

## Cálculo II — Lista 3.

1. Ache e esboce o domínio das funções:

(a)  $f(x, y) = \sqrt{y-x} + \sqrt{y-1}$ ,

(b)  $f(x, y) = \ln y - x^3$ ,

(c)  $f(x, y) = \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}$ ,

(d)  $f(x, y) = \sqrt{x+y} + \sqrt{x-y}$ ,

(e)  $f(x, y) = \frac{1}{\sqrt{x+y}} + \frac{1}{\sqrt{x-y}}$ ,

(f)  $f(x, y) = \arcsen \frac{y-1}{x}$ ,

(g)  $f(x, y) = \ln xy$ ,

(h)  $f(x, y) = \sqrt{x - \sqrt{y}}$ ,

(i)  $f(x, y) = \sqrt{x - \sqrt{y}}$ ,

(j)  $f(x, y) = \frac{\sqrt{4x - y^2}}{\ln(1 - x^2 - y^2)}$ ,

(k)  $f(x, y) = \ln x - \ln \operatorname{sen} y$ .

(l)  $f(x, y) = \sqrt{x \operatorname{sen} y}$ .

2. Esboce as curvas de nível de:

(a)  $f(x, y) = 3x + 4y$ ,

(b)  $f(x, y) = xy$ ,

(c)  $f(x, y) = \frac{1}{x^2 + y^2}$ ,

(d)  $f(x, y) = y(x^2 + 1)$ ,

(e)  $f(x, y) = \frac{xy - 1}{x^2}$ ,

(f)  $f(x, y) = \frac{x + y}{x - y}$ ,

(g)  $f(x, y) = (x^2 + y^2)^2 - 2(x^2 - y^2)$ ,  
para níveis  $c = -1, -\frac{1}{2}, 0, -\frac{1}{2}, 1$ .

3. Esboce os gráficos de:

(a)  $f(x, y) = 1 - 2x - 3y$ ,

(b)  $f(x, y) = 9x^2 + 4y^2$ ,

(c)  $f(x, y) = x^2 - y^2$ ,

(d)  $f(x, y) = y^2 + 1$ ,

(e)  $f(x, y) = \sqrt{4x^2 + y^2}$ ,

(f)  $f(x, y) = x^2 + 2x + y^2 + 4y$ ,

(g)  $f(x, y) = e^{\sqrt{x^2 + y^2}}$ ,

(h)  $f(x, y) = \frac{x}{x^2 + 1}$ .

4. Calcule as seguintes limites, caso existam. Se não existirem, explique o porquê:

(a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{\sqrt{x+1} - \sqrt{1}}{xy + y}$ ,

(b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$ ,

(c)  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$ ,

(d)  $\lim_{(x,y) \rightarrow (0,0)} \frac{\operatorname{sen}(x^2 + y^2)}{x^2 + y^2}$ ,

(e)  $\lim_{(x,y) \rightarrow (0,0)} \frac{1 - \cos(x^2 + y^2)}{(x^2 + y^2)x^2y^2}$ ,

(f)  $\lim_{(x,y) \rightarrow (0,0)} \frac{(x+y)^3}{x^2 + y^2}$ ,

(g)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + y^3}{x^2 + y^2}$ ,

(h)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + 3xy + y^2}{3x^2 + 4y^2}$ ,

(i)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + y^3}{x^2 - y^2}$ ,

(j)  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^3 - y^3}$ ,

5. Determine o conjunto dos pontos de continuidade. Justifique a resposta:

$$(a) f(x, y) = \frac{1}{a^2 - x^2 - y^2},$$

$$(b) f(x, y) = \frac{xy}{y - x^2},$$

$$(c) f(x, y) = \ln \frac{x - y}{x^2 + y^2},$$

$$(d) f(x, y) = \begin{cases} \frac{x - 3y}{x^2 + y^2}, & \text{se } (x, y) \neq (0, 0) \\ 0 & \text{se } (x, y) = (0, 0) \end{cases}$$

$$(e) f(x, y) = \begin{cases} \frac{\text{sen}(x^2 + y^2)}{x^2 + y^2}, & \text{se } (x, y) \neq (0, 0) \\ 1 & \text{se } (x, y) = (0, 0) \end{cases}$$

$$(f) f(x, y) = \begin{cases} \frac{1 - \cos(\sqrt{x^2 + y^2})}{x^2 + y^2}, & \text{se } (x, y) \neq (0, 0) \\ 1 & \text{se } (x, y) = (0, 0) \end{cases}$$