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**MAT1351 — Lista 4**  
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1. a)  $= \lim_{x \rightarrow 0} \frac{3 \cdot \text{sen}(3x)}{3x} = 3$

b)  $= \lim_{x \rightarrow 0} \frac{\text{sen}(kx)}{x} = k$

c)  $= \lim_{x \rightarrow 0} \frac{mx \cdot \text{sen}(nx) \cdot n}{\text{sen}(mx) \cdot nx \cdot m} = \frac{n}{m}$

d)  $\frac{3}{5}$

e) 1

f)  $\frac{3}{4}$

g) Não existe;

h)  $= \lim_{x \rightarrow 0} \frac{\frac{1-\cos(x)}{x} + \frac{\text{sen}(x)}{x}}{\frac{1-\cos(x)}{x} - \frac{\text{sen}(x)}{x}} = \frac{0-1}{0+1} = -1$

i)  $= \lim_{x \rightarrow 0} \frac{\text{sen}(x)(\frac{1}{\cos(x)} - 1)}{x \cdot x^2} = \lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x^2} = \frac{1}{2}$

j) Não existe.

k) Seja  $t = \pi - x$ , assim  $\lim \dots = \pi^2 \lim_{t \rightarrow 0} \frac{\text{sen}(\pi - t)}{t(2\pi + t)} = \frac{\pi}{2} \lim_{t \rightarrow 0} \frac{\text{sen}(t)}{t} = \frac{\pi}{2}$ .

l) Seja  $t = \frac{\pi}{2} - x$ , assim  $\lim \dots = \lim_{t \rightarrow 0} t \frac{\text{sen}(\frac{\pi}{2} - t)}{\cos(\frac{\pi}{2} - t)} = \lim_{t \rightarrow 0} t \frac{\cos(t)}{\text{sen}(t)} = 1$ .

m)  $= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x - \text{sen } x}{(\cos x - \text{sen } x)(\cos x + \text{sen } x)} = \frac{1}{\sqrt{2}}$ .

n)  $-\frac{a}{\pi}$

o) Se  $\beta = 0$  assim  $\lim = 1$ . Seja  $\beta \neq 0$  e  $x = \alpha - \beta$ , assim  $\alpha = x + \beta$ . Temos

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{(\text{sen}(x + \beta) - \text{sen } \beta)(\text{sen}(x + \beta) + \text{sen } \beta)}{x(x + 2\beta)} \\ &= \frac{\text{sen } \beta}{\beta} \lim_{x \rightarrow 0} \left( \frac{\text{sen}(x) \cos \beta}{x} + \frac{\cos(x) \text{sen}(\beta) - \text{sen}(\beta)}{x} \right) = \frac{\text{sen}(\beta) \cos(\beta)}{2}. \end{aligned} \quad (1)$$

2. a) 0;

b) 0;

c)  $= \lim_{x \rightarrow \pm\infty} \frac{x(x^2 + 1 - x^2)}{\sqrt{x^2 + 1} + x} = \lim_{x \rightarrow \pm\infty} \frac{1}{\sqrt{1 + \frac{1}{x^2}} + 1} = \frac{1}{2}$ ;

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- d) 0;
- e)  $\infty$ ;
- f) 2;
- g)  $\frac{1}{3}$ ;
- h)  $\frac{5}{4}$ ;
- i) 0;
- j) 0;
- k)  $\sqrt[3]{5}$ ;
- l)  $\infty$ ;
- m)  $-\infty$ ;
- n)  $\frac{5}{6}$ ;
- o)  $\infty$ ;
- p)  $\lim_{x \rightarrow \infty} \frac{\sqrt{\frac{1}{x} + \frac{1}{x}}}{1 + \frac{3}{x}} = 0$ ;
- q)  $\frac{1}{2}$ ;
- r)  $= \lim_{x \rightarrow \infty} \frac{(3x^2 - 3)}{2x + \sqrt{x^2 + 3}} = \lim_{x \rightarrow \infty} \frac{3x - \frac{3}{x}}{2 + \sqrt{1 + \frac{3}{x^2}}} = \infty$ ;
- s)  $\frac{1}{2}$ ;
- t)  $\infty$ ;
- u) 0.
3. a)  $\lim_{x \rightarrow 1^-} f(x) = 4$  e  $\lim_{x \rightarrow 1^+} f(x) = 2$
- b)
- c)  $f(x)g(x) = \begin{cases} x^2(x^2 + 3), \text{ se } x \leq 1 \\ 2(x + 1), \text{ se } x > 1 \end{cases}$
- d)  $\lim_{x \rightarrow 1^-} f(x)g(x) = 4 = \lim_{x \rightarrow 1^+} f(x)g(x)$ .
4. a)  $f(2) = \lim_{x \rightarrow 2} f(x) = 4$
- b)  $f(0) = \lim_{x \rightarrow 0} f(x) = -1$
- c) Não existe, pois  $\lim_{x \rightarrow 0^-} f(x) = -1$  e  $\lim_{x \rightarrow 0^+} f(x) = 1$
- d)  $f(3) = \lim_{x \rightarrow 3} f(x) = 6$

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5. a)  $\lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x - 1} = \lim_{x \rightarrow \frac{1}{2}} \frac{(2x - 1)(2x + 1)}{2x - 1} = 2$

b) 0

c)  $\frac{1}{3\sqrt[3]{3^2}}$

d)  $\frac{\sqrt{5}}{2}$

6. a)  $L = f(3) = \lim_{x \rightarrow 3} f(x) = 27;$

b)  $L = f(2) = \lim_{x \rightarrow 2} f(x) = \frac{1}{2\sqrt{2}}.$

7. Veja anotações das aulas.

8. Veja anotações das aulas.

9. Veja anotações das aulas.