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# FBST for Unit Root Problems

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**Abstract.** This paper presents the Full Bayesian Significance Test for unit roots in auto-regressive time series, and compares it to other approaches on a benchmark of 14 econometric series.

**Keywords.** ARMA models, e-values, FBST, Unit roots.

## INTRODUCTION

Testing for unit roots in ARMA time series models is a problem that presents well known and documented difficulties for standard Bayes Factor methodologies, see [1], [2], [4], [6], [10], and [13] to [16]. In [2, p.159], the authors state:

*“Testing for unit root is a Bayesian framework in one of the most controversial topics in the economic literature. There are several reasons for this:*

*- First ... [the use of] information that is not contained in the likelihood function and this violates the likelihood principle to which Bayesians stick.*

*- Secondly, the unit root hypothesis is a point hypothesis and Bayesians do not like testing point hypothesis because it is not natural to compare an interval which receives a positive probability with a point null hypothesis of zero mass.*

*- Finally, classical and Bayesian unit root tests do not give the same answer. This is a striking example where it is not possible to recover the classical results using a non-informative prior.”*

We will show that the FBST, or Full Bayesian Significance Test, presented in [9], easily overcomes all these difficulties, see also [7], and [17]. Moreover, the FBST e-values are computed following the absolutely standard form of FBST formalism, using non-informative priors, and in strict observance of the likelihood principle. Finally, the FBST analysis agrees with the classic analysis on a benchmark of 14 time series commonly used in the econometric literature.

The first section describes the problem and the general model used to test for unit roots and derives the posterior distribution used in the present work to perform the FBST. Afterwards, we describe the numerical procedures to calculate the e-values. Concluding, we compare the classical and bayesian procedures, pointing the FBST advantages.

## FBST REVIEW

The FBST was specially designed to give an epistemic value, or value of evidence, supporting a sharp hypothesis  $H$ . This support function is the  $e$ -value,  $ev(H)$ . Furthermore,

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the  $e$ -value has many necessary or desirable properties for a statistical support function, such as:

(I) Give an intuitive and simple measure of significance for the hypothesis in test, ideally, a *probability* defined directly in the original or *natural parameter space*.

(II) Have an intrinsically geometric definition, independent of any non-geometric aspect, like the particular parameterization of the (manifold representing the) null hypothesis being tested, or the particular coordinate system chosen for the parameter space, i.e., be an *invariant* procedure.

(III) Give a measure of significance that is smooth, i.e. *continuous and differentiable*, on the hypothesis parameters and sample statistics, under appropriate regularity conditions of the model.

(IV) Obey the *likelihood principle*, i.e., the information gathered from observations should be represented by, and only by, the likelihood function.

(V) Require *no ad hoc artifice* like assigning a positive prior probability to zero measure sets, or setting an arbitrary initial belief ratio between hypotheses.

(VI) Be a *possibilistic* support function, where the support of a logical disjunction is the maximum support among the support of the disjuncts.

(VII) Be able to provide a *consistent* test for a given sharp hypothesis.

(VIII) Be able to provide *compositionality* operations in complex models.

(IX) Be an *exact* procedure, i.e., make no use of “large sample” asymptotic approximations when computing the  $e$ -value.

(X) Allow the incorporation of previous experience or expert’s opinion via (subjective) *prior distributions*.

The objective of this section is to provide a very short review of the FBST theoretical framework, summarizing the most important statistical properties of its support function, the  $e$ -value. It also summarizes the logical (algebraic) properties of the  $e$ -value, and its relations to other classical support calculi, including possibilistic calculus and logic, paraconsistent and classical. Further details, demonstrations of theoretical properties, comparison with other statistical tests for sharp hypotheses, and an extensive list of references can be found in the author’s previous papers.

Let  $\theta \in \Theta \subseteq R^p$  be a vector parameter of interest, and  $L(\theta|x)$  be the likelihood associated to the observed data  $x$ , a standard statistical model. Under the Bayesian paradigm the posterior density,  $p_n(\theta)$ , is proportional to the product of the likelihood and a prior density. The (null) hypothesis  $H$  states that the parameter lies in the null set, defined by inequality and equality constraints given by vector functions  $g$  and  $h$  in the parameter space.

$$p_n(\theta) \propto L(\theta|X) p_0(\theta), \quad \Theta_H = \{\theta \in \Theta | g(\theta) \leq \mathbf{0} \wedge h(\theta) = \mathbf{0}\}.$$

From now on, we use a relaxed notation, writing  $H$  instead of  $\Theta_H$ . We are particularly interested in sharp (precise) hypotheses, i.e., those in which  $\dim(H) < \dim(\Theta)$ , i.e. there is at least one equality constraint.

The FBST defines  $ev(H)$ , the  $e$ -value, the epistemic value or value of (presented or observed) evidence supporting (in favor of) the hypothesis  $H$ , and  $\overline{ev}(H)$ , the  $e$ -value

against  $H$ , as

$$s(\theta) = \frac{p_n(\theta)}{r(\theta)}, \quad s^* = s(\theta^*) = \sup_{\theta \in H} s(\theta), \quad \hat{s} = s(\hat{\theta}) = \sup_{\theta \in \Theta} s(\theta),$$

$$T(v) = \{\theta \in \Theta | s(\theta) \leq v\}, \quad W(v) = \int_{T(v)} p_n(\theta) d\theta, \quad \text{ev}(H) = W(s^*),$$

$$\bar{T}(v) = \Theta - T(v), \quad \bar{W}(v) = 1 - W(v), \quad \bar{\text{ev}}(H) = \bar{W}(s^*) = 1 - \text{ev}(H).$$

The function  $s(\theta)$  is known as the posterior surprise relative to a given reference density,  $r(\theta)$ .  $W(v)$  is the cumulative surprise distribution. The surprise function was used, among other statisticians, by Good, Evans and Royall. Its role in the FBST is to make  $\text{ev}(H)$  explicitly invariant under suitable transformations on the coordinate system of the parameter space, see next section.

The tangential (to the hypothesis) set  $\bar{T} = \bar{T}(s^*)$ , is a Highest Relative Surprise Set (HRSS). It contains the points of the parameter space with higher surprise, relative to the reference density, than any point in the null set  $H$ . When  $r(\theta) \propto 1$ , the possibly improper uniform density,  $\bar{T}$  is the Posterior's Highest Density Probability Set (HDPS) tangential to the null set  $H$ . Small values of  $\bar{\text{ev}}(H)$  indicate that the hypothesis traverses high density regions, favoring the hypothesis.

In the FBST the role of the reference density,  $r(\theta)$  is to make  $\bar{\text{ev}}(H)$  explicitly invariant under suitable transformations of the coordinate system. Invariance, as used in statistics, is a metric concept. The reference density can be interpreted as a compact and interpretable representation for the reference metric in the original parameter space. This metric is given by the geodesic distance on the density surface. The natural choice of reference density is an uninformative prior, interpreted as a representation of no information in the parameter space, or the limit prior for no observations, or the neutral ground state for the Bayesian operation. Standard (possibly improper) uninformative priors include the uniform and maximum entropy densities.

Let us consider the cumulative distribution of the  $e$ -value against the hypothesis,  $\bar{V}(c) = \Pr(\bar{\text{ev}} \leq c)$ , given  $\theta^0$ , the true value of the parameter. Under appropriate regularity conditions, for increasing sample size,  $n \rightarrow \infty$ , we can say the following:

- If  $H$  is false,  $\theta^0 \notin H$ , then  $\bar{\text{ev}}$  converges (in probability) to 1, that is,  $\bar{V}(0 \leq c < 1) \rightarrow 0$ .
- If  $H$  is true,  $\theta^0 \in H$ , then  $\bar{V}(c)$ , the confidence level, is approximated by the function

$$QQ(t, h, c) = Q(t - h, Q^{-1}(t, c)), \quad \text{where}$$

$$Q(k, x) = \Gamma(k/2, x/2) / \Gamma(k/2, \infty), \quad \Gamma(k, x) = \int_0^x y^{k-1} e^{-y} dy,$$

$t = \dim(\Theta)$ ,  $h = \dim(H)$  and  $Q(k, x)$  is the cumulative chi-square distribution with  $k$  degrees of freedom. Figure 2.2 portrays  $QQ(t, h, c) = Q(t - h, Q^{-1}(t, c))$  for  $t = 2 \dots 4$  and  $h = 0 \dots t - 1$ .

Under the same regularity conditions, an appropriate choice of threshold or critical level,  $c(n)$ , provides a consistent test,  $\tau_c$ , that rejects the hypothesis if  $\bar{\text{ev}}(H) > c$ .

The empirical power analysis developed in [7] and [18], provides critical levels that are consistent and also effective for small samples.

## THE AUTO REGRESSIVE TIME SERIES MODEL

The  $AR(1)$  process

$$y_t = \phi y_{t-1} + \varepsilon_t$$

where  $\varepsilon_t \sim i.i.d.(0, \sigma^2)$ , has a unit root if  $\phi = 1$ . In this case its mean and its variance do not exist. If  $|\phi| < 1$ , then the mean of  $y_t$  is zero and its variance  $\sigma^2/(1 - \phi^2)$  and the process has a strong tendency to return to its mean value after a shock. However, if the process has a unit root, a shock has an everlasting effect. This can be seen if  $y_t$  is expressed as the cumulated sum of past errors, each with the same weight. Therefore, test for a unit root consists in testing the precise hypothesis  $H_0 : \phi = 1$ .

The economic and econometric literature has given great importance to the development of unit root tests in the past two and a half decades. It is very important to know if, for instance, economic recessions have permanent consequences for the level of future GNP, or instead represent just a temporary downturn with the output lost eventually made up during recovery. Nelson and Plosser, [10], argued that many economic series are better characterized by unit roots than by deterministic trends.

However, in the development of the tests difficulties arised because the asymptotic distribution of the ordinary least squares estimators presents a discontinuity at  $\phi = 1$ . The ADF test is the most used in unit root tests and assumes, in its more general form, that the data generating process has a constant, a deterministic trend and follows an  $AR(p)$  structure with *i.i.d.* errors. Below we introduce this model assuming gaussian disturbances to develop the bayesian inference.

The  $AR(p)$ , or order  $p$  auto-regressive time series model with white Gaussian noise and deterministic intercept and trend, is written as:

$$y_t = \mu + \delta t + \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \varepsilon_t$$

where  $\varepsilon_t \sim N(0, \sigma^2) \forall t = 1, \dots, T$ . This series can also be written in the differenced or correction form:

$$\Delta y_t = \mu + \delta t + \Gamma_0 y_{t-1} + \Gamma_1 \Delta y_{t-1} + \dots + \Gamma_{p-1} \Delta y_{t-p+1} + \varepsilon_t$$

where  $\Delta y_t = y_t - y_{t-1}$ ,  $\Gamma_0 = \phi_1 + \dots + \phi_p - 1$  and  $\Gamma_i = -\sum_{j=i+1}^p \phi_j$ , for  $i = 1, \dots, p-1$ .

If using this parametrization, the series has a unit root if  $\Gamma_0 = 0$ . The ADF tests this hypothesis against  $\Gamma_0 \leq 0$ , but if  $\Gamma_0 \geq 0$  the process is non-stationary.

This model can also be written in standard regression form, using the parameter vector  $\theta = [\beta, \sigma]$ , where  $\beta$  is a vector with all the linear parameters,  $Y_p = [y_1 \dots y_p]$  is the vector of the first  $p$  observations, and  $Y$  is the vector of all remaining observations:

$$Y = X\beta + e, \text{ where}$$

$$\theta = \begin{bmatrix} \mu \\ \delta \\ \Gamma_0 \\ \Gamma_1 \\ \dots \\ \Gamma_{p-1} \end{bmatrix}, \quad Y = \begin{bmatrix} \Delta y_{p+1} \\ \Delta y_{p+2} \\ \dots \\ \Delta y_T \end{bmatrix}, \quad X = \begin{bmatrix} 1 & 1 & y_p & \Delta y_p & \dots & \Delta y_2 \\ 1 & 2 & y_{p+1} & \Delta y_{p+1} & \dots & \Delta y_3 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & T & y_{T-1} & \Delta y_{T-1} & \dots & \Delta y_{T-p+1} \end{bmatrix}$$

The dimensions of these matrices are, respectively,  $p + 2 \times 1$  for  $\beta$ ,  $T - p \times 1$  for  $Y$ , and  $T - p \times p + 2$  for  $X$ .

Using the matrix regression form, it is easy to see that the ML estimator of  $\beta$ , the predicted ML observations, and the sum of squared errors is given by

$$\hat{\beta} = (X'X)^{-1}X'Y, \quad \hat{Y} = X\hat{\beta}, \quad \text{and}$$

$$e'e = (Y - X\hat{\beta})'(Y - X\hat{\beta}) = (Y - \hat{Y})'(Y - \hat{Y}) + (\beta - \hat{\beta})'X'X(\beta - \hat{\beta}).$$

Using the standard non-informative prior  $f(\beta, \sigma) \propto 1/\sigma$ , the model posterior can be written as:

$$f(\beta, \sigma | Y, Y_p) \propto \sigma^{-(T-p+1)} \exp\left(-\frac{1}{2\sigma^2} \left( (Y - \hat{Y})'(Y - \hat{Y}) + (\theta - \hat{\theta})'X'X(\theta - \hat{\theta}) \right)\right)$$

## NUMERICAL EXPERIMENTS AND RESULTS

After the model derived above we tested for unit roots 14 U.S. macroeconomic time series first mentioned in Nelson and Plosser, [10]. Here we use the extended series, used in Schotman and van Dijk, [14].

The following table shows the e-values and ADF test for the aforementioned econometric time series. The ADF, Augmented Dickey and Fuller test, based on the Frisch-Waugh-Lovell theorem, is arguably the most used unit root test in econometrics. We have used the computer procedure described in James MacKinnon, at Queen's University, [8]. All numerical time series follow the specification in Bauwens et al. [2], so that the results are comparable.

As can be seen from the posterior expression, the conditional posteriors are  $\pi(\theta | \sigma, Y, Y_p) \propto N(\hat{\theta}, \sigma^2 V)$  and  $\pi(1/\sigma^2 | \theta, Y, Y_p) \propto \Gamma(T - p/2, B)$ , where  $B = 0.5(Y - \hat{Y})'(Y - \hat{Y}) + (\theta - \hat{\theta})'X'X(\theta - \hat{\theta})$  and  $V = (X'X)^{-1}$ . For the FBST computations, several solvers can be used in the optimization step, as [3] or [5], and standard Monte Carlo sampling is used in the integration step, see [7].

In table 1 we can see that the non-stationary posterior probability is quite distant from the ADF p-value. These results were highlighted by Sims, [15] and Sims and Uhlig, [16]. Considering the simplest  $AR(1)$  model, they argued that, once classical inference is based on the distribution of  $\hat{\phi} | \phi = 1$ , it reaches counterintuitive conclusions because the referred distribution is skewed. Bayesian inference, they conclude, uses the distribution of  $\phi | \hat{\phi}, y_1, \dots, y_T$  which is not skewed.

Phillips, [13] claims that the difference in results between classical and bayesian approaches is due to the flat prior that puts much weight on the stationary region. He proposed the use of Jeffreys priors, which restored the conclusions drawn by the classical test. Phillips argued that the flat prior was, actually, informative when used in time series models like those for unit root tests. He made simulations that show the

*“ [the use of a] flat prior has a tendency to bias the posterior towards stationarity. ... even when [the estimator] is close to unity, there may still be a non negligible downward bias in the [flat] posterior probabilities ”.*

**TABLE 1.** Unit root tests for Nelson and Plosser data

Series	start	p	trend	ADF	p-value	$P(\Gamma_0 \geq 0 Y)$	e-value
Real GNP	1909	2	yes	-3.52	0.044	0.0005	0.045
Nominal GNP	1909	2	yes	-2.06	0.559	0.0238	0.542
Real GNP per capita	1909	2	yes	-3.59	0.037	0.0004	0.039
Industrial prod.	1860	2	yes	-3.62	0.032	0.0003	0.031
Employment	1890	2	yes	-3.47	0.048	0.0004	0.049
Unemployment rate	1890	4	no	-4.04	0.019	0.0001	0.023
GNP deflator	1889	2	yes	-1.62	0.778	0.0584	0.771
Consumer prices	1860	4	yes	-1.22	0.902	0.1154	0.984
Nominal wages	1900	2	yes	-2.40	0.377	0.0106	0.361
Real wages	1900	2	yes	-1.71	0.739	0.0475	0.724
Money stock	1889	2	yes	-2.91	0.164	0.0029	0.157
Velocity	1869	2	yes	-1.62	0.779	0.0620	0.784
Bond yield	1900	4	no	-1.35	0.602	0.0962	0.941
Stock prices	1871	2	yes	-2.44	0.357	0.0103	0.363

**TABLE 2.** MLE under  $H_0 : \Gamma_0 = 0$ 

Parameters	Real GNP	Ind. Prod.	GNP def.	Wage
$\mu$	0.01543	0.049427	0.00187	0.01494
$\delta$	0.00011	-0.00014	0.00027	0.00020
$\Gamma_1$	0.33146	0.03636	0.44992	0.46687
$\sigma$	0.05558	0.09682	0.04364	0.05545

**TABLE 3.** Standard error of MLE under  $H_0 : \Gamma_0 = 0$ 

Parameters	Real GNP	Ind. Prod.	GNP def.	Wage
$\mu$	0.01320	0.01806	0.00902	0.01247
$\delta$	0.00028	0.00024	0.00016	0.00024
$\Gamma_1$	0.10895	0.08966	0.09163	0.09661

**TABLE 4.** MLE - unrestricted model

Parameters	Real GNP	Ind. Prod.	GNP def.	Wage
$\mu$	0.81849	0.05221	0.09086	0.39792
$\delta$	0.00567	0.00718	0.00112	0.00309
$\Gamma_0$	-0.17631	-0.17658	-0.03164	-0.06494
$\Gamma_1$	0.41106	0.12432	0.46979	0.50130
$\sigma$	0.05193	0.09252	0.04329	0.05392

Tables 2 and 3 display some ML estimators and the respective standard errors assuming unit roots. Table 4 and 5 show the ML estimators for the same series for the unrestricted model. Table 6 and 7 give the number of series which rejected the unit root hypothesis in 100 generated samples assuming that there was (table 6) or not (table 7) a unit root. We used three criteria to reject the hypothesis: the ADF asymptotic p-value for 5% significance, the exact ADF p-value for 5% significance and the e-value set in 0.05.

It is important to remember that finite sample critical values for unit root tests depend

**TABLE 5.** Standard error of MLE - unrestricted model

Parameters	Real GNP	Ind. Prod.	GNP def.	Wage
$\mu$	0.23279	0.01727	0.05667	0.16301
$\delta$	0.00163	0.00206	0.00056	0.00125
$\Gamma_0$	0.05104	0.04941	0.01990	0.02756
$\Gamma_1$	0.10436	0.08915	0.09175	0.09522

**TABLE 6.** Simulated series rejecting  $H_0$  in hundred generated assuming  $H_0$ 

Series	$< ADF_{5\%}(\infty)$	$< ADF_{5\%}(ex.)$	$ev < 0.05$
Real GNP	4	3	3
Ind. Prod.	4	4	4
GNP def.	7	6	6
Wage	4	4	4

**TABLE 7.** Simulated series rejecting  $H_0$  in hundred generated assuming the unrestricted model

Series	$< ADF_{5\%}(\infty)$	$< ADF_{5\%}(ex.)$	$ev < 0.05$
Real GNP	73	67	64
Ind. Prod.	85	82	84
GNP def.	20	18	18
Wage	29	27	27

on the assumption that the error terms are  $NID(0, \sigma^2)$  once these values were generated by simulations that use this assumption. The asymptotic critical values are valid much more generally, since they do not require normality or homoskedasticity. Therefore, for small samples, it is safer to rely on asymptotic critical values.

Table 6 shows that the FBST, even using the flat prior, has a power similar to the ADF test. Hence, the argument used by Phillips to criticize conclusions based on posterior probabilities when flat priors were used is not valid for the FBST.

As mentioned in the first section, Bayes Factor tests for unit roots have had many difficulties to deal with time series presented in the field of econometrics. Several alternative Bayes Factor tests have been proposed in order to overcome these difficulties. However, their performance is still in question. For example, [1] concludes:

*“In two Monte Carlo simulations, however, we find that the ‘objective’ Bayesian test have relatively low power in distinguishing between plausible alternatives, making it difficult to draw any conclusions concerning long-run [performance]. We conclude that, at least for the ‘objective’ Bayesian test, the Bayesian approach is not necessary better than the classical ADF approach.”*

Based on simulation studies, [6] suggests that practitioners must assign a high probability to the value to be tested in order to get high power when using Bayes Factor tests, although this means to increase the non-stationary weight when testing for unit root.

There have also been other tests based on or using specially designed priors, that show a better performance. However, the use of such priors departs from some basic para-



digns of Bayesian statistics, like the Likelihood Principle. Moreover, these techniques have to be fine tuned to each particular problem type or application.

In contrast, the FBST e-value derivation and implementation is straightforward from its general definition, using absolutely no ad hoc artifice, like a special prior, or a measure on the hypothesis set induced by some special parameterization, or an arbitrary initial likelihood ratio. It respects the Likelihood Principle and does not need to eliminate nuisance parameters.

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