

MAT0103 — Lista 7

1. a) ∞
b) 2
c) 0
d) $\ln(\cos(1))$
e) $-\frac{1}{8}$
f) ∞
g) $\frac{1}{2}$
h) não existe.
2. a) $2x^2 + k$
b) $5x + k$
c) $\frac{x^3}{3} + x^2 + x + k$
d) $\frac{5x^4}{4} + 2x^2 + 7x + k$
e) $\frac{x^2}{2} + \ln|x| - \frac{1}{x} + k$
f) $\frac{10x^{\frac{13}{10}}}{13} + k$
g) $-\cos(x) + \sin(x) - 2e^x - \frac{4^x}{\ln(4)} + k$
h) $\frac{x^4}{4} - x^4 + x^3 - \frac{x^2}{2} + 5x - 2\ln|x| + k$;
i) $\frac{x^4}{2} + \frac{1}{5x^5} + k$
3. a) 2
b) 6
c) $\frac{2}{3}$
d) $-\frac{5}{2}$
e) $\frac{6}{7}$
f) $\frac{13}{10}$
g) $-\frac{67}{36}$
h) $\frac{1}{3}$
i) $\frac{1}{3}$
j) $-\frac{5}{24}$
k) 9
l) 0
m) $\frac{9}{2} + \ln(2)$
n) $\frac{2}{3}$

- o) $\frac{\pi}{4}$
 p) $\frac{1}{2}(e^2 - 1)$
4. a) $\frac{5}{4}$
 b) $\frac{\pi}{4}$
 c) $\frac{\pi}{4}$
 d) 1
 e) $1 - \frac{\pi}{4}$
5. a) $\int_1^3 (x^3) dx = 20$
 b) $\int_1^4 (\sqrt{x}) dx = \frac{14}{3}$
 c) $\int_1^{-1} (x^2 - 1) dx = \frac{4}{3}$
 d) $\int_{-2}^2 (4 - x^2) dx = \frac{32}{3}$
 e) $\int_0^2 |\sin(x)| dx = 2 \sin^2(1)$
 f) $\int_{-1}^1 (3 - 2x - x^2) dx - \int_1^2 (3 - 2x - x^2) dx = \frac{16}{3} + \frac{7}{3} = \frac{23}{3}$
 g) $\int_{-1}^2 (x^2 + 2x + 5) dx = 21$;
 h) $= -\int_0^1 (x^3 - x) dx + \int_1^2 (x^3 - x) dx = \frac{1}{4} + \frac{9}{4} = \frac{5}{2}$;
 i) $= 2 \int_0^{\frac{\pi}{2}} \cos(x) dx = 2$;
 j) $\int_0^1 (3 - \sqrt{x}) dx = \frac{7}{3}$;
 k) $= 2 \int_0^{\frac{\pi}{4}} (\cos(x) - \sin(x)) dx = 2(\sqrt{2} - 1)$.
 l) Os graficos das funções $f(x) = x^2 + 1$ e $g(x) = x + 1$ interceptam em $x = 0$ e $x = 1$. Temos

$$\int_0^1 ((x + 1) - (x^2 + 1)) dx = \frac{1}{6}.$$

- m) Os graficos das funções $f(x) = x^2 - 1$ e $g(x) = x + 1$ interceptam em $x = -1$ e $x = 2$. Temos

$$\int_{-1}^2 ((x + 1) - (x^2 - 1)) dx = \frac{9}{2}.$$

- n) Os graficos das funções $f(x) = \cos(x)$ e $g(x) = 1 - \cos(x)$ interceptam em $x = \frac{\pi}{3}$ no intervalo $[0, \frac{\pi}{2}]$. Assim

$$\int_0^{\frac{\pi}{3}} \cos(x) - (1 - \cos(x)) dx - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos(x) - (1 - \cos(x)) dx = (\sqrt{3} - \frac{\pi}{3}) + (-2 + \sqrt{3} + \frac{\pi}{6}).$$

o) Os graficos das funções $f(x) = x^3 - x$ e $g(x) = -x^2 + 5x$ interceptam em $x = 2$. Temos

$$\int_0^2 ((-x^2 + 5x) - (x^3 - x)) dx = \frac{16}{3}.$$

6. a) $e^x x - e^x + k$

b) $\sin(x) - x \cos(x) + k$

c) $x^2 e^x - 2(e^x x - e^x) + k$

d) $\frac{1}{2} x^2 \ln(x) - \frac{x^2}{4} + k$

e) $x \ln(x) - x + k$

f) $\frac{1}{3} x^3 \ln(x) - \frac{x^3}{9} + k$

g) $x \tan(x) + \ln|\cos(x)| + k$

h) $x \ln^2(x) - 2(x \ln(x) - x) + k$

i) $e^2 \sin(x)$

j) $\frac{1}{2} (e^{x^2} x^2 - e^{x^2}) + k$

k) $\frac{1}{2} (x^2 \sin(x^2) + \cos(x^2)) + k$

l) $-\frac{1}{5} e^{-x} (\cos(2x) - 2 \sin(2x)) + k$

m) $2(x \sin(x) + \cos(x)) - x^2 \cos(x) + k$

7. a) Temos

$$\int \frac{1}{x^2 - \alpha^2} dx = \frac{1}{2\alpha} \left(\int \frac{dx}{x - \alpha} - \int \frac{dx}{x + \alpha} \right) = \frac{1}{2\alpha} \ln \left| \frac{x - \alpha}{x + \alpha} \right| + k.$$

b) Escrevemos

$$\int \frac{1}{\alpha^2 + (x + \beta)^2} dx = \frac{1}{\alpha^2} \int \frac{1}{1 + \frac{(x + \beta)^2}{\alpha^2}} dx.$$

Fazendo mudança da variação $u = \frac{x + \beta}{\alpha}$. Temos

$$\int \frac{1}{1 + \frac{(x + \beta)^2}{\alpha^2}} dx = \alpha \int \frac{1}{1 + u^2} du = \alpha \arctan u + k = \alpha \arctan \frac{x + \beta}{\alpha} + k.$$

c) Temos

$$\int \frac{mu + n}{1 + u^2} du = m \int \frac{u}{1 + u^2} du + n \int \frac{1}{1 + u^2} du.$$

Mas $\int \frac{1}{1 + u^2} du = \arctan(u) + k$ e $\int \frac{u}{1 + u^2} = \frac{1}{2} \ln|1 + u^2| + k.$

8. a) $\frac{1}{2} \ln|x - 1| - \frac{1}{2} \ln|x + 1| + k$

b) $\frac{7}{2} \ln|x - 2| - \frac{3}{2} \ln|x| + k$

c) $\frac{1}{2} \ln|x^2 - 4| + k$

d) $8 \ln|x - 2| - 3 \ln|x - 1| + k$

e) $\ln|x-2| + k$

f) $\frac{1}{2} \ln|x+1| - \frac{1}{2} \ln|x+3| + k$

g) $\frac{2}{\sqrt{3}} \arctan(\frac{2x+1}{\sqrt{3}})$

h) $\frac{1}{27}(\frac{3(7x-1)}{x^2+x-1} + 7 \ln \frac{|x-1|}{|x+2|}) + k$

i) $\frac{1}{450}(\frac{60}{x+3} + 27 \ln|x-2| - 25 \ln|x| - 2 \ln|x+3|) + k$

j) $\frac{1}{2}(x^2 + \ln|x^2-1| + 2 \ln \frac{|x-1|}{|x|}) + k$

k) $\frac{11-8x}{2(x-2)^2} + \ln|x-2| + k$

l) $\frac{1}{2} \ln|x^2+2x+3| + x + 2 \ln|x-1| + \frac{1}{\sqrt{2}} \arctan(\frac{x+1}{\sqrt{2}}) + k$

m) $\frac{1}{6} \left(3x^2 - 5 \ln|x^2+2x+4| + 12x + 10 \ln|x-2| - 10\sqrt{3} \arctan(\frac{x+1}{\sqrt{2}}) \right) + k$