Resolução da P2

Q1 $\varphi : U \to S$ $\quad I = E\,du^2 + G\,dv^2 \quad II = 2\,du\,dv$

$e = g = 0 \quad f = 1$

$F = 0 \Rightarrow \Gamma_{11}^1 = \dfrac{E_u}{2E}, \quad \Gamma_{12}^2 = \dfrac{G_u}{2G}$

$$\left(\frac{E}{G}\right)_u = \frac{E_u G - E G_u}{G^2} = 2\frac{E}{G}\left(\Gamma_{11}^1 - \Gamma_{12}^2\right) = 0$$

Codazzi - Mainardi $\Rightarrow$

$$\cancel{e_v} - \cancel{f_u} = \cancel{e\Gamma_{12}^1} + f(\Gamma_{12}^2 - \Gamma_{11}^1) - \cancel{g\Gamma_{11}^2}$$

$\Rightarrow \Gamma_{12}^2 - \Gamma_{11}^1 = 0$

Analoga/e $(E/G)_v = 0$

Q2 a. $\varphi(u,v) = (u, v, uv)$ gráfico

b. $E = \varphi_u \cdot \varphi_u = 1 + v^2 \quad F = \varphi_u \cdot \varphi_v = uv \quad G = \varphi_v \cdot \varphi_v = 1 + u^2$

$$\begin{cases} \Gamma_{11}^1 E + \Gamma_{11}^2 F = \frac{1}{2} E_u = 0 \\ \Gamma_{11}^1 F + \Gamma_{11}^2 G = F_u - \frac{1}{2} E_v = 0 \end{cases} \qquad \begin{vmatrix} E & F \\ F & G \end{vmatrix} \neq 0$$

$\Rightarrow \Gamma_{11}^1 = \Gamma_{11}^2 = 0$

Analoga/e $\Gamma_{22}^1 = \Gamma_{22}^2 = 0$

$$\begin{cases} \Gamma_{12}^1 E + \Gamma_{12}^2 F = \frac{1}{2} E_v = v \\ \Gamma_{12}^1 F + \Gamma_{12}^2 G = \frac{1}{2} G_u = u \end{cases} \qquad \Gamma_{12}^1 = \frac{\begin{vmatrix} v & uv \\ u & 1+u^2 \end{vmatrix}}{\begin{vmatrix} 1+v^2 & uv \\ uv & 1+u^2 \end{vmatrix}}$$

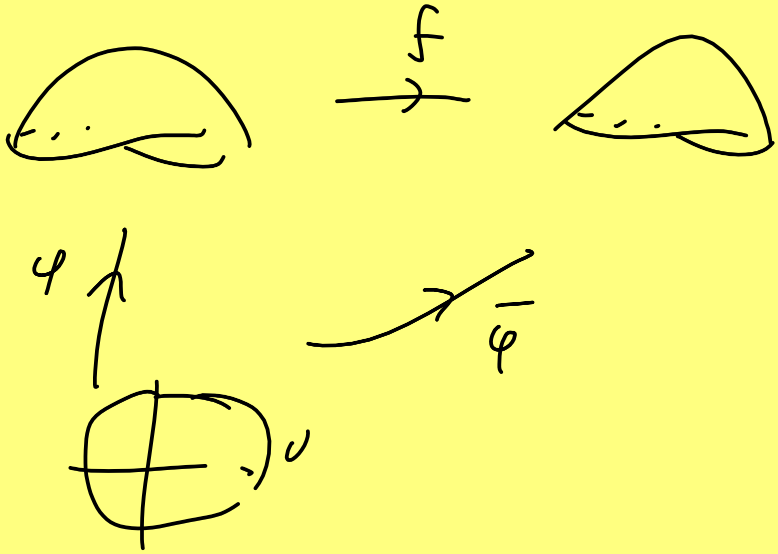
$$= \frac{v}{1+u^2+v^2} \qquad \Gamma_{12}^2 = \frac{u}{1+u^2+v^2}$$

$$\begin{cases} (1+u^2+v^2)\,u'' + 2u'v'v = 0 \\ (1+u^2+v^2)\,v'' + 2u'v'u = 0 \end{cases}$$ Eqs. das geodésicas

c_0 $\quad u=0,\ v=t \quad$ e $\quad u=t,\ v=0$

são duas soluções

Q3



a. $\bar{\nu}\circ f(p) = df_p \cdot \nu_p \Rightarrow d\bar{\nu}_{f(p)} \circ df_p = df_p \circ d\nu_p$

$\Rightarrow \bar{A}_{f(p)} \circ df_p = df_p \circ A_p \Rightarrow \bar{A}_{f(p)} = df_p \circ A_p \circ (df_p)^{-1}$

$\Rightarrow \bar{A}_{f(p)}, A_p$ têm mesmo det e traço.

b. Analoga/e $\quad \bar{\nu}\circ f(p) = -df_p \circ \nu_p$

$\Rightarrow \bar{A}_{f(p)} = -df_p \circ A_p \circ (df_p)^{-1}$

$\Rightarrow \det \bar{A}_{f(p)} = (-1)^2 \det A_p , \quad \operatorname{tr} \bar{A}_{f(p)} = -\operatorname{tr} A_p .$

c. $\bar{\nu} \circ f(p) = \nu_p \Rightarrow d\bar{\nu}_{f(p)} \circ df_p = d\nu_p$

$\Rightarrow \bar{A}_{f(p)} \cdot \lambda I = A_p \Rightarrow \bar{A}_{f(p)} = \frac{1}{\lambda} A_p$

$\Rightarrow \det \bar{A}_{f(p)} = \frac{1}{\lambda^2} \det A_p , \quad \operatorname{tr} \bar{A}_{f(p)} = \frac{1}{\lambda} \operatorname{tr} A_p$

<u>Q4</u> $S: \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad (a,b,c > 0)$

(não é sup. de revolução em geral)

Os planos coordenados $x=0$, $y=0$, $z=0$ são 3 planos de simetria do elipsóide S, logo cortam S em 3 geodésicas fechadas.