South American Journal of Logic
Vol. 4, n. 1, pp. 219–252, 2018
ISSN: 2446-6719

Karl Pearson and the Logic of Science: Renouncing Causal Understanding (the Bride) and Inverted Spinozism

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Abstract

Karl Pearson is the leading figure of XX century statistics. He and his co-workers crafted the core of the theory, methods and language of *frequentist* or classical statistics – the prevalent inductive logic of contemporary science. However, before working in statistics, K. Pearson had other interests in life, namely, in this order, philosophy, physics, and biological heredity. Key concepts of his philosophical and epistemological system of anti-Spinozism (a form of transcendental idealism) are carried over to his subsequent works on the logic of scientific discovery.

This article's main goal is to analyze K. Pearson early philosophical and theological ideas and to investigate how the same ideas came to influence contemporary science, either directly or indirectly – by the use of variant theories, methods and dialects of statistics, corresponding to variant statistical inference procedures and their specific belief calculi.

Keywords: Causality and natural laws, Inductive logic and probabilistic inference, Phenomenology, Logical Positivism, Karl Pearson, Baruch Spinoza.
Mathematics Subject Classification (2010): 01-02; 03-03, 62-03.

Scientific research can reduce superstition by encouraging people to think and view things in terms of cause and effect. Certain it is that a conviction, akin to religious feeling, of the rationality and intelligibility of the world lies behind all scientific work of a higher order. This firm belief, a belief bound up with a deep feeling, in a superior mind that reveals itself in world of experience, represents my conception of God. In common parlance this may be described as "pantheistic" (Spinoza).

Albert Einstein, Ideas And Opinions, (1988, p.46, 262).

לְכָה דוֹדִי לִקְרַאת כַּלָּה
יָשִׂישׂ עָלַיִךְ אֱלֹהָיִךְ כִּמְשׂוֹשׂ חָתָן עַל כַּלָּה
מֵרֹאשׁ מִקֶּדֶם נְסוּכָה סוֹף מַעֲשֶׂה בְּמַחֲשָׁבָה תְּחִלָּה

Let's go, beloved, let's meet the bride:
Your God will rejoice over you, As the groom rejoices over his bride;
Before the beginning she was offered, Planned from start, Created at end.

XVI century verses by Rabbi Shlomo Halevi Alkabetz

1 Introduction

Karl Pearson (1857-1936) is the leading figure of XX century statistics. Under his direct tutelage or influence, George Udny Yule (1871-1951), Ronald A. Fisher (1890-1962), Jerzy Neyman (1894-1981), Egon S. Pearson (1895-1980), and many others defined the methods, language and epistemology of the frequentist school of mathematical statistics, that is, of XX century mainstream statistics – the prevalent belief calculus used for inductive reasoning in the practice of contemporary science. This formalism translates K. Pearson's epistemological and philosophical position on the logic of scientific discovery, a position he calls: "Inverted-Spinozism – a Spinozism modified by Fichte".

Today, K. Pearson is known for his work in Statistics. However, his intellectual journey has many roots. Before working in Statistics, he studied and worked in philosophy, theology, physics, heredity and eugenics. Moreover, his work in these fields had a compelling influence in his later work in statistics. Nowadays, many didactic text-books present K. Pearson's philosophical positions very unfaithfully, in watered-down pseudo-positivist or in sterilized decision-theoretic versions. Like the roots of a tall tree, the origins of K. Pearson's philosophy are now deeply buried and often concealed, Nevertheless, like the branches of the same three, its epistemological and methodological consequences are clearly seen and its influence strongly felt all over the realm of contemporary statistical science. Moreover, since statistical test of hypotheses became the *de facto* standard for validating scientific research, this influence is exerted much further. In this and following articles we analyze the historical and conceptual development of K. Pearson philosophy, and consider how it has influenced statistical inference procedures and the logic of corresponding belief-calculi.

The main goal of this paper is to present Pearson's epistemological views, as he presented them in academic articles and books, and also as presented allegorically in his novel *The New Werther*. Section 2 presents my interpretation of K. Pearson's mystical journey of "Renouncing the bride", as narrated in *The New Werther*. Section 3 details more technical aspects of his philosophical

concept of Inverted-Spinozism. Sections 4 and 5 are conceived as a bridge, offering a connection to K. Pearson ideas concerning ether physics, heredity and eugenics, examining how these ideas were strongly influenced by his previous work in philosophy, and how they directly influenced his later work in statistics. Following articles will discuss in detail the concepts of direct vs. inverse probabilities, analyzing its historical roots, K. Pearson's ideas on this topic, and their profound influence in XX century mathematical statistics. Section 6 presents some final remarks, including other directions for future research, some conclusions and acknowledgments.

2 Sophia, Maimonides, Spinoza, Fichte, and Locki

K. Pearson arrives at his philosophical concept of Inverted-Spinozism after a religious and spiritual crisis, and presents his positions in several forms, including: His influential book *The Grammar of Science* (1892); some review articles about the philosophy of Spinoza (1880, 1883) and, most importantly in our context; his novel *The New Werther – by Locki* (1880). This novel presents his philosophy in a romantic and mystical context, that we explore in Section 2.2 via Spinoza's philosophical roots in Jewish philosophical and mystical writings by Moshe ben Maimon (1135-1204), Abraham Abulafia (1240-1291) and Joseph Gikatilla (1248-1310). Section 2.3 presents some of the motivations and some epistemological consequences of K. Pearson's Inverted-Spinozism.

2.1 Love and Sex (or lack thereof) in *The New Werther*

K. Pearson describes the style of *The New Werther* as schwärmerisch romanticism, using the term Schwärmerei in the sense of enthusiastic fervor. In German, this term usually means rapture or infatuation, and it is also used in religious literature to denote zealotry, in the sense of utmost commitment to ethical values and self-renunciation, see Levine (2010). In the next subsections, I present a philosophical and mystical interpretation of this outstanding and unusual work, that I see as an act of true philosophical and spiritual renunciation, namely, *Renouncing the Bride*, as explained in the sequel. I also explain why I see *The New Werther* as the point of departure and the fulcrum for all of K. Pearson latter works in science and statistics. Nevertheless, the reader should be aware that my interpretation differs from existing ones, specially from that of K. Pearson's main biographer, Theodore Porter (2004, Ch.3, p.43-68).

The New Werther describes the journey of Arthur, a young Englishmen in Germany, his adventures visiting Heidelberg university, and his wandering about the scenic routes of the Black Forest. The novel's full title, *The New*

Werther – by Locki, reveals its fictional author, Locki – named after the shape shifter, trickster and subversive Germanic god – that plays the role of a Faustian Mephistopheles, the scatterer and forgerer of lies, מֵפִיץ תֹּפֶל שָׁקֶר, as in Job 13:4. This epistolary novel involves three main characters: Arthur; Ethel, Arthur's fiance; and Raphael, Arthur's friend at the university.

Arthur unequivocally represents K. Pearson himself. According to Porter (2004, p.54-55) well documented interpretation, Raphael is a fictional character partially based on Raphael Wertheimer, K. Pearson's friend and Law student at Heidelberg. Furthermore, Porter (2004, p.55-56) concludes that "the role of Ethel was an elaboration of Pearson's friendship with Robert Parker", K. Pearson's study partner and colleague at Cambridge. Furthermore Porter considers their relationship suggestive of the "upper-class male relationships at school in the Victorian era, and the efflorescence of homosexual activity at King's College", pondering however that "there is no evidence to support, and much reason to doubt, that this friendship was ever explicitly sexual".

In the next subsections I offer an alternative reading of *The New Werther*, including an alternative interpretation of the characters Ethel and Raphael. I do not deny the validity of Porter's interpretation, not least because I do not have access to K. Pearson correspondence and other sources used by Porter in his biographical studies. Moreover, I do not consider these interpretations to be mutually exclusive. Perhaps, all sexual tensions so accurately alluded by Porter could be considered as earthly affairs contrasting, and in this way complementing and highlighting, the spiritual and philosophical concerns discussed in the next subsections. After all, in the human soul ($\psi\upsilon\chi\eta$, Psyche), several aspects of sex, love and thought and their representations appear to be deeply intertwined, as suggested by the ancient Greek myths of $\epsilon\rho o\varsigma$, Eros, in his two forms: either the playful young god of love, or his older form, $\phi\alpha\nu\eta\varsigma$ – Phanes, the shiny god of revelation. Hence, it should not be surprising to see all these aspects emerge together in the turmoil of K. Pearson's religious and spiritual crisis.

2.2 Spinoza's Epistemological Principles and its Precursors

The New Werther presents K. Pearson's doctrine of Inverted-Spinozism in the aforementioned schwärmerisch-romantic style, and is permeated by mystical elements. As an example, let us quote from the first encounter of Arthur and Raphael:

> *I was in the library, looking for a copy of Maimonides, to whom my study of Spinoza had led me, when I found a queer-looking person busily reading in a corner the very book I wanted. He had a distinctly Jewish face, and yet beneath his raven-black and straggling locks there was an apostolic nobility*

and depth. He seemed to know by inspiration what I had come for, and with a salutation offered me the book. Pearson (NW, p.22).

K. Pearson structures his philosophical doctrine of inverted-Sponizism around (and against) three major principles of Spinozian philosophy and epistemology, namely:
(1) *Deus sive natura*;
(2) *Cognitione causae* and *Leges naturae universales*;
(3) *Amor Dei intellectualis.*

These epistemological principles are presented in axiomatic form at Baruch Spinoza's magnum opus, namely, his *Ethica – Ordine Geometrico Demonstrata* of 1677. K. Pearson had great familiarity, and also strong opinions about Spinoza's works and his commentators, even publishing in 1880 a review of *Pollock's Spinoza* at the *Cambridge Review.* K. Pearson was also well aware of precursors of these epistemological principles rooted in medieval Jewish philosophy, as clearly attested by his article *Maimonides and Spinoza* published at *Mind* in 1883. As examined in this section, these medieval philosophical precursors are expressed in a language and permeated by forms of argumentation that are perceived by contemporary readers as essentially mystical in nature, for further comments see Stern (2017) and also Fraenkel (2006), Harvey (2007) and Idel (2000).

In our context, the mystical route to Spinoza's epistemological principles offers some advantages over the axiomatic (ordine geometrico) exposition, among others: (a) It is in the best spirit of *The New Werther*, a main object of study in this article; (b) It offers efficient short-cuts, allowing for close and effective connections with and between topics concerning causal analysis in natural science to be studied in the next sections. Having the aforementioned objectives and justifications in mind, let us explore Spinoza's epistemological principles via the Maimon-to-Spinoza mystic route.

Deus sive natura

Spinoza's principle of *Deus sive natura* – God, or equivalently, nature – finds a precursor in Moshe ben Maimon (1135-1204) classical formula: *ha-peʿulot ha-ʾelohiyoth*, *ha-peʿulot ha-tebʿayoth*, הַפְּעֻלּוֹת הָאֱלֹהִיּוֹת הַפְּעֻלּוֹת הַטֶּבַעִיּוֹת ; meaning: In the world we live in – Actions of God (are just) actions of nature.

Abraham Gikatilla (1248-1310) explains Maimonides' formula using forms of sylleptic argumentation that are typical of Jewish medieval philosophy and mysticism, see Stern (2017). His argument is based on the careful examination of the שֹׁרֶשׁ, *shoresh*, root or stem **טבע**. On one hand, this stem generates the word טֶבַע, *teba*, a substantive meaning nature or substance; On the other hand, this stem generates the word טָבַע, *tabʿa*, a verb meaning sank, stamped,

coined or formulated; including the derived form מַטְבֵּעַ, *matbeʿa*, coin, type, formula. Also of interest is the expression טְבִיעוּת־עַיִן, *tebiyoth–ʿayn*, meaning intuition or insight, literally, eye-impression; see Klein (1987, p.239-240, 337. Gikatilla conclusion from this philological analysis is that "things", as they present themselves in nature, are coined, patterned, and also behave according to ideal types or abstract formulas, types and formulas that convey the will of God.

Interestingly, the Latin words *causare, causa*, (to) cause; are etymologically related to *cudere, cusum*, (to) strike, hammer, forge, stamp, coin; suggesting an analogy that parallels Maimonides formulation and its later interpretation by Gikatilla. The same analogy is repeated in the Portuguese (the language of Spinoza's country of origin) words *causa/ cousa*, the cause (of something)/ (some)thing (caused).

Cognitione causae and Leges naturae universales

The preceding analysis of Spinoza's first epistemological principle, Deus sive natura, brings us directly to his second principle – The understanding of causes and universal natural laws. In the next paragraphs we reproduce this principle in abridged form, for the full version see Ethics (1677, Part I, Axiom 4; Pref. for Part III & IV).

> So, the cause or reason why God, or nature, acts, and the cause or reason why He exists, are therefore one and the same. ... The eternal and infinite Being, which we call God or Nature, acts by the same necessity as that whereby it exists. ... The knowledge of an effect involves and depends on the knowledge of its cause. ... and therefore, one and the same should be the method of understanding the nature of all things whatsoever, namely, through nature's universal laws and rules.[1]

Amor Dei intellectualis

Once the first and second principles are stated and accepted, one question presents itself: Can mankind, even if in approximate form, have knowledge of the ideal types or abstract formulas that regulate nature? Spinoza's third principle is to state the answer in the affirmative, this statement being also

[1] *Ratio igitur seu causa, cur Deus seu natura agit et cur existit, una eademque est. ... Aeternum namque illud et infinitum ens, quod Deum seu naturam appellamus, eadem, qua existit necessitate agit. ... Effectus cognitio a cognitione causae dependet et eandem involvit. ... atque adeo una eademque etiam debet esse ratio rerum qualiumcumque naturam intelligendi, nempe per leges et regulas naturae universales.*

an affirmation of God's love. From ancient times to medieval Jewish philosophy and kabalistic exegesis, this affirmative principle of God's love has been associated to a rich imagery based on Genesis 28:12:

וַיַּחֲלֹם וְהִנֵּה סֻלָּם מֻצָּב אַרְצָה וְרֹאשׁוֹ מַגִּיעַ הַשָּׁמָיְמָה וְהִנֵּה מַלְאֲכֵי אֱלֹהִים עֹלִים וְיֹרְדִים בּוֹ׃

He [Jacob] had a dream, a ladder was set on the earth with its top reaching to heaven; and behold, the angels of God were ascending and descending on it.

Abraham Abulafia, in his book Light of the Intellect (1285), אוֹר הַשֵּׂכֶל, *Or ha-Sekhel*, analyzes Maimonides philosophy in general and this third epistemological principle in particular, and reiterates the nature of the ascending and descending movements at Jacob's ladder, namely, this (e)motion is love! Love of Divine insight, *ahabah elohut sikhlut*, אַהֲבַה אֱלֹהוּת שִׂכְלוּת, meeting the love of human understanding, *ahabah enoshut sikhlut*, אַהֲבַה אֱנוֹשׁוּת שִׂכְלוּת.

Furthermore, in his book Life of the Soul (1275), סֵפֶר חַיֵּי הַנֶּפֶשׁ, *sepher chayei hanephesh*, Abulafia compares the joy of this Divine and human intellectual encounter at Jacob's ladder to the delight, *thaʿanug*, תַּעֲנוּג, of the groom and the bride, *ha-chathan ve-ha-kalah*, הַחָתָן וְהַכַּלָּה.

Abulafia's romantic analogy conceals sylleptic arguments typical of Jewish medieval philosophy and mysticism, see Stern (2017). On one hand, the verb חוֹתֵן, *chothen*, (to) marry, also carries the meanings of (to) circumcise and (to) join, connect. On the other hand, the word for bride, כַּלָּה, also means – to be completed, that is related to the roots כֹּל, all, whole; כָּלַל, (to) complete, perfect, generalize; and כְּלָל general rule, principle or law.[2]

More detailed discussion of the topics presented in this subsection can be found in Fraenkel (2006), Harvey (2007) and Idel (2000). These articles give very erudite views of these topics, commenting several aspects beyond the scope of the present paper. For example, some of these articles show how to rephrase philological and sylleptic arguments into gematrical calculations, a recourse typical of medieval kabalah.

2.3 Renouncing the Bride: Werther and Mephisto

After studying in depth the philosophy of Spinoza and its precursors, K. Pearson begins to depart from its core principles, finally arriving at his "Inverted-Spinozism – a Spinozism modified by Fichte". As he so honestly acknowledges,

[2]Gesenius (1906, p.368, 477-483) and Klein (1987, p.237, 276-278) offer etymological analyses based on classical methods of comparative linguistics. Clark (1999, p.118-119 p.293-301), Gesenius (1910, p.99-102), Frajzyngier (1979) and Horowitz (1993) suggest possible derivations based on formation pathways from biliteral bases to triliteral roots, or from a geminate root to its gradational variants.

these modifications take him so far away from Spinoza that their philosophies become, in many aspects, antithetic; hence the name Inverted-Spinozism. More technical aspects of this philosophical position are analyzed in Section 3, see also Jacobi (1994, p.502), Limnatis (2008, p.119-120) and Schelling (1994, p.107-108). In this section we pay close attention to *The New Werther* narrative.

My reading of *The New Werther* tells a tale of a groom, Arthur – representing K. Pearson, that rejects his bride. In this reading, Raphael becomes a spokesman for the philosopher Baruch Spinoza, while Ethel, Arthur's beloved bride, becomes a representation of Greek Sophia ($\sigma o \phi \iota \alpha$, wisdom) or Hebrew Shekinah (from the root שכן shochen, to dwell within – meaning Divine understanding as it can and does dwell within a human being), that is often symbolically represented in the kabalistic literature as the 'cosmic bride'. That is, Ethel becomes a representation of human profound understanding or intuitive insight that can be achieved by God's grace. In his scientific and philosophical papers, K. Pearson rejects the possibility of this kind of understanding, whereas, in *The New Werther*, Arthur rejects the love of Ethel, doing so in the most dramatic way and with the most tragic consequences.

The extent to which K. Pearson accepts "Inverted Spinozism" and, therefore, breaks away from the love of Ethel-Sophia, with all its philosophical and spiritual consequences, becomes clearer as we read *The New Werther* according to my proposed interpretation. The following key passages, that I for convenience have labeled (i) to (vii), provide stepping stones for reading *The New Werther* according to this interpretation.

In the first quotation, (i), Arthur presents his reasons for staying away from his (formerly) beloved Ethel. His motives still have an exploratory or tentative character, but his arguments will mature as the novel progresses, strengthening the determination of his decisions, and also the gravity of its consequences.

(i) *Let us show that the two great schools of materialism and idealism, which have divided the world against itself, are really at one; that the inexorable laws under which science asserts that the universe must for ever roll on, are not empiric, but deducible from the pure reason; and that, though the sway of the intellect shall thus be extended from the logical to the empirical, yet that the intellectual, the manly, shall itself be so bound up with feeling, the womanly, that the two shall be united in one being and in one life, as we have been. Let us prove that the Ideal is not a world invented by the painter and the poet, but that it exists in every Actual; that the Deity is not a cause outside and separate from material man; that the cause must not be sought outside, but rather in the effect – nay, perhaps, is that effect itself. With this end, best of friends* [Ethel, Sophia]*, have*

we renounced each other. (NW, p.11).

The next set of quotations, labeled (ii) to (v), give key snapshots of this novel climax, turning points and anti-climax. In the next scene of interest, he Germans are celebrating Walpurgisnacht, also known as Hexennacht – the night of the witches. At this ancient Germanic pagan festival, celebrated around bonfires at the hilltops, Arthur has a cathartic moment. At (ii), Arthur (K. Pearson) tries to convince Raphael (Spinoza) to turn his philosophy upside-down, to replace the absolute God by an absolute Ego – an Ego that is capable of ripping the veil from the face of nature. Raphael's answers (iii) with sarcastic questions: How could a mere mortal take command of the universe? How could a finite human being acquire infinite knowledge? How could such a claim be something more than arrogant hubris? Arthur is incapable of answering the unanswerable, but promises to seek justification for his position within the theoretical framework of German Idealism. As an indication of Pearson's commitment to German culture it is interesting to mention that, during his trip to Germany, he replaced his English given name by its German version – from Carl to Karl. Finally, at (v), Arthur expresses a feeling of guilt, as if he had done something terribly wrong: His heart senses he betrayed the love of Ethel (Sophia), and anticipates the dangers that lay ahead; Nevertheless, his mind is already committed to follow the way of departure, renouncing his promised bride.

(ii) *Do you not feel, said I* [Arthur, to Raphael]*, on a night like this, a titanic fire burning in your soul; that Nature is for the moment, your slave; that the Spirit of the Universe is at hand, and that you can compel it to raise its veil?*

(iii) *Raphael: But how can you, who believe in the pantheism of Spinoza, allow that the part can possibly compel the whole; that man, an infinitesimal portion of the Godhead, can command the Spirit of the Universe?* (NW, p.25-26).

(iv) *I [Arthur] have determined to read those German reasoners, and then again to discuss the matter with Raphael [Spinoza].* (NW, p.28).

(v) *The morning was cool, yet my head felt on fire. It seemed to me as if someone had told me I had wronged you, Ethel [Sophia]; and I knew it was not true.* (NW, p.29).

A few pages later, Arthur presents the core of his philosophy in a concise and well articulated form. The similarities with Fichte's philosophical position and the influence of the Inverted-Spinozism metaphor, as defined by Jacobi and expanded by Schelling, are fully acknowledged.

(vi) *The spirit, to be an "I", must have a nature of its own; a nature denotes constancy; constancy we term a law. The I, in order to exist, must follow this law; the permanency of forms which this law gives to our perceptions we term space and time. The thought-law which dictates permanency to the perceptions, compels us to look upon space and time as infinite and eternal. Thus we see how Eternity and Infinity, which stood before as twin giants, laughing at and ready at once to crush and obliterate the feeble actions of a finite humanity, are but the necessary creations of our own inner nature.* Pearson (NW, p.39).

Nothing good will come from following the advice of Locki, the trickster, no light will be found at the end of a path set by the father of Hell. Arthur is no longer interested in Rafael's friendship, and suggests that Raphael makes a visit to Ethel, who lives in Paris, the city of light (and love). For the next months, Arthur wanders trough the black forest in an individuation journey of sorts, that he makes having as sole companion his loyal dog, Gaspar, see Franz (1990, p.137; 2000, p.59).

Emerging from the forest, he goes to Paris, only to find Ethel (Sophia) and Raphael (Spinoza) deeply in love with each other. Arthur has an enraged tantrum, and after an outburst of jealousy and a melodramatic speech of false forgiveness, commits suicide. At the last pages of *The New Werther*, its fictional author, Locki, assumes the narrative, for poor Arthur is no longer in condition to fulfill this role. The very last words of *The New Werther* bring Locki's concluding message (vii), that he delivers pronouncing the words of the character Mephistopheles in Goethe's Faust:

(vii) *Ich bin der Geist, der stets verneint!*
Und das mit Recht; denn alles, was entsteht,
Ist werth, daß es zu Grunde geht;
Drum besser wär's, daß nichts entstünde.
[I am the spirit that all negates!
And rightly so, for all that originates,
Should rightly to its destruction run;
'Twere better then that nothing were begun.]

3 Inverted Spinozism: Science for the Absolute Ego

In order to fully understand the consequences of the alternative interpretation of *The New Werther* presented in the last section, we have to examine the origin and interpretations of the expression "Inverted Spinozism", an expression coined and used by Friedrich Heinrich Jacobi (1743-1819) to criticize, and also

describe – in a nutshell, the philosophy of Johann Gottlieb Fichte (1762-1814). The same expression is recycled by K. Pearson to label his own philosophical position as "Inverted-Spinozism – a Spinozism modified by Fichte". This expression appears for the first time in an open letter from Jacobi (1799) to Fichte from which, in the sequel, we quote some key paragraphs in abridged form, see Jacobi (1994, p.501-502):

> *"I am" and "There are things outside me". Speculative philosophy had to try to subordinate one of these propositions to the other; to derive the former from the latter or the latter from the former – exhaustively, in the end – so that there would be but one being and one truth before its eye, the all-seeing one! ... Thus the two main avenues, materialism and idealism, or the attempt to explain everything from a self-determining matter alone or from a self-determining intelligence, have the same aim. Their opposing courses do not take them apart at all, but rather bring them gradually nearer to each other until they finally touch.*
>
> *Strange, that the thought never occurred to Spinoza of inverting his philosophical cube; of making the upper side, the side of thought which he called the objective, into the lower, which he called the subjective or formal; and then of investigating whether his cube still remained the same thing; still for him the one and only true philosophical shape of reality. Everything would have transformed itself without fail under his hands at the experiment. The cube that had hitherto been "substance" for him – the one matter of two entirely different beings – would have disappeared before his eyes, and in exchange a pure flame would have flared up, burning all by itself, with no need of place or material to nourish it: Transcendental Idealism! ... I chose this image because I first found entry into the Doctrine of Science* [Wissenschaftslere (1813)] *through the representation of an inverted Spinozism.*

The expression Inverted-Spinozism was further disseminated by Friedrich Wilhelm Joseph von Schelling (1775-1854), and by several commentators ever since; In Schelling (1827, 1994, p.108) we find the following extended explanation of why Fichte's philosophy can be well characterized as an Inverted Spinozism:

> *There was no alternative there, if one did not wish to move once again into the absolute object, witch destroys everything free in the subject, than to move to the opposite – to the all-destroying subject, which was now no longer the empirical subject of Descartes, but only the absolute subject, the transcendental I. ...*

Since this I was not the empirical I, then for Fichte the I am*, which he made into the highest principle of philosophy, could also not be an empirical fact – Fichte declared it to be an action (Thathandlung, literally 'deed-action') and showed how the I could in no way exist independently of this action as a dead, immobile* thing*, but only in this act of self-positing [Selbstsetzung] in which he recognized not just a temporal, and also not just a transitory beginning which had begun the movement at some time, but the beginning which was always equally eternal – thus that, wherever and whenever one wanted to begin, this act of self-positing always had to be the beginning. Fichte's idealism thus is the complete opposite of Spinozism or is an* inverted *Spinozism, because it opposes to Spinoza's absolute object, which destroys everything subjective, the subject in its absoluteness, opposes the* deed [Thathandlung, deed-action] *to the merely immobile being (seyn) of Spinoza; the I is for Fichte, not as it is for Descartes, just something assumed for the purposes of philosophising, but the real, the true being, the absolute* Prius *of everything.*

The next sub-sections examine how K. Pearson faithfully and fully accepts the implications of being a philosopher of Inverse-Spinozism. Sections 4, 5, and following articles will examine how he carried these positions to his way of doing science in general and statistics in particular. The next subsections focus on two specific aspects of great importance, namely, the epistemological or ontological status of the external world, and the role (or lack thereof) of causal or metaphysical explanations in the practice of science.

3.1 Human Mind and Reality of "the" External World: A Contrast with the Constructivism of Paul Volkmann

Pearson restates the same views expressed at the last quotations of *The New Werther* using a strictly academic voice, see next quotation.

We require in fact a kind of inverted Spinozism, a Spinozism modified by Fichte. ... There is no need of the pre-established Harmony ... The outer world, as we conceive it, is the production of the conceiving 'Ego', not an objective reality enforcing its laws upon the subjective sensitive centre. When we talk about a law of physical nature, we only mean a necessary law of thought, any such law is only an intellectual law, the necessary method in which we are compelled to view our sensations. When we talk of wave theories, molecular theories, laws of attraction, etc., they are nothing independent of ourselves but intellectual constructions, necessary to simplify the complices of sensation which we call light, heat, planetary motion, etc. Pearson (1880), Pollock's Spinoza, p.95.

Jacobi's metaphor of Inverted Spinozism presents Transcendental Idealism as – the flaring up of a pure flame, burning all by itself, with no need of place or material to nourish it. An external cosmos would provide such a place for the Transcendental Ego's flame to exist, and natural laws governing material entities in this cosmos would provide this flame with a base or nourishment.

Nowadays, many didactic texts in statistics present K. Pearson philosophical positions as a form of mild subjectivism, a subjectivism that, on one hand, opposes naive or dogmatic forms of realism but, on the other hand, still allows the scientist to (in some form) relate to the external reality or to connect to existing objects in the world-out-there. In other words, a subjectivism that allows the scientist to establish some form of harmony between the laws governing the way objects "are" in the field of study of his science, a realm that stands by itself, and the rules of thought used to think about the same objects. This is absolutely not the position taken by K. Pearson, as it could be deduced from either Fichte's definition or Schelling's explanation of the expression Inverse Spinozism.

Further explanations concerning the error of seeking some harmony between rules of though and laws of nature, or between internal representations and external order, can be found in Pearson (1897), Philosophy of Natural Science, see following quotations, where K. Pearson criticizes the ideas of Paul Volkmann concerning this topic. For further details on the physical and epistemological ideas of Paul Volkmann, see Corry (2004, p.61-63) and Volkmann (1896, 1900). Volkmann is not only a scientist, but also a physics' professor greatly concerned with good didactic practices. As such, he comes to the conclusion that teaching physics as an axiomatic system is a bad idea, for it is neither a faithful depiction of how science is actually built, nor a way of presentation that favors its understanding. Instead, Volkmann suggests an iterative approach as the best option for both understanding and explaining the role played by the several elements that constitute a scientific theory.

Volkmann presents his epistemological ideas using the metaphor of – *The scientific system as a vaulted arch*: As the stones in a vaulted dome, the elements of a scientific system mutually support each other; as the constructive pieces of an arched bridge, the elements of a scientific work of architecture are kept in place by their mutual interaction. This beautiful metaphor is further explained in Volkmann's own words in the following quotation:

> *You can immediately ask for the deeper underlying reason concerning why there is a need, in the physical sciences, for an iterated cycle of knowledge. The reason can easily be seen in the form of emergence and internal operation of the physical sciences: The conceptual system of physics should not be understood as a system that is constructed in the manner of a building,*

from the bottom-up. Rather, it should be understood as a system of cross-references that is constructed like a vaulted dome or the arch of a bridge. In this way, it is a system that demands various mutual and reciprocal references, in which future results should be known from the start and, on the other way around, several previously stipulated conditions must be sustained retrospectively. Physics, in short, is a conceptual system that is retroactively consolidated.[3]

Paul Volkmann's epistemological position is far from any form of naïve or dogmatic realism. Instead, perhaps way ahead of his time, I would characterize him as a protoconstructivist. Nevertheless, Vokmann is also far away of K. Pearson's science of the Absolute Ego. Accordingly, K. Pearson dissects Volkann's work, pin-pointing what he considers to be his fundamental mistake. This fine critical exercise forces him to express his points of view in a very clear and candid fashion; this adversarial exercise forces K. Pearson to make it abundantly clear how far he is willing to walk his path of Inverted-Spinozism or Transcendental Idealism. Specifically, K. Pearson emphasizes the epistemological error of conferring an ontological status or simply acknowledging any thing-in-itself standing alone outside the conceiving Ego, an Ego that ultimately conceives everything into existence. According to K. Pearson, any attempt to represent, to connect to, or even to learn from an external order of things in themselves, leads to a dead end.

We might pass to Dr. Volkmann and show the vagueness of his definitions, the unphilosophical character of his epistemology, and indicate the danger which arises when loose analogies drawn from natural science are applied to other fields of thought. ... For him [Volkmann] *natural laws like the law of gravitation lie outside us while the conclusions of mathematics are thought-laws which lie inside us: "Diese Naturnothwendigkeiten ausser uns nirgends in Widerspruch treten mit den Denknothwendigkeiten in uns."* [These necessities of nature outside us, nowhere contradict the necessities

[3] *Sie können gleich hier nach dem tiefer liegenden Grunde fragen, warum für die physikalische Wissenschaft ein wiederholter Kreislauf der Erkenntniss nothwendig ist. Dieser Grund ist nach Entstehung und Betrieb der physikalischen Wissenschaft einfach darin zu sehen, dass das physikalische Begriffssystem nicht etwa aufzufassen ist als ein System, welches nach Art eines Gebäudes von unten aufgeführt wird, sondern als ein durch und durch gegenseitiges Bezugssystem, welches nach Art eines Gewölbes oder eines Brückenbogens aufgeführt wird und fordert, dass ebenso die mannigfaltigsten Bezugnahmen auf künftige Resultate bis zu einem gewissen Grade von vorneherein vorweg genommen werden müssen, wie umgekehrt bei späteren Ausführungen die mannigfaltigsten Zurückverweisungen auf frühere Verfügungen und Festsetzungen statthaben müssen. Die Physik ist kurz ein Begriffssystem mit rückwirkender Verfestigung.* My translation of Volkmann (1896, 1910, p.113-114), as restated in Volkmann (1900, p.3-4), see also Volkmann (1896, 1910, p.245) and Corry (2004, p.61-63).

of thought in us.][4]

This arises apparently from a pre-established harmony the source of which is accounted for in a manner which the writer tells us is the "Kernpunkt meiner erkenntnistheoretischen Studien auf naturwissenschaftlichem Boden." [The core of my epistemological studies concerning the foundations of natural sciences.]

It [the harmony] *lies namely in this: "dass die Logik in uns ihren Ursprung in dem gesetzmässigen Geschehen der Dinge ausser uns hat, dass die äussere Nothwendigkeit des Naturgeschehens unsere erste und recht eigentliche Lehrmeisterin ist."* [That the logic in us has its origin in the regular happenings of things outside us, that the external necessity of the natural events actually is our genuine and true instructor.] *We are only given this sentence, without one word more description of the process by which such harmony has been established!*

In conclusion, taking Jacobi's metaphor of *Inverted Spinozism* in its full extent, we can understand how K. Pearson reached the conclusion, see *The Grammar of Science* (1911, p.109-110), that the science of Transcendental Idealism can neither be an internal version or representation of external laws, nor the product of a harmony between internal rules of reasoning and external laws governing the cosmos. Instead, so-called laws of nature must be the product of internal harmonizations of distinct aspects or faculties within the conceiving Ego. That is, according to K. Pearson, a scientist should recognize that the laws of science result from...

...the harmony between his perceptive and reasoning faculties... Thus both the material and the laws of science are inherent in ourselves rather than in an outside world. Our groups of perceptions form for us reality, and the results of our reasoning on these perceptions and the conceptions deduced from them form our only genuine knowledge.

3.2 Phenomenological Science vs. Causal Theories

If it is an error to seek for "laws of nature", even if such laws are derived from a pre-existent or constructed harmony between rules of thought and rules governing objects of an external order, then it must be an even greater mistake to ask for causal explanations, that is, to seek valid answers to questions asking why "things" behave the way they do (according to a given law of nature). The next quotation, from Pearson (1901, The Prostitution of Science, p.50),

[4]Pearson (1896) gives only the German text; my translations are added in square brackets.

further analyzes the absurdity of any metaphysical quest, that is, the nonsense of searching for such causal explanations.

> *To argue from the harmony existing among my sensations to a like harmony and order in the* Dinge an sich *[things-in-themselves] is to multiply needlessly the causes of natural phenomena... If the human perceptive faculty is capable of so co-ordinating sensations that all the groups maintain their own sequence, and are in perfect harmony with each other, shortly that 'order' and 'design' appear in natural phenomena, what advantage do we gain by needlessly multiplying causes and throwing back the 'order' and harmony of our sensations upon the* Dinge an sich*, and an unknowable intellectuial faculty behind them?*

In a very condescending manner, K. Pearson speaks to the addicted to metaphysics, the seekers of causal explanations, trying to save them from wandering their erroneous path:

> *In all these cases we are dealing with the sequences of various types of motion, into which we analyze or reduce a variety of sense-impressions. Just as in the special case of gravitation, we can also describe these sequences and can frequently give a measure to the motions which we conceive to take place, but we are still wholly unable to state why these motions occur. We may talk, if we please, about the forces [and causes, and parameters] ... but in using such phrases we do not introduce an iota of new knowledge, but too often a whole alphabet of obscurity. We hide the fact that all knowledge is concise description, all cause is routine.* Pearson (1911, Ch.IV Cause and Effect - Probability, p.133).

Of course, many sensible scientists and philosophers were not so easily convinced to abandon the mission of explaining and understanding laws of nature, for they knew full well how important causal reasoning can be in the process of applying these laws to real problems in their fields of expertise, in conceiving new scientific hypotheses or theories, in implementing new experiments or research projects, etc. Lord Arthur James Balfour (1894, 1902) was among those totally unconvinced by K. Pearson's anti-metaphysical arguments. K. Pearson (1897) answers to lord Balfour are very illuminating. Once more, the adversarial style used in this disputation makes it clear how committed he is in following his path of Inverted-Spinozism.

> *Mr. Balfour speaks contemptuously of those who regard the Universe as a "mere collection of hypostatised sense-perceptions packed side by side in space and following each other with blind uniformity in time." He wants*

"ideas of wider sweep and richer content", and considers that the work of Science would be beneath contempt if it only provided a machinery by which the re-occurrence of feelings and ideas might be adequately accounted for. Pearson (ChD, p.195).

It does not matter whether it be Spinoza with his Infinite Substance, or Kant with his Dinge-an-sich, or the naturalist with his molecule, or the theologian with his personal God – one and all can tell us nothing of the real mode of action of his idolum specus. Pearson (ChD, p.193).

The physicist, who projects his concepts into the unknowable beyond sense-impression, is as unphilosophical and as dogmatic as the metaphysician or theologian. Pearson (ChD, p.202).

If the product of scientific research shall never be projected outside the conceiving Ego, if science can not go beyond accommodations between distinct properties of the perceiving, reasoning and conceiving Ego, what shall be the purpose of science? According to K. Pearson, the purpose of science is purely descriptive. This descriptive function encompasses past and future observations, that is, this role of description includes prediction, and this must be the sole motivation of scientific research. This point is made unequivocally clear in K. Pearson essay answering Balfour objections:

...how is Science related to the phenomenal world? Simply as providing comprehensive descriptive formula – so-called laws – summed up in a conceptual model which more or less completely figures past and rehearses future experience. The symbols of Science are not "things in themselves," nor are they perceptions – nay, as a rule, they do not even stand as equivalents for concrete and actual phenomena. Pearson (ChD, p.203).

[T]he mission of Science is not to explain but to describe; to discover a descriptive formula which will enable men to predict the nature of future perceptions; such descriptive formulae are, in the only consistent sense of the word, knowledge, they form that "economy of thought", which is the name happily devised by a philosophical physicist to describe and define Science. Pearson (ChD, p.200).

This last conclusion can be taken as the net result of K. Pearson's approach to philosophy of science. The role played by descriptive/ predictive formulas in science offers, on one hand, a touchstone for validation criteria and, on the other hand, avoids any metaphysical involvements and complications. This is K. Pearson's key to open the realm of epistemology. K. Pearson's motivations in the philosophy of Spinoza and Fichte are today almost forgotten. Nevertheless, the descriptive/ predictive approach to scientific hypotheses gained

wide acceptance, being developed over the first four decades of the XX century into the philosophical basis for the Frequentist school of statistics. In latter years, K. Pearson himself gave little emphasis to his early undertakings in Transcendental Idealism. Instead, he concentrated his efforts to advance the more pragmatical thesis of description/ prediction as the ultimate goal of scientific hypotheses, and to construct (Frequentist) Statistics as a language designed as a tool commissioned to best achieve this goal. The last quotation of this section, stated and restated by K. Pearson in his last years, is typical of this pragmatic approach:

> *The Laws of Nature are only constructs of our minds; none of them can be asserted to be true or to be false, they are good in so far as they give good fits to our observations of Nature, and are liable to be replaced by a better 'fit'...* Pearson (1935) as quoted in Inman (1994, p.6).

For K. Pearson, Goodness-of-fit, and nothing else, becomes the ultimate criterion of good science, a criterion that motivates and directs the development of Frequentist statistics. In contrast, according to K. Pearson, causal reasoning, laws of nature and their explanation, and similar efforts for metaphysical understanding have no legitimate role to play in the development of contemporary science, see Section 4 for further comments.

3.3 Auguste Comte's Positivism and Logical-Positivism

Auguste Comte (1798-1857) had a multifaceted life and personality, and so is his philosophical work and legacy, see Lacerda (2009), Manuel (1962) and Scharff (1995). As mentioned in the introduction, nowadays many text-books in statistics present K. Pearson's philosophy as akin to Positivism. K. Pearson's own opinion was quite different: On one hand, he gave Comte the merit of indicating the vanity of causal reasoning but, on the other hand, he contended Comte was never able to completely break away from metaphysical forms of thinking. In K. Pearson's candid evaluation of the historical influence and importance of Comte's philosophical ideas:

> *...the writings of Comte have at the very least acted as a stimulus – if only of the irritant kind* – Pearson (1911; The Grammar of Science, p.570).

As in previous sections, I use K. Pearson's adversarial arguments to highlight and clarify his full commitment to his Inverted-Spinozism. For K. Pearson, Comte relapses into metaphysics in (at least) two crucial occasions, namely, (a) the *Religion of Humanity*; and (b) the *Scala Intellectus* schema for classification of scientific disciplines.

(a) The Religion of Humanity is an enterprise of Comte's late life, only marginally successful in Comte's native France and some other countries. Today the Religion of Humanity has in Rio de Janeiro and Porto Alegre, Brasil, its last working temples, see Lins (1967) and Valentin (2010). Comte's religious enterprise got him in trouble with many of his own followers, and was disregarded without further consideration by later versions of Positivism, including the prestigious Logical Positivism movement of the Wiener Kreis. For K. Pearson, the Religion of Humanity was not only incompatible with his anti-metaphysical principles, but also did not sit well with his moral and political ideals.

> – *we act morally, that is, socially. Positivism has recognised in a vague impracticable fashion this, the only possible basis of a rational morality; it places the progress of mankind in the centre of its creed, and venerates a personified Humanity.* Pearson (1901; The Moral Basis of Socialism, p.303).

(b) K. Pearson's objections to Comte's *Scala Intellectus* or staircase of the intellect schema for classification of scientific disciplines is far more important for the purposes of this paper. In this schema, reminiscent of the Jacob's Ladder studied in Stern (2017), scientific disciplines are ranked according to a progressive order of complexity of their fields of interest, each being indispensable for the study of the next one in ascending order. Specific disciplines are ranked at a seven-step-ladder, starting from mathematics and progressing to astronomy, physics, chemistry, biology, sociology, and ethics. According to Pearson (1892, 1911; The Grammar of Science, The Classification of Sciences, p.592):

> *From Comte [we learn] that there is in reality an interdependence in the sciences, so that a clear understanding of one may necessitate a previous study of several others.*

For K. Pearson, this "interdependence in the sciences" must be unacceptable, for it implicitly opens the door for causality relations, even if in concealed forms or in indirect ways, between the objects of study of the distinct disciplines, following the very same links according to which these disciplines are hierarchically chained into a great chain of being. Furthermore, only getting rid of Comte's artificial hierarchy, can K. Pearson allow and justify crafting of good-fitting descriptive formulae for phenomena of interest of a given discipline to be a task that can and should be accomplished without recourse to concepts of lower ranking disciplines in the same hierarchy. Therefore, it is easy to understand his conclusion stated in Pearson (1892, 1911; The Grammar of Science, The Classification of Sciences, p.570):

It is clear that we have in Comte's staircase of the intellect [scala intellectus] a purely fanciful scheme, which, like the rest of his System of Positive Polity, is worthless from the standpoint of modern science.

Finally, it is worth mentioning that, as it should be expected, also in the topic at hand (concerning causal chains of being) we find K. Pearson's Inverted-Spinozism standing diametrically opposed to the philosophy of Spinoza. This contrast is made clear in Pearson (1901, Ethics of Renunciation, p.84):

In his [Spinoza] system, God, we have seen, is identified with the reality of things, not things regarded as phenomena, but as links in an infinite chain of intellectual causality. He is the λογος which dwells in and is all existence; 'laws of nature' are only the sensuous expression of the laws of the divine intellect; the story of the world is only the phenomenalising of the successive steps in the logic of pure thought.

This and previous sections' main goal was to understand key aspects of K. Pearson's philosophical studies, starting from Spinoza and his medieval precursors, and ending at a Fichtean type of Inverted-Spinozism. Furthermore, some consequences of K. Pearson's particular version of Inverted-Spinozism have been highlighted by comparison and contrast with views and ideas of other thinkers, like Friedrich Jacobi, Friedrich von Schelling, Paul Volkmann and Auguste Comte.

4 From Philosophy to Statistical inference and the Purging of Causal Agencies

This section gives a brief overview of K. Pearson work in science, from the time of his spiritual crisis and its subsequent philosophical solution to the end of his life. K. Pearson's work in science and philosophy proper lost nowadays most of its prestige and influence. Nevertheless, this section is intended as a bridge, providing historical and logical links for the next section, dedicated to K. Pearson subsequent work in developing the methods and language of *classical* or *frequentist statistics*. Nowadays, statistical science provides the standard accepted tools for hypothesis testing and validation, providing in this sense a pragmatic logic for scientific inference. Therefore, it is important to analyze and understand how the language of classical statistics continues to propagate K. Pearson's epistemological ideas, even if some of its underlying philosophical principles are now discredited, outdated or just forgotten.

4.1 Ether Physics and Anti-Atomism

In a series of papers from 1884 to 1891, K. Pearson develops some research topics in the area of Ether Physics. Today this area of research is all but forgotten, being only of historical interest, see Schaffner (1972) and Whittaker (1953). However, during the XIX century, Ether or Aether physics was an important field of study, and a main battle ground between the proponents of atomic or molecular theories and their antagonists. One of the main goals of Ether physics was to replace physical theories explained by effects caused by hypothetical atomic or molecular entities by purely phenomenological models based on the hydrodynamic or elastic properties of an imponderable medium called Ether.

K. Pearson sees the Atom as an archetypical agent of causality. For example, at physics intra-murus, atoms are agents of causality linking the realm of classical mechanics to that of thermodynamics, the causal link itself being explained and understood via the tools of statistical physics. Extra-murus, atoms and molecules are agents of causality linking the discipline of physics to that of chemistry, justifying a link in the great chain of being of Auguste Comte's hierarchical schema of *Scala Intellectus*. Therefore, K. Pearson's motivations and commitment to the development of Ether Physics are easy to understand. Notice that although modern statistical physics was developed in the XX century, atomic models explaining thermodynamic effects are as old as Robert Boyle's (1627-1691) volume-temperature law of gases, see Brush (1983), Needham (2004), Newman (1996, 2006) and Rosenfeld (1953).

Unfortunately for K. Pearson, the whole subject of Ether Physics was at that time already loosing prestige, until it came to an abrupt end in the *Annus mirabilis* of 1905, the year of Albert Einstein's papers in *Annalen der Physik* concerning Special Relativity, the photoelectric effect and Brownian motion. The autopsy of Ether physics reveals two causes for its sudden death: On one hand, Einstein's papers on Special Relativity replace Galileo's transformations by Lorentz transformations as the fundamental invariance group of physical theory, see Stern (2011) and references therein. This elegant substitution solves and explains with astonishing simplicity a host of problems that the Ether physics program was for many decades unsuccessfully trying to circumvent using always more complex and cumbersome phenomenological models. On the other hand, Einstein's papers concerning the photoelectric effect and Brownian motion, with the support of subsequent experimental work by Jean Perrin (1908, 1911) and Robert Millikan (1914, 1916), gave sufficient reasons for the overwhelming majority of the scientific community to abandon Ether models in favor of atomic or molecular theories, for further details and references see Stern (2014, 2017).

4.2 Heredity: Mendelian Genetics vs. Statistical Biometry

In the early twentieth century, the Grammar was understood by many, including by Pearson himself, as a philosophical rationale for statistics, though in fact he took up statistics only after completing its first edition [1892]. Thereafter, right to the end of his life, Pearson would make it his mission to reshape science using the tools of statistical mathematics. From 1893 to about 1905 he published a series of papers that gave a new direction to the field of statistics. In 1901 he founded, in collaboration with Francis Galton and W.F.R. Weldon, the journal Biometrika, which was dedicated to this project. Porter (2004, p7,8)

The short quotation opening this subsection describes the chronology of a transition period of K.Person's career. On one hand, his *The Grammar of Science* became a resounding success. Far from the romantic and mystical style of *The New Werther*, and avoiding the most peculiar eccentricities of Transcendental Idealism, this book was written to present his epistemological ideas to the working scientist. On the other hand, the scientific research project he had chosen to show-case his philosophy, Ether physics, was a complete failure. Therefore, at this point in time, K. Pearson desperately needed a new research project that could be used to demonstrate the usefulness of his ideas, and could also provide a refuge for himself as a working scientist.

K. Pearson was able to find a safe harbor in the fields of biometry, heredity and evolution biology. In 1891 he starts to collaborate with the zoologist Walter Frank Raphael Weldon (1860-1906), who soon introduces him to Francis Galton (1822-1911) – from then on K. Pearson's great protector, benefactor and role model in life. A common interest of these scientists was to study the inheritance of measurable characteristics in biological populations. How to best approach this problem was at that time a matter of great controversy, concerning the existence (or not) and the role played by "genes" – (hypothetical) corpuscula carrying elementary units of genetic information and/or causing the inheritance of specific characteristics.

On one hand, the role played by genes in biology is, from an epistemological point of view, quite similar to the role played by atoms in physics or molecules in chemistry. On the other hand, the chronologies of acceptance of the "molecular hypothesis" in physics and chemistry, and the "genetic hypothesis" in biology are quite different. Indirect evidence for the inheritance of discrete genes controlling specific characteristics of biological organisms was available since the work of Gregor Johann Mendel (1822-1884). However, Mendel (1865) work, done in seclusion at a Moravian monastery, was completely forgotten until his hypotheses postulating discrete genetic coding units were rediscovered by Hugo de Vries (1901, 1903). Even so, the nature of these (at that time

hypothetical) genetic units and their coding mechanisms remained unknown, eluding successive generations of chemists and biologists until J.D. Watson and F.H. Crick (1953) cracked the puzzle of DNA's double-helix structure, see also Maddox (2003) and Fuller (2003).

Galton's initial working hypotheses assume the existence of gemmules – genetic corpuscula transmitting hereditary traits. In order to support this field of research, Galton develops key concepts and modeling techniques at the core of modern mathematical statistics, including the definition of *correlation coefficients* and *linear regression* models. For historical accounts, see Cowan (1972), Fancher (1989), Gorroochurn (2016a,b) and Kevles (1985). Galton developed these techniques in order to study the simplest non-deterministic connections between cause and effect, namely, connections that are normally distributed, linear and unidirectional. For his satisfaction, Galton ascertains that such linear models produce a very good fit to his populational data banks. Nevertheless, for his great surprise, Galton finds the same linear models to be applicable either forwards or backwards in time, contradicting his concepts of temporal unidirectionality for cause and effect. Galton also realizes that, both conceptually and mathematically, such a system could be conceived as driven entirely by a Gaussian stochastic process, with no need to an additional force or agency causing the system to drift into a preset direction. Consequently, Galton felt the need for a paradigm shift – abandoning genetic causal theories in favor of non-causal and phenomenological statistical models for biological heredity.

It is at this juncture that Galton and K. Pearson start their work together. Each man had gone trough a crisis of his own, from which they emerged ready to make the same sacrifice: Renouncing the bride. After a conceptual crisis, both men were committed to build up a science that is purely descriptive-predictive in nature and strictly non-metaphysical, with no place for causal entities or explanations.

K. Pearson was a talented mathematician and, when the time came, he was ready to help Galton and make progress in the development of mathematical statistics. K. Pearson was also a skilful (and sometimes ruthless) administrator of human resources. In time, he would regiment many bright minds to work for the cause (no pun intended), always running a tight ship and keeping a steady course in pursuit of his and Galton's basic programmatic goals. The next quotation, from 1896, presents K. Pearson's pledge of allegiance to join forces with Galton. The main objective of following articles will be to show how the Frequentist school of mathematical statistics developed its means and methods so that they were tailor made to perfectly suit the aforementioned programmatic goals.

A considerable portion of the present memoir will be devoted to the expansion and fuller development of Mr. Galton's ideas, particularly their application to the problem of bi-parental inheritance. ... The causes in any individual case of inheritance are far too complex to admit of exact treatment; and up to the present the classification of the circumstances under which greater or less degrees of correlation between special groups of parents and offspring may be expected has made but little progress. This is largely owing to a certain prevalence of almost metaphysical speculation as to the causes of heredity, which has usurped the place of that careful collection and elaborate experiment by which alone sufficient data might have been accumulated, with a view to ultimately narrowing and specializing the circumstances under which correlation was measured. We must proceed from inheritance in the mass to inheritance in narrower and narrower classes, rather than attempt to build up general rules on the observation of individual instances. Shortly, we must proceed by the method of statistics, rather than by the consideration of typical cases. Person (1896, p.255). MCTE

4.3 Eugenics: Scientific & Individual Utility, Racial Control

One of the traditional goals and most exalted accomplishments of science is to give mankind understanding about the world, to provide deep insights on how it works, or some intuition on why it is the way it is. Not for K. Pearson: His conception of science is strictly phenomenological, its only goal being accurate description and prediction of sense-impressions. But if not for wisdom and understanding, what is science worth?

I am afraid I am a scientific heretic – an outcast from the true orthodox faith – I do not believe in science for its own sake. I believe only in science for man's sake. ... The first condition for State support is that we [anthropologists] show our science to be utile by turning to the problems of racial efficiency, of race-psychology, and to all those tasks that Galton described as the first duty of a nation – 'in short, to make every individual efficient both through Nature and by Nurture.' K. Pearson (1920, pp.136, 148)

Pearson's conception of science is utilitarian to the core. In the case of his scientific programs in biometry and heredity, the intended use was eugenics:

The term National Eugenics is here defined as the study of the agencies under social control that may improve or impair the racial qualities of future generations either physically or mentally. K. Pearson (1930, p.222).

The eugenics project reveals a secondary motivation for K. Pearson, Galton and the biometics school dislike of heredity theories based on hidden causal entities, like genes. In such theories, the manifested characteristics of an individual, its phenotype, are derived from its genotype, expressed through complex non-linear interactions of multiple genes, in a process that may be further coupled with epigenetic factors. In the context of a genetic theory of heredity, a responsible manipulation of population genetics would require deep understanding of highly complex genomic networks – also taking into account the ecological effects of genetic diversity and many other aspects far beyond XX century science, for further comments, see Kevles (1985, p.145). Galton and K. Pearson's approach dismissed the aforementioned obstacles using the following hyper-simplified definition of heredity.

> *Heredity: Given any organ in a parent and the same or any other organ in its offspring, the mathematical measure of heredity is the correlation of these organs for pairs of parent and offspring. The word organ here must be taken to include any characteristic which can be quantitatively measured. If tile organs are not those of parent and offspring, but of any two individuals with a given degree of blood relationship, the correlation of the two organs will still be the proper measure of the strength of heredity for the given degree of relationship.* Pearson (1896b p.259).

K. Pearson's strictly phenomenological and uninhibitedly utilitarian view of science may have been one of the contributing factors that lead him to pledge his support to most unscrupulous political projects, like in the following quotation from a speech from 1934 delivered at University College, London. Pearson died two years later, in 1936. Had he lived a few years longer, he may have recognized in this political project the hand of Mephistopheles – the scatterer and forgerer of lies – the fictional author of The New Werther, the book of romantic dreams of his youth.

> *The Royal Society Council having passed a resolution that mathematics and biology should not be mixed, Biometrika was founded with Galton as consultant and Weldon and myself as joint editors. Buccaneer expeditions in too many fields followed; fights took place on many seas, but whether we had right or wrong, whether we lost or won, we did produce some effect. The climax culminated in Galton's preaching of Eugenics and his foundation of the Eugenics Professorship. Did I say 'culmination'? No, that lies rather in the future, perhaps with* Reichskanzler *Hitler and his proposals to regenerate the German people. In Germany a vast experiment is in hand, and some of you may live to see its results. If it fails it will not be for want of enthusiasm, but rather because the Germans are only just starting the*

study of mathematical statistics in the modern sense! K. Pearson (1934, p.23), also quoted in Senn (2003, p.144, 238).

5 Inverted-Spinozism Lives! The Language of Classical Statistics

The main objective of this section is to indicate how K. Pearson's philosophy ultimately engenders the epistemological foundations of *classical* or *frequentist statistics* (XX century mainstream school), strongly influences the evolution of the theoretical framework of statistical science in general (directly for the frequentist school, and indirectly for the Bayesian school), and imposes implicit guide-lines for the development of well-conforming operational methods and models. Moreover, the language and dialects developed by these statistical schools were specifically designed to facilitate the acceptance of K. Pearson's philosophical principles in the practice of science, and to induce "epistemologically correct" forms of expression and communication in scientific research.

Although many of K. Pearson's philosophical and scientific ideas are now considered obsolete or discredited, the language of classical statistics, crafted by him and his co-workers, not only survives but thrives up to the present days. Since the second half of the XX century, statistical significance measures (following the nomenclature of frequentist statistics) became the accepted standard by which scientific hypotheses must be judged. Therefore, we can understand how K. Pearson's philosophy ultimately spreads its influence far and wide, even if rigorous philosophical principles got diluted along the way, hence exerting their influence in subtle, sometimes almost subliminal forms. Indeed, on one hand, mainstream contemporary statistics clearly upholds the descriptive/ predictive and non-explanatory nature of statistical models in particular and of science in general while, on the other hand, downplays the historical origins and epistemological foundations of the same philosophy, rooted in K. Pearson's version of Inverted-Spinozism. In the language of statistics, K. Pearson's Inverted-Spinozism is translated as deprecation of inverse-probabilities, as explained in the following paragraphs.

Statistical models distinguish two classes of variables, namely: On one hand, there are variables in the sample-space. These variables are associated with observable phenomena, quantities of interest to which we assign direct probabilities. On the other hand, there are variables in the parameter-space. These variables are associated with latent or non-observable quantities that often correspond to hidden causes of the observed phenomena. The most basic learning mechanism in statistical models, *Bayes rule*, is a formula used to update inverse probabilities according to newly available observations.

Bayes rule was first described in the article *An Essay towards solving a problem in the Doctrine of Chances*, published at The Philosophical Transactions of the Royal Society in 1763. In this article, the ideas of the Presbyterian minister Rev. Thomas Bayes (1701-1761) were communicated posthumously by Rev. Richard Price (1723-1791), nominated by Bayes as his literary executor. The motivations for Bayes work are clearly stated in his work, see next quotation. At the time, the expression *inverse-probability* was not yet in use, instead, since parameters often stand for hidden causes for observed phenomena, the expression *probabilities of causes* was commonly used, see Fienberg (2006).

> *The purpose, is to shew [show] what reason we have for believing that there are in the constitution of things, fixed laws according to which events happen, and that, therefore, the frame of the world must be the effect of the wisdom and power of an intelligent cause; and thus to confirm the argument taken from final causes for the existence of the Deity [...and...] it will be easy to see that the problem solved in this essay is more directly applicable to this purpose; for it shews [shows] us, with distinctness and precision, in every case of any particular order or recurrency of events, what reason there is to think that such recurrency or order is derived from stable causes or regulations in nature, and not from any of the irregularities of chance.* Bayes (1763), as quoted in Barker (2001, p.84).

Reading Bayes explanations, as presented by Price, is seems that their research perfectly conforms to the Spinozian program of *cognitione causae et leges naturae universales.* In subsequent generations, probabilists like Pierre-Simon de Laplace (1749-1827) and George Boole (1815-1864) champion the efforts for modeling causal relations and computing probabilities of possible causes, developing the mathematical techniques needed to build and formally present the first workable statistical models. As explained in Sections 2, 3 and 4, this course of development was turned around by the work of K. Pearson.

Methodologically, the frequentist school can be characterized by allowing the use of direct probability statements, that is, by considering observables as random variables, while strictly forbidding inverse probability statements, that is, by never considering random variables in the parameter-space. The Frequentist school's deprecation of inverse-probabilities is a 180° turn, a complete reversal of a long-standing tradition in the history of probability and statistics, for inverse probability methods had been developed by leading figures of preceding generations, like Bayes, Laplace and Boole.

The most powerful weapon K. Pearson developed for fighting his anti-metaphysical war was a new language, the language of the *frequentist* school (nowadays also knows as the *classic* school) of mathematical statistics. In this

development effort, K. Pearson had to fight in internal and external fronts. Externally, he had to promote "classical" statistics as the language of choice for the analysis and validation of scientific research. Internally, he had to battle heresies inside the statistical community, and assure the crafting of a language that is conducive of orthodox thinking, expressing arguments that are, by construction, free of metaphysical contamination. These battles are the primary focus of forthcoming articles, as further explained in the next section.

6 Final Comments and Further Research

In this section present some final comments, propose some additional topics for further research, and give due acknowledgments.

Spinoza, Bayes, Price, Comte, Boole and K. Pearson, all allowed their religious believes to inspire their philosophical or epistemological principles that, in turn, directly motivated their scientific work and ideas. The study of historical interfaces and influences between science and religion seems to have gone out of fashion, with scientists and academic philosophers and theologians often in accordance about the necessity to separate their domains of study, an isolation effort intended to protect each specialty from possible harmful interference. The topics discussed in the present article and in Stern (2017) make me seriously doubt the viability and validity of artificially imposing such hermetic seals. Further research in the following topics should help to clarity this position.

6.1 Statistical Languages and Philosophical Commitments

Following articles will show how K. Pearson an his coworkers cast the language of mathematical statistics in order to enforce the tacit acceptance and practical use of his philosophical and epistemological ideas. Several key choices in the construction of modern statistics can not be explained solely by logical properties and mathematical requirements. Instead, these key choices can only be understood as reflecting epistemological principles based on K. Pearson's inverted-spinozism, even if at the expense of developing a formalism with fewer or weaker functionalities and inconsistent logical properties, see Borges and Stern (2007), Esteves et al. (2016), Madruga (2001), Pereira et al. (2008), Stern (2017, 2018), Stern and Pereira (2014), and Stern et al. (2017). Moreover, following articles will show how K. Pearson managed to be very successful in his campaign, using the language of frequentist statistics to promote and consolidate the acceptance not only of his statistical methodologies, but also of his underlying (but far less advertized) philosophical principles.

6.2 Frequentist Epistemology and Logical Positivism

Later versions of the Positivist movement, known as Logical Positivism, Empirical Positivism or Logical Empiricism, were developed under the influence of the Wiener Kreis from 1907 to 1938, and subsequently around the world. These neopositivist programs abandon the goal of becoming all encompassing systems of philosophy, taken instead a more pragmatic approach, concentrating their efforts into specific epistemological questions and the development of mathematical logic as a tool to support their research program.

Rudolf Carnap (1932) stated as a main goal of the neopositivist program to be – *The Elimination of Metaphysics Through Logical Analysis of Language*. Meanwhile, K. Pearson and his school were developing Frequentist statistics as a tool for building up a science free of metaphysics. This is a far more ambitious and radical proposal since statistics, as conceived by K. Pearson, is not a tool designed to analyze science as it is and circumvent its metaphysical problems, but a tool designed for actually making scientific research in such a way that science becomes, by construction, free of metaphysics.

I have the impression that the two approaches, based on their respective supporting languages, namely, mathematical logic and mathematical statistics, had the potential of greatly benefiting from each other. However, the interaction between researchers of the two groups and their mutual influence seems to have been rather small. Nevertheless, in the second part of the XX century, science practitioners and writers of didactic textbooks have often confused ideas coming from neopositivism and Frequentist statistics, carelessly overlapping topics that would require more careful treatment. I believe that confirming these first impressions and trying to understand these parallel courses of historical development are topics that deserve future research.

6.3 Acknowledgements

The author is grateful for the support of IME-USP – the Institute of Mathematics and Statistics of the University of São Paulo; FAPESP – the State of São Paulo Research Foundation (grants CEPID 2013/07375-0 and CEPID 2014/50279-4); and CNPq – the Brazilian National Counsel of Technological and Scientific Development (grants PQ 301206/2011-2 and PQ 301892/2015-6). Finally, the author is grateful for advice and comments concerning the first version of this paper received from Rafael Bassi Stern, Rabbi Ruben Sternschein, Jean-Yves Béziau, from anonymous referees, from participants of the 2nd World Congress on Logic and Religion, held on June 18-22, 2017, at Warsaw, Poland, and from participants of the X Principia International Symposium, held on August 14-17, 2017, at Florianópolis, Brazil.

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