MAT1351 — Lista 4

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1. a) $= \lim_{x\to 0} \frac{3\cdot \mathrm{sen}(3x)}{3x} = 3$

   b) $= \lim_{x\to 0} \frac{\mathrm{sen}(kx)}{x} = k$

   c) $= \lim_{x\to 0} \frac{mx\cdot \mathrm{sen}(nx)\cdot n}{\mathrm{sen}(mx)\cdot nx\cdot m} = \frac{n}{m}$

   d) $\frac{3}{5}$

   e) 1

   f) $\frac{3}{4}$

   g) Não existe;

   h) $= \lim_{x\to 0} \frac{\frac{1-\cos(x)}{x} + \frac{\mathrm{sen}(x)}{x}}{\frac{1-\cos(x)}{x} - \frac{\mathrm{sen}(x)}{x}} = \frac{0-1}{0+1} = -1$

   i) $= \lim_{x\to 0} \frac{\mathrm{sen}(x)(\frac{1}{\cos(x)}-1)}{x\cdot x^2} = \lim_{x\to 0} \frac{1-\cos(x)}{x^2} = \frac{1}{2}$

   j) Não existe.

   k) Seja $t = \pi - x$, assim $\lim ... = \pi^2 \lim_{t\to 0} \frac{\mathrm{sen}(\pi - t)}{t(2\pi + t)} = \frac{\pi}{2} \lim_{t\to 0} \frac{\mathrm{sen}(t)}{t} = \frac{\pi}{2}$.

   l) Seja $t = \frac{\pi}{2} - x$, assim $\lim ... = \lim_{t\to 0} t\frac{\mathrm{sen}(\frac{\pi}{2}-t)}{\cos(\frac{\pi}{2}-t)} = \lim_{t\to 0} t\frac{\cos(t)}{\mathrm{sen}(t)} = 1$.

   m) $= \lim_{x\to\frac{\pi}{4}} \frac{\cos x - \mathrm{sen}\, x}{(\cos x - \mathrm{sen}\, x)(\cos x + \mathrm{sen}\, x)} = \frac{1}{\sqrt{2}}$.

   n) $-\frac{a}{\pi}$

   o) Se $\beta = 0$ assim $\lim = 1$. Seja $\beta \neq 0$ e $x = \alpha - \beta$, assim $\alpha = x + \beta$. Temos

$$\begin{aligned} &\lim_{x\to 0} \frac{(\mathrm{sen}(x+\beta) - \mathrm{sen}\,\beta)(\mathrm{sen}(x+\beta) + \mathrm{sen}\,\beta)}{x(x+2\beta)} \\ =&\frac{\mathrm{sen}\,\beta}{\beta} \lim_{x\to 0} (\frac{\mathrm{sen}(x)\cos\beta}{x} + \frac{\cos(x)\,\mathrm{sen}(\beta) - \mathrm{sen}(\beta)}{x}) = \frac{\mathrm{sen}(\beta)\cos(\beta)}{2}. \end{aligned} \tag{1}$$

2. a) 0;

   b) 0;

   c) $= \lim_{x\to\pm\infty} \frac{x(x^2+1-x^2)}{\sqrt{x^2+1}+x)} = \lim_{x\to\pm\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}}+1} = \frac{1}{2}$;

d) 0;

e) ∞;

f) 2;

g) $\dfrac{1}{3}$;

h) $\dfrac{5}{4}$;

i) 0;

j) 0;

k) $\sqrt[3]{5}$;

l) ∞;

m) $-\infty$;

n) $\dfrac{5}{6}$;

o) ∞;

p) $\lim\limits_{x\to\infty} \dfrac{\sqrt{\frac{1}{x}+\frac{1}{x}}}{1+\frac{3}{x}} = 0$;

q) $\dfrac{1}{2}$;

r) $= \lim\limits_{x\to\infty} \dfrac{(3x^2-3)}{2x+\sqrt{x^2+3)}} = \lim\limits_{x\to\infty} \dfrac{3x-\frac{3}{x}}{2+\sqrt{1+\frac{3}{x^2}}} = \infty$;

s) 0;

t) ∞;

u) 0.

3. a) $\lim\limits_{x\to 1^-} f(x) = 4$ e $\lim\limits_{x\to 1^+} f(x) = 2$

   b)

   c) $f(x)g(x) = \begin{cases} x^2(x^2+3), \text{se } x \leq 1 \\ 2(x+1), \text{se } x > 1 \end{cases}$

   d) $\lim\limits_{x\to 1^-} f(x)g(x) = 4 = \lim\limits_{x\to 1^+} f(x)g(x)$.

4. a) $f(2) = \lim\limits_{x\to 2} f(x) = 4$

   b) $f(0) = \lim\limits_{x\to 0} f(x) = -1$

   c) No existe, pois $\lim\limits_{x\to 0^-} f(x) = -1$ e $\lim\limits_{x\to 0^+} f(x) = 1$

   d) $f(3) = \lim\limits_{x\to 3} f(x) = 6$

5. a) $\lim\limits_{x\to\frac{1}{2}} \dfrac{4x^2-1}{2x-1} = \lim\limits_{x\to\frac{1}{2}} \dfrac{(2x-1)(2x+1)}{2x-1} = 2$

   b) 0

   c) $\dfrac{1}{3\sqrt[3]{3^2}}$

   d) $\dfrac{\sqrt{5}}{2}$

6. a) $L = f(3) = \lim\limits_{x\to 3} f(x) = 27;$

   b) $L = f(2) = \lim\limits_{x\to 2} f(x) = \dfrac{1}{2\sqrt{2}}.$