MAT1351 — Lista 5 — Gabarito

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1. a) 6
   b) 5
   c) 5
   d) $\frac{7}{2}$
   e) -6
   f) 1
   g) 2
   h) $\frac{5}{2}$
   i) -1
   j) $\frac{37}{2}$

2. a) $\cos(a)$
   b) $-\operatorname{sen}(a)$
   c) $\sec^2(a)$
   d) $-\operatorname{cossec}^2(a)$

3. a) $6x - 5$
   b) $4x^3 - x^2 + 5x - 0.3$
   c) $2ax + b$
   d) $\frac{1}{3\sqrt[3]{x^2}}$
   e) $\frac{1}{2}(7\sqrt{x^5} + \frac{1}{\sqrt{x} - 2})$
   f) $6(a - x)$
   g) $\frac{ax^2 - c}{(a+b)x^2}$
   h) $4x^3 - 3x^2 - 8x + 9$
   i) $6x^5 - 15x^4 + 12x^3 - 9x^2 + 2x + 3$
   j) $\frac{-x-1}{2\sqrt{x^3}}$
   k) $(\sqrt[3]{x} + 2x)(\frac{2x}{3\sqrt[3]{x^4}} + 3) + (\frac{1}{3\sqrt[3]{x^2}} + 2)(\sqrt[3]{x^2} + 3x + 1)$
   l) Igual ao anterior.
   m) $6x^5 - 56x^3 + 98x$
   n) $\frac{3\sqrt{6}x + 2(\sqrt{2} + \sqrt{3} + \sqrt{6})\sqrt{x} + \sqrt{3} + \sqrt{2} + 1}{2\sqrt{x}}$

4. $f'(a) = 3 - \frac{1}{\sqrt{a}}$

5.

6. O foco deste exercício está em aplicar a fórmula da derivada do quociente de funções.

a) $\frac{-2}{(x-1)^2}$

b) $\frac{1-x^2}{(x^2+1)^2}$

c) $\frac{3t^2-6t-1}{(t-1)^2}$

d) $\frac{v^4+2v^3+5v^2-2}{(v^2+v+1)^2}$

e) $\frac{ad-bc}{(cx+d)^2}$

f) $-\frac{4x}{3(x^2-1)^2}-3x^2+2x+1$

g) $\frac{-6x^2}{(1+x^3)^2}$

h) $\frac{-6x^2}{(x^3-1)^2}$

i) $\frac{-3x^2}{\sqrt{\pi}}$

j) $\frac{-2t-1}{(t^2+t+1)^2}$

k) $\frac{-2t+3}{(t^2-3t+6)^2}$

l) $\frac{-4(x^5-2b^2x^3)}{(b^2-x^2)^2}$

m) $\frac{-x^4-2x^3+3x^2+2x+1}{(x^3+1)^2}$

n) $\frac{6x(-5x^3+3x+1)}{(1-2x^3)^2(1-x^2)^2}$

7. a) $\cos(x)-\operatorname{sen}(x)$

b) $\frac{1-\cos(x)-x\operatorname{sen}(x)}{(1-\cos x)^2}$

c) $\frac{x\sec^2 x-\tan x}{x^2}$

d) $\operatorname{sen} x + x\cos x - \operatorname{sen} x$

e) $\frac{-\operatorname{sen} x}{x^2}+\frac{\cos x}{x}+\frac{1}{\cos x}-\frac{x\cos x}{\operatorname{sen}^2 x}$

f) $\frac{\operatorname{sen}^2 t-\operatorname{sen} t+\cos^2 t+t\cos t}{(t+\cos t)^2}$

g) $\frac{\operatorname{sen} x+\cos x-x(\cos x-\operatorname{sen} x)}{(\operatorname{sen} x+\cos x)^2}$

h) $\frac{x\cos x+x\operatorname{sen} x+\operatorname{sen} x+\operatorname{sen}(x)\tan(x)-x\tan(x)\sec(x)}{(1+\tan x)^2}$

i) $-2\cos(x)\operatorname{sen}(x)$

j) $\frac{2}{(\operatorname{sen} x+\cos x)^2}$

8. a) $12\left(x^2+x\right)^3(2x+1)-15x^2\sin\left(x^3\right)$;

b) $\frac{e^{x^4}4x^3\left(x^2+1\right)-2xe^{x^4}}{\left(x^2+1\right)^2}$;

c) $20x^4 \ln(x^2+1)(x^5+1)^3 + \dfrac{2x(x^5+1)^4}{x^2+1}$;

d) $\dfrac{2(5x^2+6x^6)(10x+36x^5)(x^2+1) - 2x(5x^2+6x^6)^2}{(x^2+1)^2}$;

e) $\dfrac{2(x+1)^3(-x(x+1)+2)}{e^{x^2}}$;

f) $-\dfrac{3(4x^3\cos(x^4) - 5x^4\sin(x^5))}{(\sin(x^4)+\cos(x^5))^2}$;

g)

h) $e^{4x^3+3x^2}(12x^2+6x) + 8x\ln(x^5+4x^4)(x^2+1)^3 + \dfrac{(5x+16)(x^2+1)^4}{x(x+4)}$;

i) $\dfrac{3x^2\sec(x^4)}{2\sqrt{x^3}} + \dfrac{4x^3\tan(x^4)\sqrt{x^3}}{\cos(x^4)}$;

j) $15e^{x^5}x^4 + \dfrac{30}{x}$;

k) $e^{(x^2+x+1)^3} 3(x^2+x+1)^2(2x+1)$;

l) $12x^2\cos(x^3) - \dfrac{5x^4}{\sin^2(x^5)}$;

m) $4(2x+6x^2) + 3\left(6e^{x^6}x^{10} + 5e^{x^6}x^4\right) + 14x^6$

n) $\dfrac{8x^2\ln(x^5)(x^2+1)^3 - 5(x^2+1)^4}{x\ln^2(x^5)}$

o) $\dfrac{4x(x^2+4)}{3\left((x^2+4)^2\right)^{\frac{2}{3}}}$

p) $\dfrac{\cos(\sin(x)) + x\sin(\sin(x))\cos(x)}{\cos^2(\sin(x))}$

q) $\dfrac{3x^5\cos(x) + 13x^4\sin(x) + 3x^4\cos(x) + 12x^3\sin(x) + 3x\cos(x) + 3\cos(x) + \sin(x)}{3(x+1)^{\frac{2}{3}}}$

r) $\dfrac{(x^2+1)(4(3x^2+2x+7)(6x+2)+5x^4) - 2x((3x^2+2x+7)^4+x^5+1)}{(x^2+1)^2}$

9. a) $3e^{x^2}x^2 + 2e^{x^2}x^4$;

b) $12\ln(x)(3x+5)^3 + \dfrac{(3x+5)^4}{x}$;

c) $2xe^{x^3}\cos(x^4) + \left(3e^{x^3}x^2\cos(x^4) - 4e^{x^3}x^3\sin(x^4)\right)x^2$;

d) $\dfrac{2e^x}{(1-e^x)^2}$;

e) $2\left(e^{x}x^{2}\ln(x)+4e^{x}x\ln(x)+\frac{e^{x}(x+1)^{2}}{x}+3e^{x}\ln(x)\right)$;

f) $\frac{(x+1)(-x\ln(x)-x-3\ln(x)-1)}{x^{4}\ln^{2}(x)}$;

g) $5(x+2x\ln(x))$;

h) $\frac{e^{x}(x^{2}+1)-2xe^{x}}{(x^{2}+1)^{2}}$;

i) $\frac{1-\ln(x)}{x^{2}}$;

j) $\frac{2(3x^{2}+2x+4)^{2}(9x-3x^{5}-5x^{4}-16x^{3}+3)}{(x^{4}+1)^{3}}$;

k) $x^{x^{2}}(2x\ln(x)+x)$

l) $x^{x^{x}}(x^{x}\ln(x)(\ln(x)+1)+x^{x-1})$

m) $x^{\sin(x)}\left(\cos(x)\ln(x)+\frac{\sin(x)}{x}\right)$

n) $x^{\frac{2\sqrt{x}-1}{2}}\ln(x)+2x^{\frac{2\sqrt{x}-1}{2}}$

o) $\frac{2e^{\ln^{2}(x)}\ln(x)}{x}$

p) $x^{\frac{1-2x}{x}}(-\ln(x)+1)$

q) $2\left(-\frac{\ln(x+1)}{x^{2}}+\frac{1}{x(x+1)}\right)(x+1)^{\frac{2}{x}}$

r) $\sin^{-1+\cos(x)}(x)\left(\cos^{2}(x)-\sin^{2}(x)\ln(\sin(x))\right)$

s) $\ln^{x}(x)\left(\ln(\ln(x))+\frac{1}{\ln(x)}\right)$

t) $\left(\frac{x}{x+1}\right)^{x}\left(\ln\left(\frac{x}{x+1}\right)+\frac{1}{x+1}\right)$

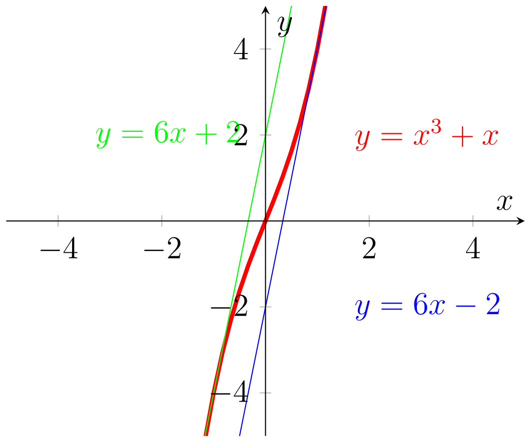
u) $\left(\cos(x)\ln(x^{2}+1)+\frac{2x\sin(x)}{x^{2}+1}\right)(x^{2}+1)^{\sin(x)}$.

10. a) Temos $f'(x)=3x^{2}+3$ assim reta tangente é paralela ao reta $y=6x-1$ em ponto x_0 se $3x_0^{2}+3=6$ ou seja $x_0=-1$ ou $x_0=1$. Assim temos duas soluções: $y-f(-1)=6(x-(-1))$ ou $y-f(1)=6(x-1)$. No primeiro caso a reta é

$$y=6x+2.$$

No segundo caso a reta é

$$y=6x-2.$$

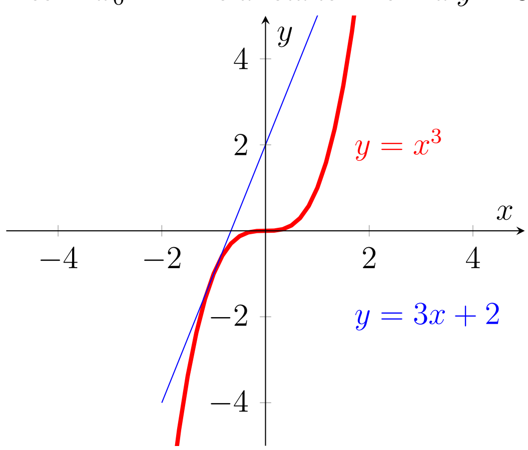


b)

c) Temos $f'(x) = 3x^2$. Como reta passa pelo $(0, 2)$ assim procuramos x_0 tal que

$$f(x_0) - 2 = 3x_0^2(x_0 - 0).$$

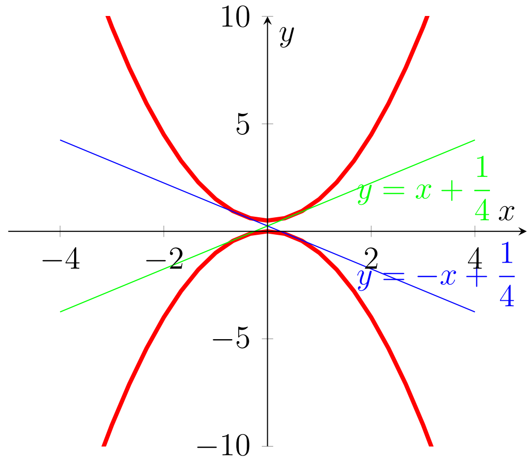
Assim $x_0 = -1$ e a reta tem forma $y = 3x + 2$.



d) Encontramos x_1 e x_2 tais que

$$f'(x_1) = g'(x_2).$$

Que implica que $x_1 = -x_2$. Outra condição é $\dfrac{f(x_1) - g(x_2)}{x_1 - x_2} = f'(x_1)$. Assim $x_1 = 1/2$ ou $x_1 = -1/2$. No primeiro caso reta é $y = -x + \dfrac{1}{4}$. No segundo é $y = x + \dfrac{1}{4}$



e) Encontramos x tais que

$$f'(x_1) = 4x^3 + 6x^2 - 4x + 8 = 8.$$

Assim $x = 0$, $x = \dfrac{-3 \pm \sqrt{17}}{4}$. Por exemplo no primeiro caso a reta tem forma

$$y = 8x + 12.$$