MAT0103 - Matemática para Administração e Contabilidade

Lista 4

1º semestre de 2025

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(1) Encontre o valor do limite e justifique:

(a) $\lim\limits_{x\to 0} \dfrac{\operatorname{sen}(3x)}{x}$; (b) $\lim\limits_{x\to 0} \dfrac{\tan(kx)}{x}$; (c) $\lim\limits_{x\to 0} \dfrac{\operatorname{sen}(nx)}{\operatorname{sen}(mx)}$;

(d) $\lim\limits_{x\to 0} \dfrac{\tan(3x)}{\operatorname{sen}(5x)}$; (e) $\lim\limits_{x\to 0} \dfrac{\operatorname{sen}(x^n)}{\operatorname{sen}(x)^n}$; (f) $\lim\limits_{x\to 0} \dfrac{1-\cos^3(x)}{x\operatorname{sen}(2x)}$;

(g) $\lim\limits_{x\to 0} \dfrac{\tan(x)}{\sqrt[3]{(1-\cos x)^2}}$; (h) $\lim\limits_{x\to 0} \dfrac{1+\operatorname{sen}(x)-\cos(x)}{1-\operatorname{sen}(x)-\cos(x)}$; (i) $\lim\limits_{h\to 0} \dfrac{\tan(x)-\operatorname{sen}(x)}{x^3}$;

(j) $\lim\limits_{x\to 0} \dfrac{(1-\cos(x))^2}{\tan^3(x)-\operatorname{sen}^3(x)}$; (k) $\lim\limits_{x\to \pi} \dfrac{\operatorname{sen}(x)}{1-\frac{x^2}{\pi^2}}$; (l) $\lim\limits_{x\to \frac{\pi}{2}} \left(\frac{\pi}{2}-x\right)\tan x$;

(m) $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{\cos x-\operatorname{sen} x}{\cos 2x}$; (n) $\lim\limits_{y\to a} \operatorname{sen}\frac{y-a}{2}\tan\frac{\pi y}{2a}$; (o) $\lim\limits_{\alpha\to \beta} \dfrac{\operatorname{sen}^2(\alpha)-\operatorname{sen}^2(\beta)}{\alpha^2-\beta^2}$.

(2) Encontre o valor do limite e justifique:

(a) $\lim\limits_{x\to\infty} (\sqrt{x+a}-\sqrt{x})$; (b) $\lim\limits_{x\to\infty} (\sqrt{x^2+1}-\sqrt{x^2-1})$;

(c) $\lim\limits_{x\to\pm\infty} x(\sqrt{x^2+1}-x)$; (d) $\lim\limits_{x\to\infty} (\sqrt{(x+a)(x+b)}-x)$;

(e) $\lim\limits_{x\to\infty} x^{\frac{2}{3}}(\sqrt{x^3+1}-\sqrt{x^3-1})$; (f) $\lim\limits_{x\to\infty} \dfrac{2x+1}{x+3}$;

(g) $\lim\limits_{x\to-\infty} \dfrac{x^2-2x+3}{3x^2+x+1}$; (h) $\lim\limits_{x\to\infty} \dfrac{5x^4-2x+1}{4x^4+3x+2}$;

(i) $\lim\limits_{x\to\infty} \dfrac{x}{x^2+3x+1}$; (j) $\lim\limits_{x\to\infty} \dfrac{2x^3+1}{x^4+2x+3}$;

(k) $\lim\limits_{x\to\infty} \sqrt[3]{5+\dfrac{2}{x}}$; (l) $\lim\limits_{x\to\infty} (x^4-3x+2)$;

(m) $\lim\limits_{x\to\infty} (5-4x+x^2-x^5)$; (n) $\lim\limits_{x\to\infty} \dfrac{5x^3-6x+1}{6x^3+2}$;

(o) $\lim\limits_{x\to\infty} \dfrac{5x^3-6x+1}{6x^2+x+3}$; (p) $\lim\limits_{x\to\infty} \dfrac{\sqrt{x}+1}{x+3}$;

(q) $\lim\limits_{x\to\infty} \dfrac{x+\sqrt{x+3}}{2x-1}$; (r) $\lim\limits_{x\to\infty} (2x-\sqrt{x^2+3})$;

(s) $\lim\limits_{x\to\infty} [\sqrt{x+\sqrt{x}}-\sqrt{x-1}]$; (t) $\lim\limits_{x\to\infty} (x-\sqrt[3]{2+3x})$;

(u) $\lim\limits_{x\to\infty} \dfrac{2-3x}{x^2-6x+9}$.

(3) Dados

$$f(x) = \begin{cases} x^2+3, \text{se } x \leq 1 \\ x+1, \text{se } x > 1 \end{cases} \quad \text{e} \quad g(x) = \begin{cases} x^2, \text{se } x \leq 1 \\ 2, \text{se } x > 1 \end{cases},$$

(a) Mostre que $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$ e portanto, $\lim_{x \to 1} f(x)$ não existe.

(b) Mostre que $\lim_{x \to 1^-} g(x) \neq \lim_{x \to 1^+} g(x)$ e portanto, $\lim_{x \to 1} g(x)$ não existe.

(c) Determine fórmulas para $f(x)g(x)$.

(d) Prove que $\lim_{x \to 1}(f(x)g(x))$ existe, mostrando que $\lim_{x \to 1^-}(f(x)g(x)) = \lim_{x \to 1^+}(f(x)g(x))$.

(4) Dê o valor $f(p)$, se existir, para que $f = f(x)$ seja contínua em p. Justifique.

(a) $f(x) = \dfrac{x^2-4}{x-2}$, $p = 2$.

(b) $f(x) = \dfrac{x^2-x}{x}$, $p = 0$.

(c) $f(x) = \dfrac{|x|}{x}$, $p = 0$.

(d) $f(x) = \begin{cases} \dfrac{x^2-9}{x-3}, \text{se } x \neq 3, \\ 4, \text{se } x = 3; \end{cases}$, $p = 3$.

(5) Calcule e justifique.

(a) $\displaystyle\lim_{x \to \frac{1}{2}} \frac{4x^2-1}{2x-1}$;

(b) $\displaystyle\lim_{x \to 3} \frac{x^2-9}{x+3}$;

(c) $\displaystyle\lim_{x \to 3} \frac{\sqrt[3]{x}-\sqrt[3]{3}}{x-3}$;

(d) $\displaystyle\lim_{x \to 1} \frac{\sqrt{x}-1}{\sqrt{2x+3}-\sqrt{5}}$.

(6) Determine L para que a função dada seja contínua no ponto dado. Justifique.

(a) $f(x) = \begin{cases} \dfrac{x^3-27}{x-3}, \text{se } x \neq 3 \\ L, \text{se } x = 3. \end{cases}$

(b) f(x) = $f(x) = \begin{cases} \dfrac{\sqrt{x}-\sqrt{2}}{x-2}, \text{se } x \neq 2 \\ L, \text{se } x = 2. \end{cases}$