MAT1351 — Lista 5

8. a) $12\left(x^2+x\right)^3(2x+1)-15x^2\sin\left(x^3\right)$;
   b) $\frac{e^{x^4}4x^3\left(x^2+1\right)-2xe^{x^4}}{\left(x^2+1\right)^2}$;
   c) $20x^4\ln\left(x^2+1\right)\left(x^5+1\right)^3+\frac{2x\left(x^5+1\right)^4}{x^2+1}$;
   d) $\frac{2\left(5x^2+6x^6\right)\left(10x+36x^5\right)\left(x^2+1\right)-2x\left(5x^2+6x^6\right)^2}{\left(x^2+1\right)^2}$;
   e) $\frac{2(x+1)^3(-x(x+1)+2)}{e^{x^2}}$;
   f) $-\frac{3\left(4x^3\cos\left(x^4\right)-5x^4\sin\left(x^5\right)\right)}{\left(\sin(x^4)+\cos(x^5)\right)^2}$;
   g)
   h) $e^{4x^3+3x^2}\left(12x^2+6x\right)+8x\ln\left(x^5+4x^4\right)\left(x^2+1\right)^3+\frac{(5x+16)\left(x^2+1\right)^4}{x(x+4)}$;
   i) $\frac{3x^2\sec\left(x^4\right)}{2\sqrt{x^3}}+\frac{4x^3\tan\left(x^4\right)\sqrt{x^3}}{\cos(x^4)}$;
   j) $15e^{x^5}x^4+\frac{30}{x}$;
   k) $e^{\left(x^2+x+1\right)^3}3\left(x^2+x+1\right)^2(2x+1)$;
   l) $12x^2\cos\left(x^3\right)-\frac{5x^4}{\sin^2(x^5)}$;
   m) $4\left(2x+6x^2\right)+3\left(6e^{x^6}x^{10}+5e^{x^6}x^4\right)+14x^6$
   n) $\frac{8x^2\ln\left(x^5\right)\left(x^2+1\right)^3-5\left(x^2+1\right)^4}{x\ln^2(x^5)}$
   o) $\frac{4x\left(x^2+4\right)}{3\left((x^2+4)^2\right)^{\frac{2}{3}}}$
   p) $\frac{\cos(\sin(x))+x\sin(\sin(x))\cos(x)}{\cos^2(\sin(x))}$
   q) $\frac{3x^5\cos(x)+13x^4\sin(x)+3x^4\cos(x)+12x^3\sin(x)+3x\cos(x)+3\cos(x)+\sin(x)}{3(x+1)^{\frac{2}{3}}}$
   r)

9. a) $3e^{x^2}x^2+2e^{x^2}x^4$;
   b) $12\ln(x)(3x+5)^3+\frac{(3x+5)^4}{x}$;
   c) $2xe^{x^3}\cos\left(x^4\right)+\left(3e^{x^3}x^2\cos\left(x^4\right)-4e^{x^3}x^3\sin\left(x^4\right)\right)x^2$;
   d) $\frac{2e^x}{(1-e^x)^2}$;
   e) $2\left(e^x x^2\ln(x)+4e^x x\ln(x)+\frac{e^x(x+1)^2}{x}+3e^x\ln(x)\right)$;

f) $\frac{(x+1)(-x\ln(x)-x-3\ln(x)-1)}{x^4\ln^2(x)}$;

g) $5\left(x+2x\ln\left(x\right)\right)$;

h) $\frac{e^x\left(x^2+1\right)-2xe^x}{\left(x^2+1\right)^2}$;

i) $\frac{1-\ln(x)}{x^2}$;

j) $\frac{2\left(3x^2+2x+4\right)^2\left(9x-3x^5-5x^4-16x^3+3\right)}{\left(x^4+1\right)^3}$;

k) $x^{x^2}\left(2x\ln\left(x\right)+x\right)$

l) $x^{x^x}\left(x^x\ln\left(x\right)\left(\ln\left(x\right)+1\right)+x^{x-1}\right)$

m) $x^{\sin(x)}\left(\cos\left(x\right)\ln\left(x\right)+\frac{\sin(x)}{x}\right)$

n) $x^{\frac{2\sqrt{x}-1}{2}}\ln\left(x\right)+2x^{\frac{2\sqrt{x}-1}{2}}$

o) $\frac{2e^{\ln^2(x)}\ln(x)}{x}$

p) $x^{\frac{1-2x}{x}}\left(-\ln\left(x\right)+1\right)$

q) $2\left(-\frac{\ln(x+1)}{x^2}+\frac{1}{x(x+1)}\right)\left(x+1\right)^{\frac{2}{x}}$

r) $\sin^{-1+\cos(x)}\left(x\right)\left(\cos^2\left(x\right)-\sin^2\left(x\right)\ln\left(\sin\left(x\right)\right)\right)$

s) $\ln^x\left(x\right)\left(\ln\left(\ln\left(x\right)\right)+\frac{1}{\ln(x)}\right)$

t) $\left(\frac{x}{x+1}\right)^x\left(\ln\left(\frac{x}{x+1}\right)+\frac{1}{x+1}\right)$

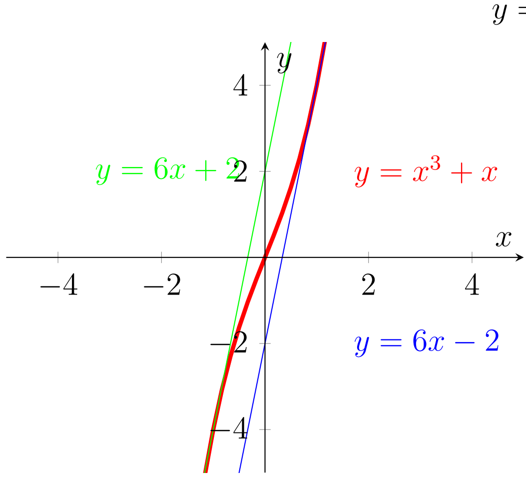
u) $\left(\cos\left(x\right)\ln\left(x^2+1\right)+\frac{2x\sin(x)}{x^2+1}\right)\left(x^2+1\right)^{\sin(x)}$.

10. a) Temos $f'(x)=3x^2+3$ assim reta tangente paralela ao reta $y=6x-1$ em ponto x_0 se $3x_0^2+3=6$ ou seja $x_0=-1$ ou $x_0=1$. Assim temos duas solues: $y-f(-1)=6(x-(-1))$ ou $y-f(1)=6(x-1)$. No primeiro caso a reta

$$y=6x+2.$$

No segundo caso a reta

$$y=6x-2.$$

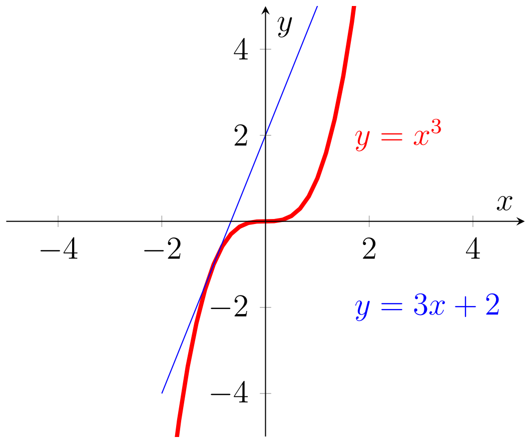


b)

c) Temos $f'(x) = 3x^2$. Como reta passa pelo $(0,2)$ assim procuramos x_0 tal que

$$f(x_0) - 2 = 3x_0^2(x_0 - 0).$$

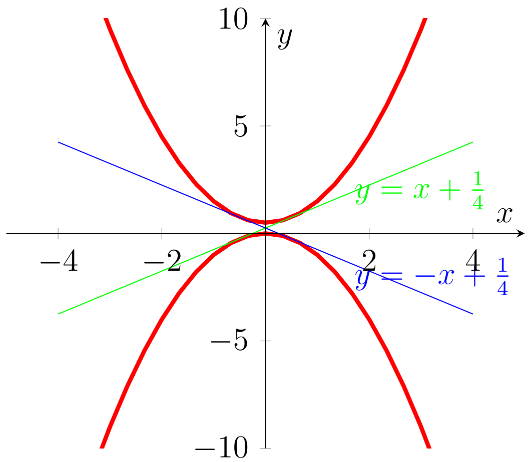
Assim $x_0 = -1$ e a reta tem forma $y = 3x + 2$.



d) Encontramos x_1 e x_2 tais que

$$f'(x_1) = g'(x_2).$$

Que implica que $x_1 = -x_2$. Outra condio $\dfrac{f(x_1) - g(x_2)}{x_1 - x_2} = f'(x_1)$. Assim $x_1 = 1/2$ ou $x_1 = -1/2$. No primeiro caso reta $y = -x + \frac{1}{4}$. No segundo $y = x + \frac{1}{4}$

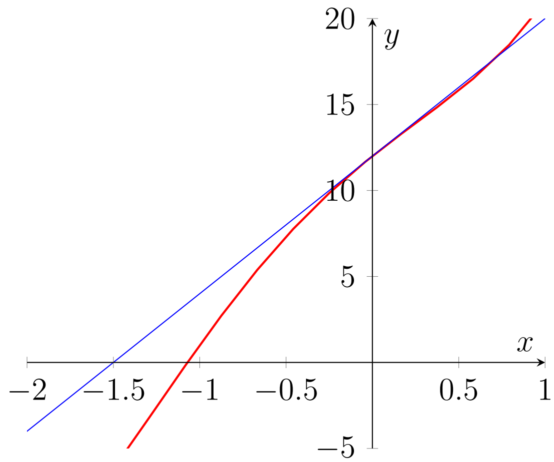


e) Encontramos x tais que

$$f'(x_1) = 4x^3 + 6x^2 - 4x + 8 = 8.$$

Assim $x = 0$, $x = \dfrac{-3 \pm \sqrt{17}}{4}$. Por exemplo no primeiro caso a reta tem forma

$$y = 8x + 12.$$


20
15
10
5
−5
y
x
−2
−1.5
−1
−0.5
0.5
1