# EXISTENCE RESULTS FOR AN ELLIPTIC EQUATION INVOLVING THE FRACTIONAL LAPLACIAN

GAETANO SICILIANO

In the talk we present recent results on the following system involving the fractional Laplacian in the whole $\mathbb{R}^N, N \geq 3$ :

$$\begin{cases} (-\Delta)^s u + u - \varphi |u|^{p-2} u = 0, \\ (-\Delta)^{\alpha/2} \varphi = \gamma(\alpha) |u|^p. \end{cases}$$

Here $\gamma(\alpha)$ is a normalizing constant, $s \in (0,1)$ and $\alpha \in (0, N)$. Finally $p$ varies in a suitable range involving $s, \alpha, N$ which permits to obtain a unique solution to the second equation, $\varphi = \varphi(u)$ for every $u \in H^s(\mathbb{R}^N)$. The system is then reduced to a single nonlocal equation.

We look for weak solutions to the above problem in the fractional Sobolev space $H^s(\mathbb{R}^N)$. These solutions can be characterized as critical points of a functional, whose geometry and compactness properties (the Palais-Smale condition) will depend on the values of the parameters. Using variational methods and concentration compactness arguments (note the invariance by translations) we are able to show the existence of a ground state which is positive and radially symmetric. Moreover we also obtain multiplicity results for radial and non-radial solutions, as well as nonexistence result when $p$ is greater then a "critical" exponent.

These results are obtained in collaboration with P. d'Avenia (Politecnico di Bari, IT) and M. Squassina (Univ. di Verona, IT).