

## TUTORIAL BOOKS – TABLES OF CONTENTS

# Lectures on Probability, Entropy and Statistical Physics

Ariel Caticha

## CONTENTS

Preface	Vii
1 Inductive Inference	1
1.1 Probability	1
1.2 Inductive reasoning	3
2 Probability	7
2.1 Consistent reasoning: degrees of belief	8
2.2 The Cox Axioms	10
2.3 Regraduation: the Product Rule	11
2.3.1 Cox's first theorem	11
2.3.2 Proof of the Associativity Theorem	12
2.3.3 Setting the range of degrees of belief	15
2.4 Further regraduation: the Sum Rule	16
2.4.1 Cox's second theorem	16
2.4.2 Proof of the Compatibility Theorem	18
2.5 Some remarks on the sum and product rules	19
2.5.1 On meaning, ignorance and randomness	19
2.5.2 The general sum rule	20
2.5.3 Independent and mutually exclusive events	20
2.5.4 Marginalization	22
2.6 The expected value	22
2.7 The binomial distribution	24
2.8 Probability vs. frequency: the law of large numbers	26
2.9 The Gaussian distribution	28
2.9.1 The de Moivre-Laplace theorem	28
2.9.2 The Central Limit Theorem	31
2.10 Updating probabilities: Bayes' rule	33
2.10.1 Formulating the problem	33
2.10.2 Minimal updating: Bayes' rule	34
2.10.3 Multiple experiments, sequential updating	38
2.10.4 Remarks on priors	39

2.11 Examples from data analysis	42
2.11.1 Parameter estimation	42
2.11.2 Curve fitting	46
2.11.3 Model selection	47
2.11.4 Maximum Likelihood	49
3 Entropy I: The Evolution of Carnot's Principle	51
3.1 Carnot: reversible engines	51
3.2 Kelvin: temperature	54
3.3 Clausius: entropy	56
3.4 Maxwell: probability	57
3.5 Gibbs: beyond heat	59
3.6 Boltzmann: entropy and probability	60
3.7 Some remarks	65
4 Entropy II: Measuring Information	67
4.1 Shannon's information measure	68
4.2 Relative entropy	73
4.3 Join entropy, additivity, and subadditivity	74
4.4 Conditional entropy and mutual information	75
4.5 Continuous distributions	76
4.6 Communication Theory	79
4.7 Assigning probabilities: MaxEnt	82
4.8 Canonical distributions	83
4.9 On constraints and relevant information	86
5 Statistical Mechanics	89
5.1 Liouville's theorem	89
5.2 Derivation of Equal a Priori Probabilities	91
5.3 The relevant constraints	93
5.4 The canonical formalism	95
5.5 The Second Law of Thermodynamics	97
5.6 The thermodynamic limit	100
5.7 Interpretation of the Second Law: Reproducibility	103
5.8 Remarks on irreversibility	104
5.9 Entropies, descriptions and the Gibbs paradox	105
6 Entropy III: Updating Probabilities	113
6.1 What is information?	116
6.2 Entropy as a tool for updating probabilities	118
6.3 The proofs	124
6.3.1 Axiom 1: Locality	124
6.3.2 Axiom 2: Coordinate invariance	126
6.3.3 Axiom 1: again	127

6.3.4 Axiom 3: Consistency for identical independent subsystems	128
6.3.5 Axiom 3: Consistency for non-identical subsystems	132
6.3.6 Axiom 3: Consistency with the law of large numbers	133
6.4 Random remarks	135
6.4.1 On deductive vs. inductive systems	135
6.4.2 On priors	135
6.4.3 Comments on other axiomatizations	136
6.5 Bayes' rule as a special case of ME	138
6.6 Commuting and non-commuting constraints	142
6.7 Information geometry	144
6.7.1 Derivation from distinguishability	144
6.7.2 Derivation from a Euclidean metric	145
6.7.3 Derivation from relative entropy	146
6.7.4 Volume elements in curved spaces	147
6.8 Maximum entropy is not required, just more probable	148
6.9 An application to fluctuations	151
6.10 Conclusion	153

# Cognitive Constructivism and the Epistemic Significance of Sharp Statistical Hypotheses

Julio Michael Stern

## CONTENTS

Preface	11
Cognitive Constructivism	12
Basic Tools for the (Home) Works	13
Acknowledgements and Final Remarks	14
1 Eigen-Solutions and Sharp Statistical Hypotheses	17
1.1 Introduction	17
1.2 Autopoiesis and Eigen-Solutions	18
1.3 Functional Differentiation	22
1.4 Eigensolutions and Scientific Hypotheses	24
1.5 Sharp Statistical Hypotheses	24
1.6 Semantic Degradation	31
1.7 Competing Sharp Hypotheses	32
1.8 Final Remarks	34
1.8.1 Noether and de Finetti Theorems	35
1.8.2 Compositionality	36
2 Language and the Self-Reference Paradox	39
2.1 Introduction	39
2.2 Eigen-solutions and Language	42
2.3 The Languages of Science	45
2.4 The Self-Reference Paradox	48
2.5 Objective Idealism and Pragmatism	49
2.6 The Philosophy of C.S.Peirce	52
2.7 Final Remarks	60
3 Decoupling, Randomization, Sparsity, and Objective Inference	63
3.1 Introduction	63
3.2 The Decoupling Principle	65
3.3 Covariance Structure Models	68
3.4 Simpson's Paradox and the Control of Confounding Variables	71
3.5 C.S.Peirce and Randomization	74
3.6 Bayesian Analysis of Randomization	76
3.7 Randomization, Epistemic Considerations	78
3.8 Final Remarks	81

4 Metaphor and Metaphysics: The Subjective Side of Science	83
4.1 Introduction	83
4.2 Statistics is Prediction. Is that all there is?	85
4.3 Rhetoric and Self-Fulfilling Prophecies	86
4.4 Language, Metaphor and Insight	89
4.5 Necessary and Best Worlds	93
4.6 Efficient and Final Causes	94
4.7 Modern Metaphysics	97
4.8 Averaging over All Possible Worlds	101
4.9 Hypothetical versus Factual Models	104
4.10 Magic, Miracles and Final Remarks	107
5 Complex Structures, Modularity, and Stochastic Evolution	111
5.1 Introduction	111
5.2 The Ergodic Path: One for All	112
5.2.1 Block Partitions	113
5.2.2 Simulated Annealing	115
5.2.3 Heuristic Perturbations	118
5.3 The Way of Sex: All for One	119
5.3.1 Functional Trees	120
5.3.2 Genetic Programming	123
5.3.3 Schemata and Parallelism	125
5.4 Simple Life: Small is Beautiful	126
5.4.1 Overfitting and Regularization	127
5.4.2 Building Blocks	130
5.5 Evolution of Theories	133
5.5.1 Nested, Mixture, Separate and Series-Parallel Models	134
5.5.2 Complementary Models	138
5.6 Varieties of Probability	141
5.6.1 Heisenberg's Uncertainty Principle	141
5.6.2 Schrödinger's Wave Equation	144
5.6.3 Classic and Quantum Probability	146
5.7 Theories of Evolution	149
5.7.1 Systemic View and Probabilistic Causation	150
5.7.2 Modularity Requires Quantization	154
5.7.3 Quantization Entails Objective Probability	158
5.8 Final Remarks	159
Epilog	161
References	163
A FBST Review	191
A.1 Introduction	191

A.2 The Epistemic e-values	192
A.3 Reference, Invariance and Consistency	194
A.4 Loss Functions	197
A.5 Belief Calculi and Support Structures	198
A.6 Sensitivity and Inconsistency	200
A.7 Complex Models and Compositionality	202
B Binomial, Dirichlet, Poisson and Related Distributions	205
B.1 Introduction and Notation	205
B.2 The Bernoulli Process	209
B.3 Multinomial Distribution	213
B.4 Multivariate Hypergeometric Distribution	215
B.5 Dirichlet Distribution	217
B.6 Dirichlet-Multinomial	221
B.7 Dirichlet of the Second Kind	224
B.8 Examples	225
B.9 Functional Characterizations	229
B.10 Final Remarks	232
C Model Miscellanea	233
C.1 Contingency Table Models	234
C.2 Weibull Wearout Model	235
C.3 The Normal-Wishart Distribution	238
C.4 Structural Models	240
C.4.1 Mean and Covariance Structure	240
C.4.2 Numerical Optimization	242
C.5 Factor Analysis	243
C.5.1 EM Algorithm	244
C.5.2 Orthogonal and Oblique Rotations	245
C.5.3 Frobenius Norm and Projections	245
C.5.4 Sparsity Optimization	247
C.6 Mixture Models	249
C.6.1 Dirichlet-Normal-Wishart Mixtures	250
C.6.2 Gibbs Sampling and Integration	251
C.6.3 EM Algorithm for ML and MAP Estimation	252
C.6.4 Experimental Tests and Final Remarks	254
C.7 REAL Classification Trees	256
C.7.1 Conviction, Loss and Discretization	257
C.7.2 Branching and Merging	259
D Deterministic Evolution and Optimization	261
D.1 Matrix Notation, Convex Sets and Polyedra	262
D.2 Linear Programming	267

D.2.1 Primal and Dual Simplex Algorithms	267
D.2.2 Decomposition Methods	274
D.3 Non-Linear Programming	276
D.3.1 GRG: Generalized Reduced Gradient	278
D.3.2 Line Search and Local Convergence	280
D.3.3 The Gradient ParTan Algorithm	282
D.3.4 Global Convergence	287
D.4 Variational Principles	289
E Entropy and Asymptotics	293
E.1 Convexity	293
E.2 Boltzmann-Gibbs-Shannon Entropy	294
E.3 Csiszar's $\alpha$ -divergence	296
E.4 Maximum Entropy under Constraints	297
E.5 Fisher's Metric and Jeffreys' Prior	299
E.6 Posterior Asymptotic Convergence	301
F Sparse Factorizations	305
F.1 Cholesky Factorization	305
F.2 Sparsity and Graphs	307
F.3 Sparse Cholesky Factorization	309
F.4 Bayesian Networks	313
G Monte Carlo Miscellanea	317
G.1 Pseudo, Quasi and Subjective Randomness	317
G.2 Integration and Variance Reduction	324
G.3 Estimation of Ratios	327
G.4 Monte Carlo for Linear Systems	330
H Stochastic Evolution and Optimization	333
H.1 Simulated Annealing	333
H.2 Genetic Programming	340
H.3 Ontogenetic Development	341
I Research Projects	345
J Image and Art Gallery	349

# Symmetry Studies of Entropy: An Introduction to Canonical Decompositions

Marlos A. G. Viana

## CONTENTS

Preface	V
Chapter 1. Symmetry, Classification, and the Analysis of Structured Data	1
1.1. Introduction	1
1.2. Symmetry and classification	1
1.3. Data indexed by symmetries	2
1.4. Symmetry and data reduction	4
1.5. Statistical aspects	6
1.6. Algebraic aspects	7
1.7. Structured data	9
1.8. Canonical projections	12
1.9. Summary	12
Appendix A	14
Chapter 2. Algebraic Aspects	15
2.1. Introduction	15
2.2. Permutations	15
2.3. Groups and homomorphisms	17
2.4. Group actions	19
2.5. Actions on mappings	21
2.6. Burnside's Lemma	22
2.7. Representations	22
2.8. Unitary representations	24
2.9. Regular representations and group algebras	25
2.10. Tensor representations	25
2.11. Reducibility	26
2.12. Schur's Lemma	29
2.13. Characters of a linear representation	31
2.14. Orthogonality relations for characters	31
2.15. The canonical projections	38
2.16. The standard decomposition	41
Chapter 3. Canonical decompositions of entropy data	45
3.1. Introduction	45
3.2. The Standard Decomposition of Entropy	45

3.3. Invariant plots	45
3.4. The n-component case	46
3.5. The standard decomposition of the entropy of the Sloan fonts	47
3.6. Geological compositions	47
3.7. The regular decomposition of entropy	48
Glossary of Selected Symbols, Notations, and Terms	51
Bibliography	53
Index	55

## MAXENT 2008 GROUP PHOTOS





# Photo Gallery

## TUTORIALS



Nestor Caticha



Julio Stern



Ariel Caticha



Marlos Viana

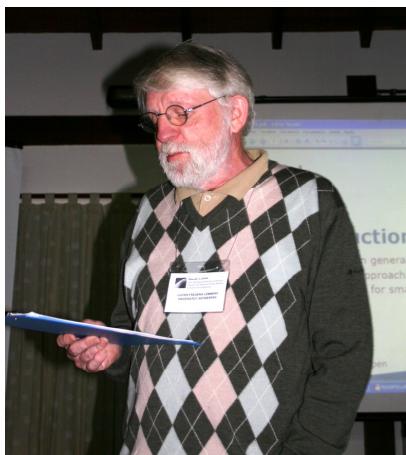
## INVITED TALKS AND ORAL PRESENTATIONS



**Kevin Knuth**



**Julian Center**



**Lucien Frederik Lemmens**



**Ali Mohammad-Djafari**



**Andre Sesboue**



**TJ. Romke Bontekoe, John Skilling, Paul Goggans**



**Márcio Diniz**



**John Skilling**



**Pericchi, Carlinhos and Ali**



**Nestor and Ariel Caticha and some students**



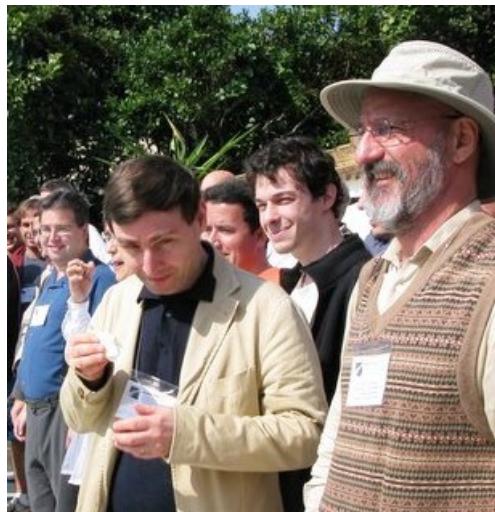
**Deniz 1 X Watermelon 0**



**Carla Koike**



**Nabin Malakar**

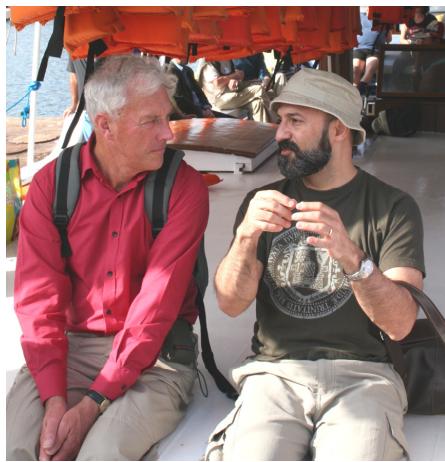


**Petitjean, Rafael and Pericchi**

## **EXCURSION – BOAT TRIP**



**Nabin Malakar**



**John Skilling and Ariel Caticha**



**Shu-Kun Lin and his son, Di-Fan**



**Hacheme Ayasso, Dirk Dodt and Shu-Kun Lin**



**Atlantic Forest landscape view from the boat**



**MaxEnt2008 swimmers:**  
**Ariel Caticha, Udo Toussaint, Ali Mohammad-Djafari and John Skilling**



**MaxEnt2008 Organizing Committee:  
Carlos, Sylvia, Nestor, Elvira, Marcelo, Julio, Requena**



**Goodbye, my friends**



**‘Bandeirinha’, by Rockne Knuth**



**See you at Mississippi in 2009**

## AUTHOR INDEX

### A

Abbas, A. E., 3  
Ayasso, H., 243

### F

Fekih-Salem, S., 243  
Ferreira, F. H., 173  
Figueiredo, A., 207

### B

Barraza, N. R., 103  
Barriga, G. D. C., 111  
Bernardini Papalia, R., 325  
Bernardo, J. M., 340  
Bilich, F., 252  
Boekema, C., 260  
Bonassi, F. V., 8  
Borges, W. S., 268  
Browne, M. C., 260

### G

Gençaga, D., 286  
Girardin, V., 181  
Goggans, P. M., 294  
Gori, S., 348  
Gselmann, E., 3

### C

Cafaro, C., 16  
Cancho, V. G., 111  
Carbon, D. F., 286  
Caticha, A., 23, 361  
Center, Jr., J. L., 119, 127  
Chan, C.-Y., 294  
Colla, E. C., 136  
Cook, J. D., 278  
Coque, Jr., M. A., 215

Knuth, K. H., 35, 286  
Koike, C. C., 301  
Kolev, N. V., 173  
Kyo, K., 309

### L

Lauretto, M., 317  
Lemmens, L. F., 189  
Lin, S.-K., 49  
Louzada-Neto, F., 111

### D

DaSilva, R., 252  
de B. Pereira, C. A., 90, 96, 215  
de O. Siqueira, J., 332  
Dias, T. C. M., 144  
Diniz, C. A. R., 144, 150  
Diniz, M., 157

### M

Maksa, G., 3  
Marques F., P. C., 96  
Martins, C. B., 340  
Mendes, F. M., 207  
Mohammad-Djafari, A., 197, 243  
Muravchik, C. H., 165

### E

Esteves, L. G., 268

### N

Nakano, F., 317  
Noda, H., 309

**P**

- Pereira, C. A. B., 157, 317  
Pericchi G., L. R., 278  
Petitjean, M., 61  
Polpo, A., 215  
Preuss, L., 224

**R**

- Rifo, L. L. R., 232  
Rodrigues, C. P., 150

**S**

- Schwarz-Selinger, T., 348  
Sesboüé, A., 181  
Skilling, J., 67  
Solana, V., 80  
Solana-Ortega, A., 80

- Stern, J. M., 136, 157, 317, 364  
Stern, R. B., 8, 90  
Sun, Z., 3

**T**

- Takada, H. H., 332  
Tomazella, V. L. D., 340  
Torres, S., 232

**V**

- Viana, M. A. G., 368  
von Ellenrieder, N., 165  
von Toussaint, U., 348

**W**

- Wechsler, S., 8, 96, 268