ($n \to \infty$ beads) $\Rightarrow$ trigonometric functions *basis*, harmonic* frequencies ($f = v/\lambda$), Hilbert space rules, ...





- Musical scales & harmonic** chords: Like (a jigsaw?) puzzle!
- “Known” by men & wrens without mathematical formalisms;
- Perceived & used by essential properties of eigen-solutions;
- Eigen-Solutions (relations) can be named!

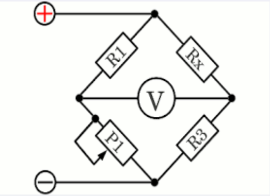
Objects are Tokens for Eigen-Solutions

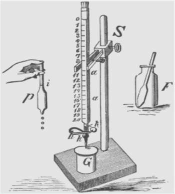
Theoretical		Metaphysical		Experimental
Mathematical formalization	$\Rightarrow$	Causal explanation	$\Rightarrow$	Hypotheses formulation
$\Uparrow$				$\Downarrow$
Speculative interpretation		Eigen-solution verification*		Experiment (trial) design
$\Uparrow$				$\Downarrow$
Statistical modeling	$\Leftarrow$	Data acquisition	$\Leftarrow$	Technological implementation
Parameter space		Operational		Sample space

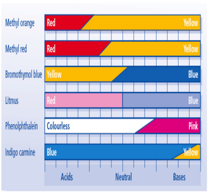
The Scientific production diagram.

- *Verification: Essential properties & overall puzzle quality;
- Good ones (true, objective) deserve names (labels, words),
- so to be re-presented in language.

Theoretical and Empirical Instruments for Verification







- Invariant quantities & also experimental means & theoretical methods are all part of the scientific production cycle, & hence must be represented in the pertinent specialized *Ontology* ≈ dictionary of *emerging* invariant quantities & auxiliary objects.

* Ohm & Kirchhoff (linear system) circuit theory +measurements on Wheatstone bridge +doped glass ion-specific electrodes +electronic amplifier = Beckman pH-meter (1936); Old volumetric titration equipment.

Ontology: Extended *Bases* for Communication

- Scientific ontology:

• Controlled language (vocabulary, grammar, semantics) used to describe concepts, means and methods of a given discipline;
• Grammar, or language articulation rules, must reflect the compositionality properties of the corresponding objects.

- Symbol grounding:

• Eigen-solutions correspond to key entities in an ontology;
• Statistical models represent them as *Precise Hypotheses*;
• There are *statistical significance measures* specially designed to access their quality (truthfulness, objectivity);
• Ex: *e*-value – $\text{ev}(H \mid X)$ – the *Bayesian epistemic value* of (sharp) hypothesis H given the (evidence) observed data X.

- IME-USP FBST research program:

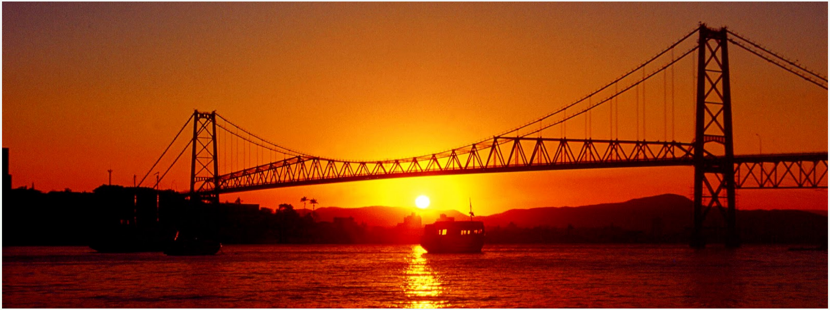
• $\text{ev}(H \mid X)$ – theory, logic, comput. methods and applications;
• Appropriate epistemological interpretations, like Popperian falsification & *p*-values, Decision theory & Bayes factors, etc.

Thank you! Grato! Gracias! Merci! Danke!

Daí, o senhor veja: tanto trabalho, ainda, por causa de uns metros de água mansinha, só por falta duma ponte.

There you see, my lord: So much work, still, just in order to overcome some friendly waters, for the lack of a good bridge.

João Guimarães Rosa; Grande Sertão: Veredas.



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FAQ1: Economic Eigen-values and Tokens

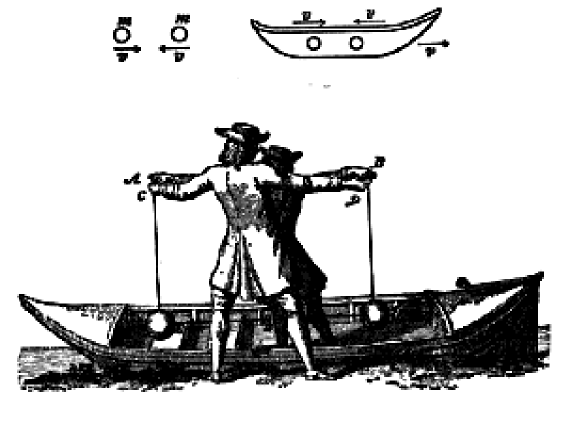
- Efficient markets exhibit *objective* prices (exchange rates);
- Nominated in a currency; Hard = convertible; (to what?)*
- Inefficient markets are unstable, imprecise, prone to arbitrage;
- Severe inefficiency can lead to a collapse of civilization! ⇒
- We have a political and social responsibility to take care...

* Owl of Athens, 450 BC, gold foil; Euro, 2002; Greek Drachma, 1973, weakly convertible; Notgeld, 1944, 1920's, non-convertible.

FAQ2: Ontological alignments

- Newton: $F = md^2x/dt^2$, $F = gm_1m_2/r^2$; Einstein: $E = mc^2$;
- 22/07/2015: 1 GBP (British Pound) $\simeq$ 6 ILS (Israeli Shekel);
- pH meter (1936) $\approx$ Litmus indicator (middle age Alchemists);
- Affinity of Geoffroy (1718) $\approx$ Affinity of de Donder (1923);
- Mephite Barytes $\approx$ $BaCO_3$; Thermodyn. $\approx$ Stat.Phys. P, T...

- Are we equating (using the same name) distinct concepts?
- Do mass, acidity, money, etc. have an invariant meaning?
- If not (strictly) so, do they have a compatible meaning?
- How can we access and measure such a compatibility?
- Distinguish synchronic/ diachronic, horizontal/ vertical, etc.



Emmy Noether's Theorem (1918): There is a correspondence between Symmetries of a theory, its Conserved quantities and its Invariant (ontological) objects, see Stern (2011b).
Figure by Christiaan Huygens (1656).

⓪ $v_0 \longrightarrow$ ① $v_1 \rightarrow$ $\Rightarrow$ ⓪ $w_0 \rightarrow$ ① $w_1 \longrightarrow$

Simple application: Collision between two bodies

- Galilean transformation (GT) as an Invariance Group:

• Consider Reference Frame $\boxed{F'}$ moving at $f' \longrightarrow$ uniform velocity relative to this slide (F); Galilean Relativity states that:
$>$ Velocities in F' are: $v'_0 = v_0 - f', v'_1 = v_1 - f', w'_0 = w_0 - f', \ldots$
$>$ Laws of Physics should be the same in F or F'

- Velocities u, v are not invariant by GT; Nevertheless, $\Delta_0 = (w_0 - v_0)$ and $\Delta_1 = (w_1 - v_1)$ are invariants.

- $\exists\ m_1(u) \in R \mid 1\Delta_0 = -m_1(u)\,\Delta_1$;
where u is the scale (unit of measure) for velocities
- Scale invariance (no privileged unit) $\Rightarrow m_1(u) = m_1$;
- Constant m_1 is the (collision) inertial <mass> of particle p_1, relative to the standard unit mass $m_0 = 1$ of particle p_0.

FAQ3: Symmetry / Invariance / Conservation Laws

- **GT** relativity + **Scale** invar. $\Rightarrow$ $1(w_0 - v_0) = -m_1(w_1 - v_1)$ is valid for any initial condition v_0, v_1 ; Assuming additional + **Compositionality** rules for relative inertia $\propto$ masses $\Rightarrow$

  ① $v_1 \longrightarrow$ ② $v_2 \rightarrow$ $\Rightarrow$ ① $w_1 \rightarrow$ ② $w_2 \longrightarrow$

  $\Rightarrow$ Momentum Conservation law:

  <MC> $m_1 v_1 + m_2 v_2 = m_1 w_1 + m_2 w_2$, where

  m_1 and m_2 are the (collision) inertial masses of particles p_1 and p_2, relative to p_0 with standard unitary mass $m_0 = 1$.

- Time Reversibility (TR) symmetry (elastic collision): $(v_1 - v_2) = -(w_1 - w_2)$;

- <MC> + **TR** $\Rightarrow$ Energy Conservation law:

  <EC> $m_1 v_1^2 + m_2 v_2^2 = m_1 w_1^2 + m_2 w_2^2$.

See A.P. French (1971). Newtonian Mechanics. MIT Press.