MAT0103 - Matemática para Administração e Contabilidade

Lista 7

1º semestre de 2025

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(1) Encontre o valor do limite utilizando a regra de L'Hospital:

(a) $\displaystyle\lim_{x\to 3} \frac{e^{3x}-e^3}{x-3}$;

(b) $\displaystyle\lim_{x\to 0} \frac{e^{x}-e^{-x}-x^2}{2x-\operatorname{sen}(x)}$;

(c) $\displaystyle\lim_{x\to 0} \frac{x-\operatorname{sen}(x)}{3x^2}$;

(d) $\displaystyle\lim_{x\to 1} \frac{\ln(\cos(1-x))}{1-\operatorname{sen}\frac{\pi x}{2}}$;

(e) $\displaystyle\lim_{x\to \frac{\pi}{2}} \frac{\ln(\operatorname{sen}(x))}{(\pi-2x)^2}$;

(f) $\displaystyle\lim_{x\to} x^2e^x$;

(g) $\displaystyle\lim_{x\to} \sqrt{x^2+x}-x$;

(h) $\displaystyle\lim_{x\to} \frac{x+\tan(x)}{\operatorname{sen}(x)}$.

(2) Calcule:

(a) $\displaystyle\int 4x\,dx$

(b) $\displaystyle\int 5\,dx$

(c) $\displaystyle\int (x^2+2x+1)\,dx$

(d) $\displaystyle\int (5x^3+4x+7)\,dx$

(e) $\displaystyle\int (x+\frac{1}{x}+\frac{1}{x^2})\,dx$

(f) $\displaystyle\int \sqrt[10]{x^3}\,dx$

(g) $\displaystyle\int (\operatorname{sen}(x)+\cos(x)-2e^x-4^x)\,dx$

(h) $\displaystyle\int \frac{x^5-4x^4+3x^3-x^2+5x-2}{x}\,dx$

(i) $\displaystyle\int (2x^3-\frac{1}{x^4})\,dx$

(3) Calcule as integrais definidas abaixo:

(a) $\displaystyle\int_{-1}^{1} (2x+1)\,dx$

(b) $\displaystyle\int_{-2}^{1} (x^2+1)\,dx$

(c) $\displaystyle\int_{0}^{1} (4x^3-\frac{1}{3})\,dx$

(d) $\displaystyle\int_{1}^{0} (x+2)\,dx$

(e) $\displaystyle\int_{0}^{1} \sqrt[6]{x}\,dx$

(f) $\displaystyle\int_{0}^{1} (x+\sqrt[4]{x})\,dx$

(g) $\displaystyle\int_{1}^{0} (x^8-5x^3+3)\,dx$

(h) $\displaystyle\int_{0}^{1} (x-1)^2\,dx$

(i) $\displaystyle\int_0^1 (2x-1)^2\,dx$

(j) $\displaystyle\int_1^2 \frac{1-t^2}{t^4}\,dt$

(k) $\displaystyle\int_0^3 (u^2-2u+3)\,du$

(l) $\displaystyle\int_{-1}^1 \sqrt[3]{t}\,dt$

(m) $\displaystyle\int_1^2 \frac{1+3x^2}{x}\,dx$

(n) $\displaystyle\int_0^\pi \operatorname{sen}(3x)\,dx$

(o) $\displaystyle\int_0^1 \frac{dx}{1+x^2}$

(p) $\displaystyle\int_{-1}^0 e^{-2x}\,dx$

(4) Calcule as integrais definidas:

(a) $\displaystyle\int_0^{\frac{\pi}{3}} \left(\operatorname{sen}(x)+\operatorname{sen}(2x)\right)\,dx$

(b) $\displaystyle\int_0^{\frac{\pi}{2}} \left(\frac{1}{2}+\frac{1}{2}\cos(2x)\right)\,dx$

(c) $\displaystyle\int_0^{\frac{\pi}{2}} \operatorname{sen}^2(x)\,dx$

(d) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1}{\cos^2(x)}\,dx$

(e) $\displaystyle\int_0^{\frac{\pi}{4}} \tan^2(x)\,dx$

(5) Calcule a área do conjunto dado. Esboce a região.

(a) A é limitado pelas retas $y=0$, $x=1$, $x=3$ e pelo gráfico de $y=x^3$.

(b) A é limitado pelas retas $y=0$, $x=1$, $x=4$ e pelo gráfico de $y=\sqrt{x}$.

(c) $A=\{(x,y)\mid x^2-1\le y\le 0\}$.

(d) $A=\{(x,y)\mid 0\le y\le 4-x^2\}$.

(e) $A=\{(x,y)\mid 0\le y\le |\operatorname{sen}(x)|, 0\le x\le 2\}$.

(f) A é limitado pelo eixo 0x e pelo gráfico de $y=x^2-x$, $0\le x\le 2$.

(g) A é limitado pela reta $y=0$ e pelo gráfico de $y=3-2x-x^2$, $-1\le x\le 2$.

(h) A é limitado pelas retas $x=-1, x=2, y=0$ e pelo gráfico de $y=x^2+2x+5$.

(i) A é limitado pelo eixo 0x e pelo gráfico de $y=x^3-x$, $-1\le x\le 1$.

(j) A é limitado pela reta $y=0$ e pelo gráfico de $y=x^3-x$, $0\le x\le 2$.

(k) A é limitado pelas retas $x=0$, $x=\pi$, $y=0$ e pelo gráfico de $y=\cos(x)$.

(l) $A=\{(x,y)\mid 0\le x\le 1,\ \sqrt{x}\le y\le 3\}$.

(m) A é limitado pelas retas $x=0$, $x=\frac{\pi}{2}$ pelos gráficos de $y=\operatorname{sen}(x)$ e $y=\cos(x)$.

(n) $A=\{(x,y)\mid x^2+1\le y\le x+1\}$.

(o) $A=\{(x,y)\mid x^2-1\le y\le x+1\}$.

(p) A é limitado pelas retas $x=0$, $x=\frac{\pi}{2}$ pelos gráficos de $y=\cos(x)$ e $y=1-\cos(x)$.

(q) $A=\{(x,y)\mid x\ge 0, x^3-x\le y\le -x^2+5x\}$.

(6) Encontre as primitivas:

(a) $\displaystyle\int xe^x\,dx$

(b) $\displaystyle\int x\,\mathrm{sen}(x)\,dx$

(c) $\displaystyle\int x^2e^x\,dx$

(d) $\displaystyle\int x\ln(x)\,dx$

(e) $\displaystyle\int \ln(x)\,dx$

(f) $\displaystyle\int x^2\ln(x)\,dx$

(g) $\displaystyle\int \frac{x}{\cos^2(x)}\,dx$

(h) $\displaystyle\int (\ln(x))^2\,dx$

(i) $\displaystyle\int e^2\cos(x)\,dx$

(j) $\displaystyle\int x^3e^{x^2}\,dx$

(k) $\displaystyle\int x^3\cos(x^2)\,dx$

(l) $\displaystyle\int e^{-x}\cos(2x)\,dx$

(m) $\displaystyle\int x^2\,\mathrm{sen}(x)\,dx$

(7) Sejam $\alpha \neq 0$, β, m e n constantes reais. Mostre as fórmulas:

(a) $\displaystyle\int \frac{1}{x^2-\alpha^2}\,dx = \frac{1}{2\alpha}\ln\left|\frac{x-\alpha}{x+\alpha}\right| + k.$

(b) $\displaystyle\int \frac{1}{\alpha^2+(x+\beta)^2}\,dx = \frac{1}{\alpha}\,\mathrm{arctg}\left(\frac{x+\beta}{\alpha}\right) + k.$

(c) $\displaystyle\int \frac{mu+n}{1+u^2}\,du = \frac{m}{2}\ln(1+u^2) + n\,\mathrm{arctg}\,u + k.$