

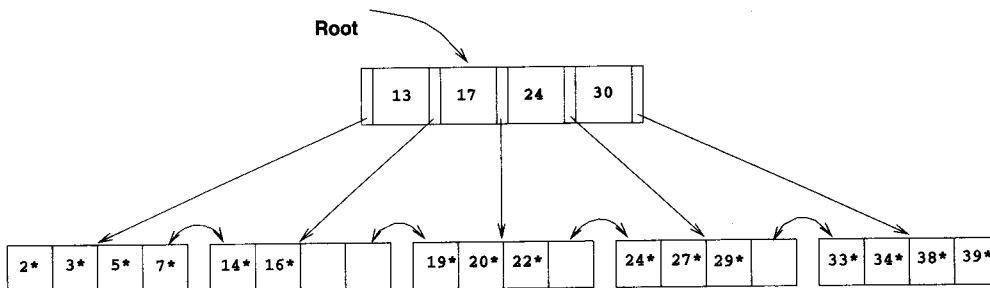
```

func find (search key value  $K$ ) returns nodepointer
// Given a search key value, finds its leaf node
return tree_search(root,  $K$ ); // searches from root
endfunc

func tree_search (nodepointer, search key value  $K$ ) returns nodepointer
// Searches tree for entry
if *nodepointer is a leaf, return nodepointer;
else,
  if  $K < K_1$  then return tree_search( $P_0$ ,  $K$ );
  else,
    if  $K \geq K_m$  then return tree_search( $P_m$ ,  $K$ ); //  $m = \#$  entries
    else,
      find  $i$  such that  $K_i \leq K < K_{i+1}$ ;
      return tree_search( $P_i$ ,  $K$ )
endfunc

```

Figure 9.9 Algorithm for Search

Figure 9.10 Example of a B+ Tree, Order $d=2$

```

proc insert (nodepointer, entry, newchildentry)
// Inserts entry into subtree with root '*nodepointer'; degree is d;
// 'newchildentry' is null initially, and null upon return unless child is split

if *nodepointer is a non-leaf node, say N,
    find  $i$  such that  $K_i \leq \text{entry's key value} < K_{i+1}$ ; // choose subtree
    insert( $P_i$ , entry, newchildentry); // recursively, insert entry
    if newchildentry is null, return; // usual case; didn't split child
    else, // we split child, must insert *newchildentry in N
        if N has space, // usual case
            put *newchildentry on it, set newchildentry to null, return;
        else, // note difference wrt splitting of leaf page!
            split N: //  $2d + 1$  key values and  $2d + 2$  nodepointers
                first  $d$  key values and  $d + 1$  nodepointers stay,
                last  $d$  keys and  $d + 1$  pointers move to new node, N2;
                // *newchildentry set to guide searches between N and N2
                newchildentry = & (<smallest key value on N2, pointer to N2>);
                if N is the root, // root node was just split
                    create new node with <pointer to N, *newchildentry>;
                    make the tree's root-node pointer point to the new node;
                return;

if *nodepointer is a leaf node, say L,
    if L has space, // usual case
        put entry on it, set newchildentry to null, and return;
    else, // once in a while, the leaf is full
        split L: first  $d$  entries stay, rest move to brand new node L2;
        newchildentry = & (<smallest key value on L2, pointer to L2>);
        set sibling pointers in L and L2;
        return;

endproc

```

widec



Figure 9.11 Algorithm for Insertion into B+ Tree of Order d

```

proc delete (parentpointer, nodepointer, entry, oldchildentry)
// Deletes entry from subtree with root '*nodepointer'; degree is d;
// 'oldchildentry' null initially, and null upon return unless child deleted
if *nodepointer is a non-leaf node, say N,
    find  $i$  such that  $K_i \leq \text{entry's key value} < K_{i+1}$ ;           // choose subtree
    delete(nodepointer,  $P_i$ , entry, oldchildentry);                 // recursive delete
    if oldchildentry is null, return;                                 // usual case: child not deleted
    else,                                                            // we discarded child node (see discussion)
        remove *oldchildentry from N, // next, check minimum occupancy
        if N has entries to spare, // usual case
            set oldchildentry to null, return; // delete doesn't go further
        else, // note difference wrt merging of leaf pages!
            get a sibling S of N: // parentpointer arg used to find S
            if S has extra entries,
                redistribute evenly between N and S through parent;
                set oldchildentry to null, return;
            else, merge N and S // call node on rhs M
                oldchildentry = & (current entry in parent for M);
                pull splitting key from parent down into node on left;
                move all entries from M to node on left;
                discard empty node M, return;

if *nodepointer is a leaf node, say L,
    if L has entries to spare, // usual case
        remove entry, set oldchildentry to null, and return;
    else, // once in a while, the leaf becomes underfull
        get a sibling S of L; // parentpointer used to find S
        if S has extra entries,
            redistribute evenly between L and S;
            find entry in parent for node on right; // call it M
            replace key value in parent entry by new low-key value in M;
            set oldchildentry to null, return;
        else, merge L and S // call node on rhs M
            oldchildentry = & (current entry in parent for M);
            move all entries from M to node on left;
            discard empty node M, adjust sibling pointers, return;

endproc

```

Figure 9.16 Algorithm for Deletion from B+ Tree of Order d