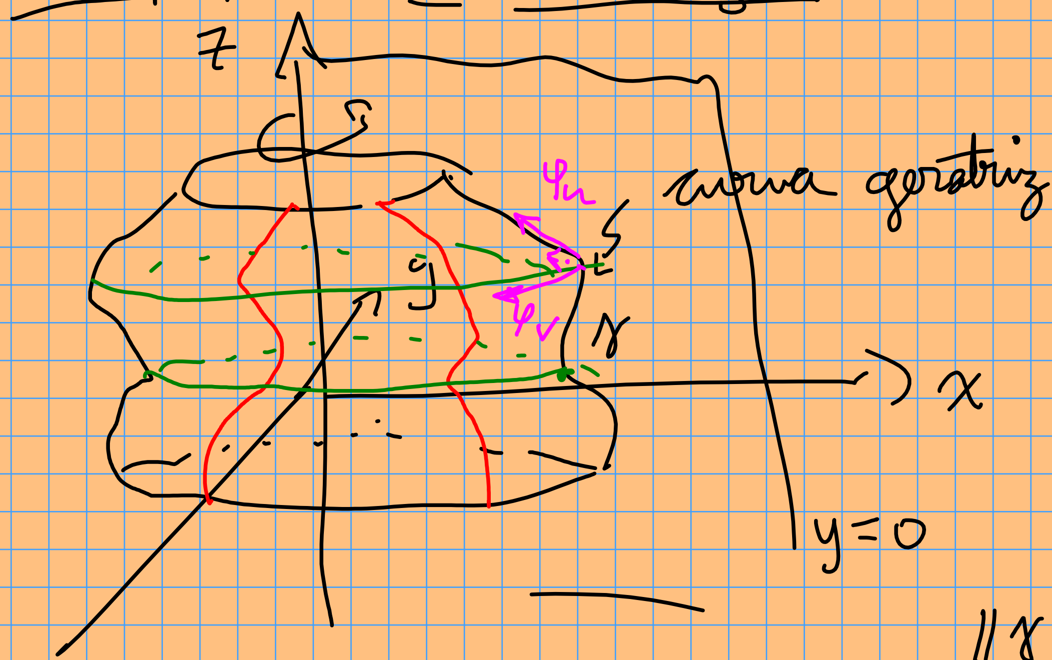
Superfície de revolução



$\|\gamma'\| = 1$

$$\gamma(u) = (f(u), 0, g(u)) \qquad u \in (a,b)$$

$f(u) > 0$

$\forall u$

$$\varphi(u,v) = (\underbrace{f(u)\cos v, f(u)\sin v}, g(u))$$

$v \in (v_0, v_0 + 2\pi)$

$$\varphi_u = (f'(u)\cos v, f'(u)\sin v, g'(u))$$

$$\varphi_v = (-f(u)\sin v, f(u)\cos v, 0)$$

$E = \varphi_u \cdot \varphi_u = f'(u)^2 + g'(u)^2 = 1$

$F = \varphi_u \cdot \varphi_v = 0$

$G = \varphi_v \cdot \varphi_v = f(u)^2$

$$(\mathrm{II}) = \begin{pmatrix} l & m \\ m & n \end{pmatrix}$$

$$\ell = N\cdot\varphi_{uu} = \frac{\varphi_u\times\varphi_v\cdot\varphi_{uu}}{\|\varphi_u\times\varphi_v\|}$$

$$= \frac{1}{\sqrt{EG-F^2}}\begin{vmatrix} f'\cos v & f'\sin v & g' \\ -f\sin v & f\cos v & 0 \\ f''\cos v & f''\sin v & g'' \end{vmatrix}$$

$$= \frac{1}{f}\Big(ff'\cos^2 v\, g'' - ff''g'\sin^2 v - ff''g'\cos^2 v + ff'g''\sin^2 v \Big)$$

$$= \frac{1}{f}\left(ff'g'' - ff''g' \right) = f'g'' - f''g' = \kappa_\gamma$$

$$m = N\cdot\varphi_{uv} = \frac{\varphi_u\times\varphi_v\cdot\varphi_{uv}}{\|\varphi_u\times\varphi_v\|}$$

$$= \frac{1}{f}\begin{vmatrix} f'\cos v & f'\sin v & g \\ -f\sin v & f\cos v & 0 \\ -f'\sin v & f'\cos v & 0 \end{vmatrix} = \underline{0}$$

$$n = N\cdot\varphi_{vv} = fg'$$

$$K = \frac{\ell n - \cancel{m^2}}{EG-\cancel{F^2}} = \frac{(f'g''-f''g')\cancel{f}g'}{f^{\cancel{2}}} = \kappa_\gamma\frac{g'}{f}$$

$$H = \frac{\ell G - \cancel{2mF} + nE}{2(EG-\cancel{F^2})} = \frac{1}{2}\left(\kappa_\gamma + \frac{g'}{f}\right)$$

$\therefore$ As curvaturas principais são

$$k_1 = k_\gamma \qquad k_2 = \frac{g'}{f}$$

φ_u e φ_v diagonalizam II $\therefore$ São as direções principais

Os <u>paralelos</u> e os <u>meridianos</u> são tangentes às direções principais, $\therefore$ são <u>linhas de curvatura da superfície</u>

Alternativamente: $f'^2 + g'^2 = 1$

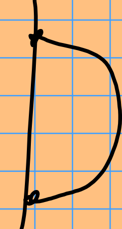
$$\Rightarrow (f'^2 + g'^2)' = 0 \Rightarrow 2f'f'' + 2g'g'' = 0 \quad (*)$$

$$K = (f'g'' - f''g')\frac{g'}{f} = \frac{f'g''g' - f''g'^2}{f}$$

$$\overset{(*)}{=} \frac{f'(-f'f'') - f''g'^2}{f}$$

$$= -\underbrace{(f'^2 + g'^2)}_{=1}\frac{f''}{f} \qquad \therefore \quad \boxed{K = -\frac{f''}{f}}$$

<u>Exemplos</u>: $S^2(1)$



$$\gamma(u) = (\underbrace{\cos u}_{=f(u)}, 0, \underbrace{\sin u}_{=g(u)})$$

$$K = \frac{-(\cos u)''}{\cos u} = \frac{-(-\cos u)}{\cos u} = 1$$

- $\gamma(u) = (\underbrace{u}_{f(u)}, 0, 0)$ gera um plano

$f'' = 0 \qquad K = 0.$

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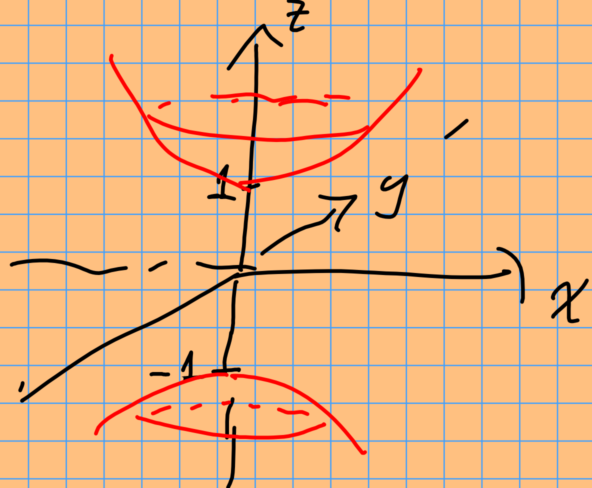
Ex. 13, p.65 Parametrizar

$$-x^2 - y^2 + z^2 = 1$$

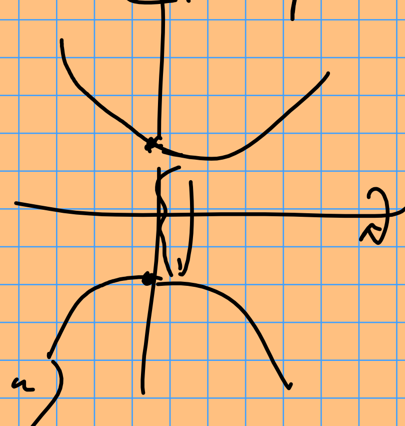
$y = 0$: $-x^2 + z^2 = 1$

$$z^2 = x^2 + y^2 + 1 \geq 1 \qquad |z| \geq 1$$



A curva geratriz é uma hipérbole.

⟶ 2 folhas

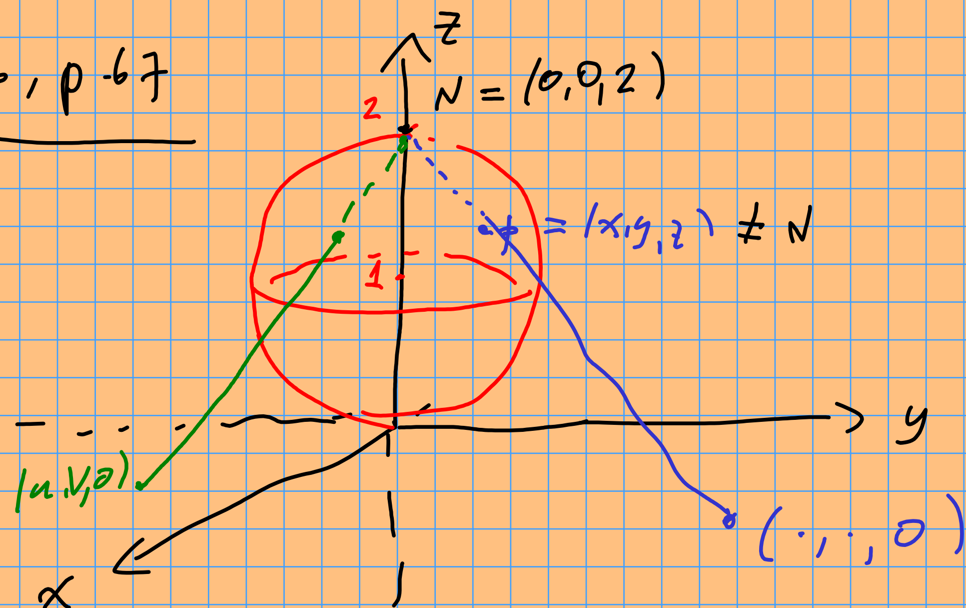


1 folha

$$\gamma(u) = (\sinh u, 0, \cosh u)$$

$$\varphi(u,v) = (\sinh u \cos v, \sinh u \sin v, \cosh u)$$

Ex. 16, p. 67



$$\pi : S^2 \setminus \{N\} \longrightarrow \mathbb{R}^2$$

$$X = (0,0,2) + t\,((x,y,z) - (0,0,2)) \qquad t \in \mathbb{R}$$

$$= (\underbrace{tx,\ ty}, \underbrace{2 + t(z-2)}_{=0 \iff t = \frac{2}{2-z}}) \qquad (z \neq 2)$$

$$\pi(x,y,z) = (tx, ty)$$

$$= \frac{2}{2-z}(x,y)$$

$$Y = (0,0,2) + s\,((u,v,0) - (0,0,2))$$

$$= (su, sv, 2-2s) \in S^2$$

$$\iff (su)^2 + (sv)^2 + (\underbrace{2-2s-1}_{=1-2s})^2 = 1$$

$$s^2(u^2+v^2) + \cancel{1} - 4s + 4s^2 = \cancel{1}$$

$$s^{\cancel{2}}(u^2+v^2+4) = 4\cancel{s}$$ $s \neq 0$ pois $s=0$ corresponde a N.

$$s = \frac{4}{u^2+v^2+4} \qquad 1-s = \frac{u^2+v^2+\cancel{4}-\cancel{4}}{u^2+v^2+4}$$

$$\pi^{-1}(u,v) = (su, sv, 2(1-s))$$

$$= \left(\frac{4u}{u^2+v^2+4}, \frac{4v}{u^2+v^2+4}, \frac{2(u^2+v^2)}{u^2+v^2+4} \right)$$