# Bayesian Portfolio Optimization for Electricity Generation Planning

**Hellinton H. Takada, Julio M. Stern, Oswaldo L. V. Costa and Celma de O. Ribeiro**

**Abstract** Nowadays, there are several electricity generation technologies based on the different sources, such as wind, biomass, gas, coal, and so on. Considering the uncertainties associated with the future costs of such technologies is crucial for planning purposes. In the literature, the allocation of resources in the available technologies have been solved as a mean-variance optimization problem using the expected costs and the correspondent covariance matrix. However, in practice, the expected values and the covariance matrix of interest are not exactly known parameters. Consequently, the optimal allocations obtained from the mean-variance optimization are not robust to possible errors in the estimation of such parameters. Additionally, there are specialists in the electricity generation technologies participating in the planning process and, obviously, the consideration of useful prior information based on their previous experience is of utmost importance. The Bayesian models consider not only the uncertainty in the parameters, but also the prior information from the specialists. In this paper, we introduce the Bayesian mean-variance optimization to solve the electricity generation planning problem using both improper and proper prior distributions for the parameters. In order to illustrate our approach, we present an application comparing the Bayesian with the naive mean-variance optimal portfolios.

**Keywords** Statistics · Inference methods · Energy analysis · Policy issues

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## 1 Introduction

In the early and middle years of the nineteenth century, the fundamental principles of electricity generation were discovered by scientists such as Alessandro Volta, André Ampère, Benjamin Franklin and Michael Faraday [5]. Since then, already in the last years of the nineteenth century, the electricity generation plants started to be built together with the transmission networks [9]. During the time, the mankind has developed several electricity generation technologies based on the different sources, such as wind, biomass, gas, coal, nuclear, and so on. Evidently, each technology has associated costs, sustainability, and security of supply characteristics, efficiency, and environmental concerns.

The worldwide demand for energy has been increasing over the last decades and it will continue to grow [10]. Consequently, for both countries and companies, the long-term planning of the electricity generation infrastructure is of utmost importance. Actually, it should be part of the central objectives of any energy policy. The achievement of an optimally designed electricity generation infrastructure bends toward a more balanced portfolio allocation among the different available technologies. In addition, it is also important to distinguish in the planning process the already existing electricity producing plants with maintenance costs from the ones desired to be built. Obviously, drastic changes of the electricity investment allocations is not feasible.

The U.S. Energy Information Administration has not only historical data on the average annual operation, maintenance, and fuel costs for existing power plants by major fuel or energy source types, but also projections for electricity generation costs [18]. However, even so, the costs have a significant uncertainty. For instance, future control on $CO_2$ emission and the corresponding mechanisms will surely impact the electricity generation costs. Precisely, the future price of an emitted ton of $CO_2$ is uncertain and this uncertainty should be taken into account in the planning process. Consequently, electricity generation policies solely relying on the evolution of historical average costs of electricity generation technologies are unsatisfactory.

Considering the costs as random variables, in the literature, the allocation of resources in the available electricity generation technologies has been solved as a mean-variance optimization problem using the expected values and covariance matrix of the technology costs in megawatt hours (see, for instance, [1, 2, 14, 15]). The mean-variance optimization, introduced by Markowitz [13], was the first mathematical formalization of investment diversification and it is part of the modern portfolio theory (MPT). The mean-variance optimized portfolios compose the called efficient frontier, a set of portfolios that dominate all other feasible portfolios in terms of their mean and variance tradeoff. Obviously, in the MPT the random variables of interest are the returns of the risky assets instead of the costs of the technologies.

In practice, the expected values and the covariance matrix of the electricity generation technology costs for a future time horizon are not exactly known. Noticeably, the usefulness of the allocations obtained from the mean-variance optimization depends on the preciseness of such parameters. For instance, in the MPT context, it was shown in [3] that small changes in the expected returns can produce large changes in asset allocation decisions. Consequently, several robust versions of the mean-variance optimization were proposed in the MPT literature to consider uncertainties on the expected returns and covariance matrix (see, for instance, [4, 8, 11]). Particularly in [6], for the first time in the electricity planning context, it was presented a robust portfolio optimization approach to deal with uncertainties in the input parameters.

In the electricity planning processes, it is usual to have the participation of specialists in the electricity generation technologies of interest. Undoubtedly, a natural way of conducting a comprehensive planning process is to take into account the available data together with the prior experience of the participant specialists. Bayesian approaches treat probability distributions as uncertain and subject to updates as new information becomes available. Consequently, the Bayesian approach has been successfully applied in the MPT context to take into account not only the beliefs of the investors but also the uncertainties in the expected returns and the correspondent covariance matrix (see, for instance, [3, 16, 17]). The Bayesian mean-variance portfolio optimizations could take into account both the estimation uncertainty and the specialist prior information.

In this paper, our objective is the introduction of the Bayesian approach to electricity generation planning. First, we give a brief review of the classical mean-variance optimization with the basic notation and fundamental concepts. Then, the Bayesian approach is presented using both improper and proper priors. For illustration purposes, an application comparing the Bayesian with the naive mean-variance optimal portfolios is given. Finally, some final comments are presented.

## 2 Classical Approach

Traditionally, the classical or naive mean-variance optimization presumes that cost and risk, the last one measured as the portfolio volatility, are known when making portfolio-selection decisions. Therefore, a rational planner would prefer a portfolio with a lower expected cost for a given level of risk. Alternatively, a preferred portfolio is one that minimizes risk for a given expected cost level. The set of portfolios that are optimal is called the efficient frontier. No rational planner would select a portfolio lying above the efficient frontier, since that would mean accepting a higher cost for the same amount of risk as an efficient portfolio. Equivalently, it would mean accepting greater risk for the same expected cost as an efficient portfolio.

As already mentioned and following [6, 12], it is important to distinguish in the planning process an already existing electricity producing plant using technology $i$, with random cost $C_i^e$ in USD/MWh, from a prospective idea of using $i$, with random cost $C_i^p$ in USD/MWh. The random vectors of costs for existing and prospective technologies when there are $N$ different technologies are given by

$$\mathbf{C}^e \equiv \left(C_1^e\ C_2^e\ \dots\ C_N^e\right)' \quad \text{and} \quad \mathbf{C}^p \equiv \left(C_1^p\ C_2^p\ \dots\ C_N^p\right)', \tag{1}$$

respectively. It is also usual to assume that the random costs are multivariate normal

$$\mathbf{C}^e|\boldsymbol{\mu}^e, \Sigma^e \sim \mathrm{N}\left(\boldsymbol{\mu}^e, \Sigma^e\right) \quad \text{and} \quad \mathbf{C}^p|\boldsymbol{\mu}^p, \Sigma^p \sim \mathrm{N}\left(\boldsymbol{\mu}^p, \Sigma^p\right), \tag{2}$$

where $\boldsymbol{\mu}^e = (\mu_i^e)_{N\times 1}$ and $\boldsymbol{\mu}^p = (\mu_i^p)_{N\times 1}$ are mean vectors and $\Sigma^e$ and $\Sigma^p$ are $N \times N$ covariance matrices. The means $\mu_i^e$ and $\mu_i^p$ are different, because maintenance costs are different from the costs of building a new plant. Additionally, the risk of maintenance $\sigma_i^e$ is also different from the risk of building a new plant $\sigma_i^p$. However, since the technology is the same, the correlation between $C_i^e$ and $C_i^p$ is equal to $\rho_{C_i^e, C_i^p} = 1$. Thus, we can write almost surely (with probability 1) that (see Proposition 1.1.2 from [7])

$$C_i^e = \frac{\sigma_i^e}{\sigma_i^p}\left(C_i^p - \mu_i^p\right) + \mu_i^e. \tag{3}$$

Essentially, the Eq. 3 says that the source of uncertainty for both $C_i^e$ and $C_i^p$ is the same. Additionally, $\Sigma^e = \mathrm{diag}\,(\boldsymbol{\sigma}^e)\,\mathbf{R}\,\mathrm{diag}\,(\boldsymbol{\sigma}^e)$ and $\Sigma^p = \mathrm{diag}\,(\boldsymbol{\sigma}^p)\,\mathbf{R}\,\mathrm{diag}\,(\boldsymbol{\sigma}^p)$, where the correlation matrix $\mathbf{R}$ is the same for both the existing and the prospective costs, $\boldsymbol{\sigma}^e = \left(\sigma_i^e\right)_{N\times 1}$ and $\boldsymbol{\sigma}^p = \left(\sigma_i^p\right)_{N\times 1}$.

Defining $\mathbf{C} = \left(\mathbf{C}^{e\prime}\ \mathbf{C}^{p\prime}\right)'$, it follows that

$$\mathbf{C}|\boldsymbol{\mu}, \Sigma \sim \mathrm{N}\left(\boldsymbol{\mu}, \Sigma\right), \tag{4}$$

where

$$\boldsymbol{\mu} = \left(\boldsymbol{\mu}^{e\prime}\ \boldsymbol{\mu}^{p\prime}\right)' \quad \text{and} \quad \Sigma = \begin{pmatrix} \Sigma^e & \mathrm{diag}\,(\boldsymbol{\sigma}^e)\,\mathbf{R}\,\mathrm{diag}\,(\boldsymbol{\sigma}^p) \\ \mathrm{diag}\,(\boldsymbol{\sigma}^e)\,\mathbf{R}\,\mathrm{diag}\,(\boldsymbol{\sigma}^p) & \Sigma^p \end{pmatrix}. \tag{5}$$

The portfolio weights are the proportions of the total budget allocated in each technology. The allocation vectors in the existent and prospective technologies are denoted by $\boldsymbol{\omega}^e = (\omega_i^e)_{N\times 1}$ and $\boldsymbol{\omega}^p = (\omega_i^p)_{N\times 1}$, respectively. Naturally, $0 \le \omega_i^e \le 1$, $\forall i = 1, 2, \ldots, N$; $0 \le \omega_i^p \le 1$, $\forall i = 1, 2, \ldots, N$; and

$$\sum_{i=1}^{N}\left(\omega_i^e + \omega_i^p\right) = 1. \tag{6}$$

Defining $\boldsymbol{\omega} = \left(\boldsymbol{\omega}^{e\prime}\ \boldsymbol{\omega}^{p\prime}\right)'$, we denote by $\Omega$ the set of admissible electricity generation mix so that we must have $\boldsymbol{\omega} \in \Omega$. The set $\Omega$ will represent constraints like Eq. 6, $\boldsymbol{\omega}'\mathbf{1}_{2N} = 1$ ($\mathbf{1}_{2N}$ is a $2N \times 1$ vector of ones), and minimum and/or maximum values for the allocations ($\boldsymbol{\omega}_{\min} \le \boldsymbol{\omega}$ and/or $\boldsymbol{\omega} \le \boldsymbol{\omega}_{\max}$). Using the $\boldsymbol{\omega}$ definition, the total cost of the portfolio is given by

$$\mathscr{C} = \boldsymbol{\omega}'\mathbf{C}. \tag{7}$$

Using the previous Eq. 7, the expected cost of the portfolio is given by

$$\mathrm{E}\left[\mathscr{C}\right] = \boldsymbol{\omega}'\boldsymbol{\mu} \tag{8}$$

and the variance of the portfolio is given by

$$\mathrm{Var}\left[\mathscr{C}\right] = \boldsymbol{\omega}'\Sigma\boldsymbol{\omega}. \tag{9}$$

For the case in which the vector of expected costs $\boldsymbol{\mu}$ and the covariance matrix $\Sigma$ are known, three kinds of mean-variance problems are usually considered in the literature. The first approach minimizes the variance of the costs conditional on a target maximum expected cost $c$. The target maximum expected cost $c \in \Re_+$ is provided by the electricity energy policy planner which represents the maximum allowable expected energy cost. Formally, the problem is written as follows

$$\min_{\boldsymbol{\omega}} \boldsymbol{\omega}'\Sigma\boldsymbol{\omega} \tag{10}$$

$$\text{s. t. } \boldsymbol{\omega}'\boldsymbol{\mu} \le c, \;\; \boldsymbol{\omega} \in \Omega. \tag{11}$$

The second approach, a dual form of the first approach, minimizes the expected cost conditional on a maximum value $s^2$ for the variance of the costs. The value $s^2 \in \Re_+$, provided by the policy planner, represents the maximum value that the variance of the cost could achieve. Formally, the problem is written as follows

$$\min_{\boldsymbol{\omega}} \boldsymbol{\omega}'\boldsymbol{\mu} \tag{12}$$

$$\text{s. t. } \boldsymbol{\omega}'\Sigma\boldsymbol{\omega} \le s^2, \;\; \boldsymbol{\omega} \in \Omega. \tag{13}$$

The third approach minimizes a combination of the expectation and variance of the costs, weighted by a risk aversion parameter $\lambda > 0$. Higher value of $\lambda$ indicates a greater risk aversion. Formally, the problem is written as follows

$$\min_{\boldsymbol{\omega}} \boldsymbol{\omega}'\boldsymbol{\mu} + \lambda\boldsymbol{\omega}'\Sigma\boldsymbol{\omega} \tag{14}$$

$$\text{s. t. } \boldsymbol{\omega} \in \Omega. \tag{15}$$

Trivially, using quadratic programming solvers, the previous three problems can be solved for the case in which the vector of expected costs $\boldsymbol{\mu}$ and covariance matrix $\Sigma$ are assumed to be known.

# 3 Bayesian Approach

In terms of modeling, the Bayesian approaches address estimation risk from a conceptually different perspective. Instead of treating the unknown parameters as constants, they are considered random. Additionally, the belief or prior knowledge of the specialist about the input parameters is combined with the observed data. The Bayesian models yield an entire distribution of predicted costs which explicitly takes into account the estimation and predictive uncertainty.

The predictive, posterior, or updated distribution of the unknown parameters $\boldsymbol{\mu}$ and $\Sigma$, according to the Bayes' theorem, is given by

$$\mathrm{p}\left(\boldsymbol{\mu}, \Sigma | \mathbf{c}_1, \ldots, \mathbf{c}_T\right) \propto \mathrm{L}\left(\boldsymbol{\mu}, \Sigma | \mathbf{c}_1, \ldots, \mathbf{c}_T\right) \pi\left(\boldsymbol{\mu}, \Sigma\right), \tag{16}$$

where $\mathbf{c}_1, \ldots, \mathbf{c}_T$ are recorded observations; $\pi\left(\cdot\right)$ is the prior distribution; and $\mathrm{L}\left(\cdot|\cdot\right)$ is the likelihood function. Particularly, the likelihood function is given by

$$\mathrm{L}\left(\boldsymbol{\mu}, \Sigma | \mathbf{c}_1, \ldots, \mathbf{c}_T\right) \propto |\Sigma|^{-\frac{T}{2}} \exp\left[-\frac{1}{2}\sum_{i=1}^{T}\left(\mathbf{c}_i - \boldsymbol{\mu}\right)' \Sigma^{-1}\left(\mathbf{c}_i - \boldsymbol{\mu}\right)\right]. \tag{17}$$

In the following subsections, we present the predictive distributions using improper and proper priors for the unknown parameters $\boldsymbol{\mu}$ and $\Sigma$.

## *3.1 Improper Prior Case*

In many cases, our prior beliefs are vague and thus difficult to translate into an informative prior. Therefore, we want to reflect our uncertainty about the model parameters without substantially influencing the predictive parameter inference. The so-called noninformative priors, also called vague or diffuse priors, are employed to that end. We consider the case when the investor is uncertain about the distribution of both parameters, $\boldsymbol{\mu}$ and $\Sigma$, and has no particular prior knowledge of them. This uncertainty can be represented by a improper or diffuse prior, which is typically taken to be the Jeffreys' prior,

$$\pi\left(\boldsymbol{\mu}, \Sigma\right) \propto |\Sigma|^{-\frac{(2N+1)}{2}}, \tag{18}$$

where $\boldsymbol{\mu}$ and $\Sigma$ are considered independent in the prior, and $\boldsymbol{\mu}$ is not restricted. The prior is noninformative in the sense that only changes in the data exert an influence on the predictive distribution of the parameters.

When the sample mean, $\hat{\boldsymbol{\mu}}$, and sample covariance matrix, $\hat{\Sigma}$, are given, it is straightforward to verify that the predictive distribution of the costs is a multivariate Student's t-distribution

$$\mathbf{C}|\tilde{\boldsymbol{\mu}}, \tilde{\Sigma} \sim \mathrm{t}_{T-2N}\left(\tilde{\boldsymbol{\mu}}, \tilde{\Sigma}\right), T - 2N \geq 2, \tag{19}$$

where the predictive mean and covariance matrix are, respectively,

$$\tilde{\boldsymbol{\mu}} = \hat{\boldsymbol{\mu}} \text{ and } \tilde{\Sigma} = \frac{\left(1 + T^{-1}\right)(T-1)}{T - 2N - 2}\hat{\Sigma}. \tag{20}$$

The predictive covariance here represents the sample covariance scaled up by a factor, reflecting estimation risk. For a given number of technologies $N$, the uncertainty $\tilde{\Sigma}$ decreases as more historical data become available. Actually, when $N$ is fixed and $T \to \infty$, we have $\tilde{\Sigma} \to \hat{\Sigma}$. On the other hand, with a fixed number of historical observations $T$, increasing the number of technologies $N$ respecting the constraint $T - 2N - 2 > 0$, leads to higher uncertainty and estimation risk, since the relative amount of available data declines. In practice, there is relevant information coming from specialists on energy costs. Consequently, in the next subsection, we present a study with proper priors.

### *3.2 Proper Prior Case*

The specialists have informative beliefs about the mean and covariance of technology costs. In this subsection, we adopt conjugate priors because it is an algebraic convenience producing a closed expression for the posterior. The conjugate prior for the unknown covariance matrix of the normal distribution is the inverse Wishart distribution while the conjugate prior for the mean vector of the normal distribution (conditional on $\Sigma$) is the multivariate normal:

$$\boldsymbol{\mu}|\Sigma \sim \mathrm{N}\left(\boldsymbol{\eta}, \frac{1}{\tau}\Sigma\right), \Sigma \sim \mathrm{W}^{-1}(\Psi, \nu), \tag{21}$$

where $\boldsymbol{\eta}$ is the vector of expected costs based on the specialist experience, $\tau \in \Re_+$ represents the strength of the confidence the specialist places on the value of $\boldsymbol{\eta}$, $\Psi$ is the covariance matrix based on the specialist experience, $\nu \in \Re$ represents the degrees of freedom of the inverse Wishart distribution reflecting the confidence about $\Psi$. Lower values of $\tau$ and $\nu$ indicates higher uncertainty about $\boldsymbol{\eta}$ and $\Psi$, respectively.

As in the improper prior case, the predictive distribution of the costs is a multivariate Student's t-distribution

$$\mathbf{C}|\breve{\boldsymbol{\mu}}, \breve{\Sigma} \sim \mathrm{t}_{T-2N}\left(\breve{\boldsymbol{\mu}}, \breve{\Sigma}\right), T - 2N \geq 2, \tag{22}$$

where the predictive mean and covariance matrix are, respectively,

$$\breve{\boldsymbol{\mu}} = \frac{\tau}{T+\tau}\boldsymbol{\eta} + \frac{T}{T+\tau}\hat{\boldsymbol{\mu}} \tag{23}$$

and

$$\breve{\Sigma} = \frac{T+1}{T\left(\nu+2N-1\right)}\left[\Psi + \left(T-1\right)\hat{\Sigma} + \frac{T\tau}{T+\tau}\left(\boldsymbol{\eta}-\hat{\boldsymbol{\mu}}\right)\left(\boldsymbol{\eta}-\hat{\boldsymbol{\mu}}\right)'\right]. \tag{24}$$

We notice that the predictive mean $\breve{\boldsymbol{\mu}}$ is a weighted average of the prior mean, $\boldsymbol{\eta}$, and the sample mean, $\hat{\boldsymbol{\mu}}$. In other words, the sample mean is shrunk toward the prior mean. Actually, the predictive mean and predictive covariance matrix are not proportional to the sample estimates. The improper prior case is appropriate to employ when we do not suspect that the sample mean or sample covariance matrix contains substantial estimation errors. Otherwise, the proper prior case is better when the planner believes that in the future the expectation and covariance matrix of the costs will differ substantially from the historical ones.

## 4 Results

In this section, we present an application to illustrate the robust Bayesian approaches. In [12], the vector of expected costs and standard deviations are given for 8 different technologies (differentiating between existent and prospective cases). Additionally, the correlation matrix of the technologies is also given. For the purpose of our application, we consider the data from [12] as the sample estimates of the parameters $\hat{\boldsymbol{\mu}}$ and $\hat{\Sigma}$. The naive mean-variance efficient frontier obtained using $\hat{\boldsymbol{\mu}}$ and $\hat{\Sigma}$ is presented in Figs. 1 and 2 (repeated in the two graphics). It is important to notice that the portfolios above the efficient frontier are inefficient and the portfolios below the efficient frontier are unrealizable.

In the improper prior case, illustrated in Fig. 1, the efficient frontier changes depending on the value of $T$. As already mentioned, the predictive covariance of the improper case is the sample covariance scaled up by a factor that approaches to one when T increases. Obviously, we do not have here $T$ representing the actual size of the sample used in the estimation. Actually, for us, $T$ is not only a proxy to the size of the sample used in the estimation but also the degree of confidence the planner has on the estimations based only on the historical data. Consequently, decreasing the value of $T$ shifts the efficient frontier to the right. The same shift to the right was observed in [6] using the robust mean-variance optimization when decreasing the degree of confidence the planner has on the estimations. However, the robust mean-variance optimization is computationally more expensive than our approach because the first requires more optimizations.

In the proper prior case, the hyperparameters $\boldsymbol{\eta}$ and $\Psi$ represent the prior information of the specialist about the expected value and covariance matrix of the technology costs, respectively. Since we do not have such parameters for the situation described in [12], we assume, for illustration purposes, that $\boldsymbol{\eta}$ and $\Psi$ are obtained increasing in 10% the parameters $\hat{\boldsymbol{\mu}}$, $\hat{\Sigma}^e$ and $\hat{\Sigma}^p$. In Fig. 2, we present the obtained efficient frontiers for different values of $\tau$ with $T = 50$ and $\nu = 34$. Noticeably, the

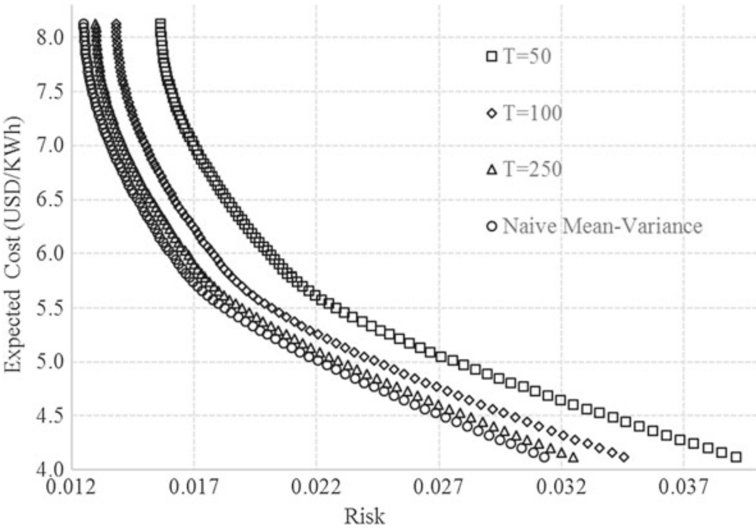


**Fig. 1** Efficient frontiers using naive and Bayesian approaches for the improper prior case for some values of $T$

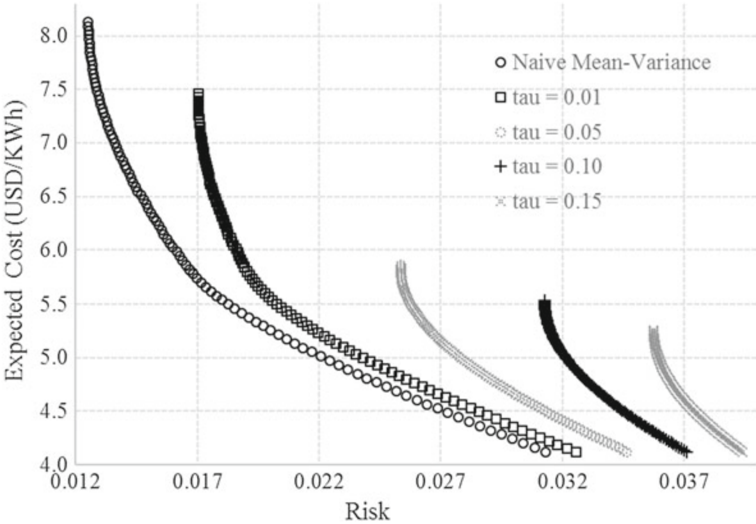


**Fig. 2** Efficient frontiers using naive and Bayesian approaches for the proper prior case for some values of $\tau$

resulting efficient frontiers are not simple shifts of the naive mean-variance frontier. Consequently, as already mentioned, the informative proper prior case is better than the improper prior when the planner believes that in the future the costs will differ substantially from the historical ones.

## 5 Final Remarks

In this paper, we introduce the use of the Bayesian mean-variance optimization in the electricity generation planning. We illustrate the application of the approach using improper and proper priors. Comparing with the existent robust approach to electricity portfolio selection, the Bayesian approach has the advantage of not only dealing with the estimation uncertainty, but also considering the prior information of the specialists in the planning process. Particularly, in the proper prior case, we have assumed that the covariance matrix of the expected value of the costs are proportional to the covariance matrix of the costs. In practice, the assumption is not necessarily valid. For future research, we suggest the investigation of changing the proper priors to give more flexibility to the electricity generation planner.

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