Exemplo: $T: \mathbb{R}^3 \to \mathbb{R}^3$

$$T(x,y,z) = (-3x+4y-2z, -2x+3y-2z, y-z)$$

$$[T]_\mathcal{E} = \begin{pmatrix} -3 & 4 & -2 \\ -2 & 3 & -2 \\ 0 & 1 & -1 \end{pmatrix}$$

$$[T-tI]_\mathcal{E} = \begin{pmatrix} -3-t & 4 & -2 \\ -2 & 3-t & -2 \\ 0 & 1 & -1-t \end{pmatrix}$$

$$\begin{aligned} P_T(t) &= (3+t)(3-t)(1+t) + 4 - 2(3+t) - 8(1+t) \\ &= (9-t^2)(1+t) - 10 - 10t \\ &= -t^3 - t^2 - t - 1 \\ &= -(t^3+t^2+t+1) \end{aligned}$$

$t=-1$ é uma raíz

$$P_T(t) = -(t+1)(t^2+1)$$

$$\Lambda_T = \{-1\}$$

OBS: Sempre que T tem "autovalor complexo" $\Rightarrow T$ NÃO é diagonalizável como transformação linear sobre $\mathbb{R}$

$$S_{-1} = \{(x,y,z) \in \mathbb{R}^3 \mid T(x,y,z) = -(x,y,z)\}$$

$$\Leftrightarrow \begin{cases} -3x+4y-2z = -x \\ -2x+3y-2z = -y \\ y-z = -z \end{cases} \Leftrightarrow \begin{cases} -2x+4y-2z = 0 \\ -2x+4y-2z = 0 \\ y = 0 \end{cases}$$

$$\Leftrightarrow \quad y=0, \quad z=-x \quad \Rightarrow \quad S_{-1} = \{(x,0,-x) \mid x \in \mathbb{R}\}$$
$$= [(1,0,-1)]$$

$$\Rightarrow d_{-1} = 1 < 3 \quad \Rightarrow \quad T \text{ Não é diagonalizável}$$

Exemplo $T:\mathbb{R}^3 \to \mathbb{R}^3$

$$T(x,y,z) = (2x-3y+2z\ ,\ 2x-3y+2z\ ,\ -x+y-z)$$

$$[T]_{\mathcal{E}} = \begin{pmatrix} 2 & -3 & 2 \\ 2 & -3 & 2 \\ -1 & 1 & -1 \end{pmatrix}$$

$$[T-tI]_{\mathcal{E}} = \begin{pmatrix} 2-t & -3 & 2 \\ 2 & -3-t & 2 \\ -1 & 1 & -1-t \end{pmatrix}$$

$$P_T(t) = (2-t)(3+t)(1+t) + 6 + 4 - 2(3+t) - 2(2-t) - 6(1+t)$$

$$= (2-t)(3+4t+t^2) - 6t - 6$$

$$= -t^3 - 2t^2 + 5t - 6t = -(t^3+2t^2+t) = -t(t^2+2t+1)$$

$$= -t(t+1)^2$$

$$\Lambda_T = \{0, -1\}$$

$$S_0 = N(T) = \{(x,y,z) \in \mathbb{R}^3 \mid T(x,y,z) = 0\}$$

$$\Leftrightarrow \begin{cases} 2x-3y+2z=0 \\ -x+y-z=0 \end{cases} \Leftrightarrow y=0\ ,\ z=-x$$

$$\Rightarrow S_0 = [(1,0,-1)]\ ,\quad d_0 = 1$$

$$S_{-1} = \{(x,y,z) \in \mathbb{R}^3 \mid T(x,y,z) = -(x,y,z)\}$$

$$\Leftrightarrow \begin{cases} 2x-3y+2z = -x \\ 2x-3y+2z = -y \\ -x+y-z = -z \end{cases} \Leftrightarrow \begin{cases} 3x-3y+2z=0 \\ 2x-2y+2z=0 \\ -x+y=0 \end{cases}$$

$$\Leftrightarrow x=y\ ,\ z=0 \Rightarrow S_{-1} = \{(x,x,0) \mid x \in \mathbb{R}\} = [(1,1,0)]$$

$$d_{-1} = 1$$

$d_0 + d_{-1} = 2 < 3 \Rightarrow T$ Não é diagonalizável

__Exemplo:__ $T: \mathbb{R}^3 \to \mathbb{R}^3$

$$T(x,y,z) = (y+2z, -x-2y-2z, x+y+z)$$

$$[T]_\varepsilon = \begin{pmatrix} 0 & 1 & 2 \\ -1 & -2 & -2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$[T-tI]_\varepsilon = \begin{pmatrix} -t & 1 & 2 \\ -1 & -2-t & -2 \\ 1 & 1 & 1-t \end{pmatrix}$$

$$p_T(t) = t(2+t)(1-t) - 2 - 2 + 2(2+t) + 1-t - 2t$$

$$= t(2+t)(1-t) + 1-t$$

$$= (1-t)\left(t(2+t)+1\right) = (1-t)(t^2+2t+1) = (1-t)(t+1)^2$$

$$\Lambda_T = \{-1, 1\}$$

$S_{-1} = \{(x,y,z) \in \mathbb{R}^3 \mid T(x,y,z) = -(x,y,z)\}$, $(x,y,z) \in S_{-1} \Leftrightarrow$

$$\Leftrightarrow \begin{cases} y+2z = -x \\ -x-2y-2z = -y \\ x+y+z = -z \end{cases} \Leftrightarrow \begin{cases} x+y+2z = 0 \\ -x-y-2z = 0 \\ x+y+2z = 0 \end{cases} \Leftrightarrow x = -y-2z$$

$$\Leftrightarrow S_{-1} = \{(-y-2z, y, z) \mid y,z \in \mathbb{R}\} = [(-1,1,0),(-2,0,1)]$$

$d_{-1} = 2$

$S_1 = \{(x,y,z) \in \mathbb{R}^3 \mid T(x,y,z) = (x,y,z)\}$, $(x,y,z) \in S_1 \Leftrightarrow$

$$\Leftrightarrow \begin{cases} y+2z = x \\ -x-2y-2z = y \\ x+y+z = z \end{cases} \Leftrightarrow \begin{cases} -x+y+2z = 0 \\ -x-3y-2z = 0 \\ x+y = 0 \end{cases}$$

$y = -x$, $z = x \Rightarrow S_1 = \{(x,-x,x) \mid x \in \mathbb{R}\} \Rightarrow S_1 = [(1,-1,1)]$

$d_1 = 1$

$d_{-1}+d_1 = 3 = \dim \mathbb{R}^3 \Rightarrow T$ é diagonalizável.

A base $\mathcal{F}=(f_1,f_2,f_3)$ com

$f_1=(-1,1,0)$, $f_2=(-2,0,1)$, $f_3=(1,-1,1)$

é base que diagonaliza T e

$$[T]_{\mathcal{F}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Exemplo $T:\mathbb{R}^3\to\mathbb{R}^3$

$$T(x,y,z)=(4x-5y+3z,\ 3x-4y+3z,\ x-y+2z)$$

$$[T-tI]_{\mathcal{E}} = \begin{pmatrix} 4-t & -5 & 3 \\ 3 & -4-t & 3 \\ 1 & -1 & 2-t \end{pmatrix}$$

$$P_T(t) = -(4-t)(4+t)(2-t) - 15 - 9 + 3(4+t) + 15(2-t) + 3(4-t)$$
$$= -(16-t^2)(2-t) - 15t + 30$$
$$= -t^3+2t^2+t-2 = -(t^3-2t^2-t+2) = -(t-1)(t+1)(t-2)$$

$\Rightarrow \Lambda_T = \{-1, 1, 2\}$

$$(x,y,z)\in S_{-1} \Leftrightarrow \begin{cases} 4x-5y+3z=-x \\ 3x-4y+3z=-y \\ x-y+2z=-z \end{cases} \Leftrightarrow \begin{cases} 5x-5y+3z=0 \\ 3x-3y+3z=0 \\ x-y+3z=0 \end{cases}$$

$(2)-(3) \Rightarrow 2x-2y=0 \Rightarrow x=y \Rightarrow z=0$

$\Rightarrow S_{-1} = [(1,1,0)]$ $d_1 = 1$

- $$(x,y,z)\in S_1 \Leftrightarrow \begin{cases} 4x-5y+3z=x \\ 3x-4y+3z=y \\ x-y+2z=z \end{cases} \Leftrightarrow \begin{cases} 3x-5y+3z=0 \\ 3x-5y+3z=0 \\ x-y+z=0 \end{cases}$$

$\Leftrightarrow x=-z,\ y=0 \Rightarrow S_{-1} = [(1,0,-1)]$, $d_{-1}=1$

$$(x,y,z) \in S_2 \iff \begin{cases} 4x - 5y + 3z = 2x \\ 3x - 4y + 3z = 2y \\ x - y + 2z = 2z \end{cases} \iff \begin{cases} 2x - 5y + 3z = 0 \\ 3x - 6y + 3z = 0 \\ x - y = 0 \end{cases}$$

$$\iff \begin{cases} -3y + 3z = 0 \\ x = y \end{cases} \iff x = y = z$$

$$\Rightarrow S_2 = [(1,1,1)] \quad , \quad d_2 = 1$$

$$d_{-1} + d_1 + d_2 = 3 = \dim \mathbb{R}^3 \Rightarrow T \text{ é diag.}$$

$$[T]_{\mathcal{F}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

onde $\mathcal{F} = (f_1, f_2, f_3)$,

$f_1 = (1, 1, 0$

$f_2 = (1, 0, -1)$

$f_3 = (1, 1, 1)$

OBS: Se $T: \mathbb{R}^n \to \mathbb{R}^n$, $\Lambda_T = \{\lambda_1, \dots, \lambda_n\}$, $\lambda_i \neq \lambda_j$ $\forall i \neq j$

$\Rightarrow T$ é diagonalizável.