Cálculo II — Lista 2.

1. Ache e esboce o domínio das funções:

   (a) $f(x,y) = \sqrt{y-x} + \sqrt{y-1}$,

   (b) $f(x,y) = \ln y - x^3$,

   (c) $f(x,y) = \sqrt{1 - \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2}}$,

   (d) $f(x,y) = \sqrt{x+y} + \sqrt{x-y}$,

   (e) $f(x,y) = \dfrac{1}{\sqrt{x+y}} + \dfrac{1}{\sqrt{x-y}}$,

   (f) $f(x,y) = \operatorname{arc\,sen} \dfrac{y-1}{x}$,

   (g) $f(x,y) = \ln xy$,

   (h) $f(x,y) = \sqrt{x - \sqrt{y}}$,

   (i) $f(x,y) = \sqrt{x - \sqrt{y}}$,

   (j) $f(x,y) = \dfrac{\sqrt{4x-y^2}}{\ln(1-x^2-y^2)}$,

   (k) $f(x,y) = \ln x - \ln \operatorname{sen} y$.

   (l) $f(x,y) = \sqrt{x \operatorname{sen} y}$.

2. Esboce as curvas de nível de:

   (a) $f(x,y) = 3x + 4y$,

   (b) $f(x,y) = xy$,

   (c) $f(x,y) = \dfrac{1}{x^2+y^2}$,

   (d) $f(x,y) = y(x^2+1)$,

   (e) $f(x,y) = \dfrac{xy-1}{x^2}$,

   (f) $f(x,y) = \dfrac{x+y}{x-y}$,

   (g) $f(x,y) = (x^2+y^2)^2 - 2(x^2-y^2)$,
   para níveis $c = -1, -\dfrac{1}{2}, 0, -\dfrac{1}{2}, 1$.

3. Esboce os gráficos de:

   (a) $f(x,y) = 1 - 2x - 3y$,

   (b) $f(x,y) = 9x^2 + 4y^2$,

   (c) $f(x,y) = x^2 - y^2$,

   (d) $f(x,y) = y^2 + 1$,

   (e) $f(x,y) = \sqrt{4x^2+y^2}$,

   (f) $f(x,y) = x^2 + 2x + y^2 + 4y$,

   (g) $f(x,y) = e^{\sqrt{x^2+y^2}}$,

   (h) $f(x,y) = \dfrac{x}{x^2+1}$.

4. Calcule os seguintes limites, caso existam. Se não existirem, explique o por quê:

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sqrt{x+1}-\sqrt{1}}{xy+y}$,

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$,

   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$,

   (d) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\operatorname{sen}(x^2+y^2)}{x^2+y^2}$,

   (e) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{(x^2+y^2)x^2y^2}$,

   (f) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(x+y)^3}{x^2+y^2}$,

   (g) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$,

   (h) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+3xy+y^2}{3x^2+4y^2}$,

   (i) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2-y^2}$,

   (j) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^3-y}$,

5. Determine o conjunto dos pontos de continuidade. Justifique a resposta:

(a) $f(x, y) = \dfrac{1}{a^2 - x^2 - y^2}$,

(b) $f(x, y) = \dfrac{xy}{y - x^2}$,

(c) $f(x, y) = \ln \dfrac{x - y}{x^2 + y^2}$,

(d) $f(x, y) = \begin{cases} \dfrac{x - 3y}{x^2 + y^2}, & se \quad (x, y) \neq (0, 0) \\ 0 & se \quad (x, y) = (0, 0) \end{cases}$

(e) $f(x, y) = \begin{cases} \dfrac{sen(x^2 + y^2)}{x^2 + y^2}, & se \quad (x, y) \neq (0, 0) \\ 1 & se \quad (x, y) = (0, 0) \end{cases}$

(f) $f(x, y) = \begin{cases} \dfrac{1 - \cos(\sqrt{x^2 + y^2})}{x^2 + y^2}, & se \quad (x, y) \neq (0, 0) \\ 1 & se \quad (x, y) = (0, 0) \end{cases}$