

TUTORIAL BOOKS – TABLES OF CONTENTS

Lectures on Probability, Entropy and Statistical Physics

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Cognitive Constructivism and the Epistemic Significance of Sharp Statistical Hypotheses

Julio Michael Stern

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Symmetry Studies of Entropy: An Introduction to Canonical Decompositions

Marlos A. G. Viana

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Nestor and Ariel Caticha and some students



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Shu-Kun Lin and his son, Di-Fan



Hacheme Ayasso, Dirk Dodt and Shu-Kun Lin



Atlantic Forest landscape view from the boat



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Goodbye, my friends



‘Bandeirinha’, by Rockne Knuth



See you at Mississippi in 2009

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