Gabaritos — Lista IV — MAT0147

1.

2.

3.

$$\frac{\partial f}{\partial x} = 3x^2 h'(3x^2+y^3) + h(3x^2+y^3)$$
$$\frac{\partial f}{\partial x}(1,1) = 3h'(4) + h(4) = -3 + 3 = 0$$

4. (a)

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = x\left(\sin\left(\frac{x}{y}\right) + \frac{x}{y}\cos\left(\frac{x}{y}\right)\right) - yx\cos\left(\frac{x}{y}\right)\frac{1}{y^2} = z$$

(b)

$$\frac{\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}\cdot\frac{\partial f}{\partial y}} = \frac{3x^2y - y^3 + x^3 - 3y^2x}{\left(3x^2y - y^3\right)\left(x^3 - 3y^2x\right)} = \frac{6-8+1-12}{(6-8)(1-12)}$$

5. Para $(x,y) \neq 0$ temos

$$\frac{\partial f}{\partial x} = \frac{\left(x^2+y^2\right) + 2y\left(x+y^4\right)}{\left(x^2+y^2\right)^2}$$

Para $(x,y) = (0,0)$ temos

$$\frac{\partial f}{\partial x}(0,0) = \lim_{t\to 0}\frac{f(t,0) - f(0,0)}{t}$$
$$= \lim_{t\to 0}\frac{\frac{1}{t}}{t}$$
$$= \lim_{t\to 0}\frac{1}{t^2} = \infty$$

6.

7.

8. Lembremos que o vetor normal ao plano será

$$\vec{n} = \left(\frac{\partial f}{\partial x}(x_0, y_0), \frac{\partial f}{\partial y}(x_0, y_0), -1\right) = \left(2x_0 + y_0, x_0, -1\right)$$

pois, $\frac{\partial f}{\partial x} = 2x + y$ e $\frac{\partial f}{\partial y} = x$. Agora como queremos que seja paralelo ao plano $z = 2x + 3y$, que temo como vetor normal $\vec{m} = (2, 3, -1)$, basta ter

$$\vec{n} = \lambda \vec{m}$$

logo temos

$$2x_0 + y_0 = 2\lambda$$
$$x_0 = 3\lambda$$
$$\lambda = 1$$

assim

$$y_0 = -4$$
$$x_0 = 3$$
$$\lambda = 1$$

e como

$$z_0 = f(3, -4) = 9 - 12 = -3$$

além disso a equação do plano tangente à superficie é

$$\frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)\left(y - y_0\right) - (z - z_0) = 0$$

então tal equação do plano é

$$2(x - 3) + 3\left(y + 4\right) - (z + 3) = 0$$

9.

10.

11.

12.

13. Tomando $x = u - v$ e $y = v - u$, temos

$$\begin{aligned}\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} \\ &= \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} - \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} \\ &= 0\end{aligned}$$

14. Reparemos que $x = t^2$ e $y = 2t$

$$\begin{aligned}\frac{\partial f}{\partial t} &= \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} \\ 3t^2 - 3 &= 2t\frac{\partial f}{\partial x} + 2\frac{\partial f}{\partial y}\end{aligned}$$

Terminamos fazendo $t = 1$