The Tropical Semiring and the Theory of Automata

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1 Overview

This series of two lectures will survey the applications of the tropical semiring to the Theory of Automata. This subject has been surveyed in two papers [18, 17] and these surveys are going to be the basis of our exposition. The full text of these surveys can be found on the World Wide Web: [18][1], and [17][2].

Other interesting references and which are more recent than the bibliography of [18] are [14, 16, 20, 21, 3, 11, 12, 10, 9, 24, 23, 22, 19, 18, 7, 29, 6, 28, 8, 5, 4, 26, 25, 27, 2, 17, 15, 13, 1]

The first talk will focus on the use of the tropical semiring in decision procedures in the Theory of Automata. We will formulate the limitedness problem and show its applications for several problems, such as the finite power property, the finite products property and the nondeterministic complexity of finite automata. Most of these applications are described in [18].

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[1] http://www.ime.usp.br/ is/papir/rmts.html
[2] http://www.liafa.jussieu.fr/ jep/PostScript/Tropical.ps.gz

The second talk will focus on the use of the tropical semiring in undecidable problems in Automaton Theory. The main reference here is a paper by Krob [11][3], which is also available on the World Wide Web. We will show here that the equivalence problem for automata with multiplicities in the tropical semiring is undecidable by showing Krob's reduction of Hilbert's tenth problem to it.

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