$$\arctan\left(\frac{1-\sin\theta}{\cos\theta}\right) = t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$ 27/09/21

$$\underbrace{\tan t}_{=\frac{\sin t}{\cos t}} = \frac{1-\sin\theta}{\cos\theta} \qquad \sin^2 t + \cos^2 t = 1$$

$$\frac{\pi}{6} \leq \theta \leq \frac{\pi}{2}$$

$$(1-\sin\theta) > 0$$

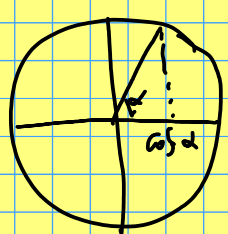
$$0 < \sin t = \frac{1-\sin\theta}{\sqrt{(1-\sin\theta)^2 + \cos^2\theta}} = \frac{1-\sin\theta}{\sqrt{2-2\sin\theta}} = \sqrt{\frac{1-\sin\theta}{2}}$$

$t \in$ 1.º Q

$$\sin^2 t = \frac{1-\sin\theta}{2} \qquad \underbrace{1-2\sin^2 t}_{\cos \underbrace{2t}_{\in(-\pi,\pi)}} = \underbrace{\sin\theta}_{=\cos\left(\frac{\pi}{2}-\theta\right)}$$

$\in$ 1.º quad

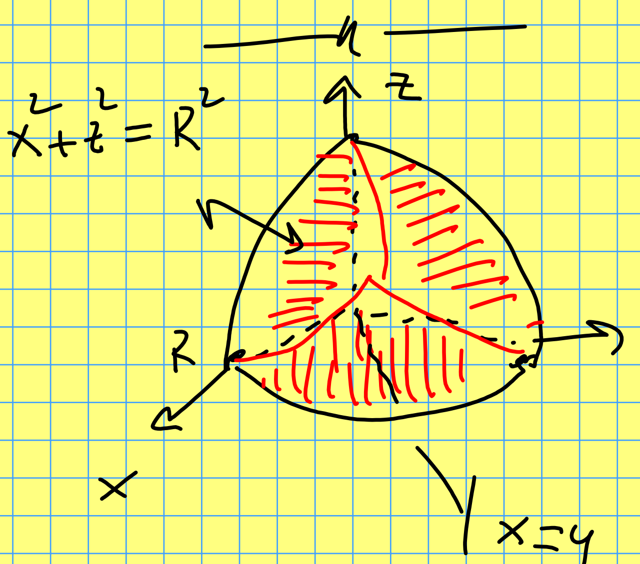
$$\frac{\pi}{6} \leq \theta \leq \frac{\pi}{2}$$

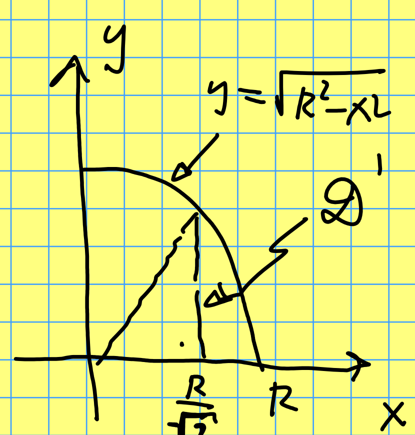


$$\cos 2t = \cos\left(\frac{\pi}{2}-\theta\right) \Rightarrow \pm 2t = \frac{\pi}{2}-\theta$$

$$2t = \frac{\pi}{2}-\theta$$

$$t = \frac{\pi}{4} - \frac{\theta}{2}$$





sólido no 1.º octante é simétrico rel plano $x=y$

$$V = 8 \cdot 2 \iint_{D'} \sqrt{R^2 - x^2}\, dA$$

$$= 16\left(\int_0^{R/\sqrt{2}} \int_0^{x} \sqrt{R^2 - x^2}\, dy\, dx + \int_{R/\sqrt{2}}^{R} \int_0^{\sqrt{R^2 - x^2}} \sqrt{R^2 - x^2}\, dy\, dx \right)$$

$$= 16\left(\int_0^{R/\sqrt{2}} x\sqrt{R^2 - x^2}\, dx + \int_{R/\sqrt{2}}^{R} R^2 - x^2\, dx \right)$$

$$= 16\left(\left[-\frac{1}{3} (R^2 - x^2)^{3/2} \right]_0^{R/\sqrt{2}} + \left[R^2 x - \frac{x^3}{3} \right]_{R/\sqrt{2}}^{R} \right)$$

$$= 16\left(\underbrace{-\frac{1}{3} \left(\frac{R^2}{2}\right)^{3/2}} + \underbrace{\frac{1}{3}(R^2)^{3/2}} + \underbrace{\frac{2R^3}{3}} - \left(\frac{R^3}{\sqrt{2}} - \frac{R^3}{3 \cdot 2\sqrt{2}} \right) \right)$$

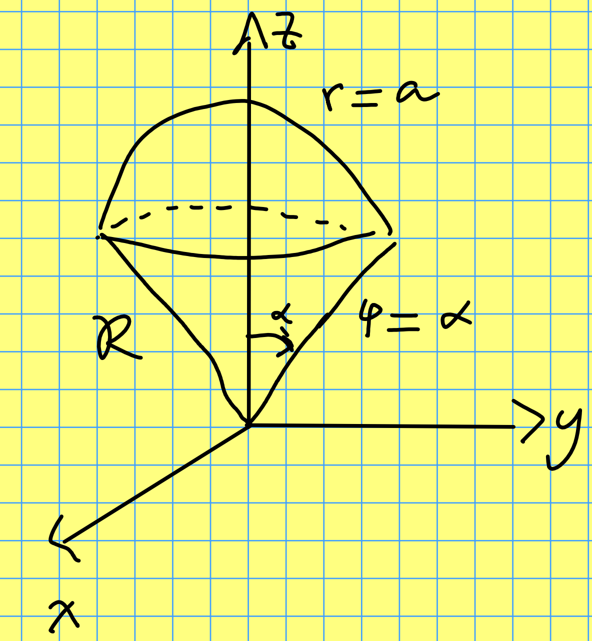
$$= 16\left(\frac{R^3}{3} - \frac{1}{3} \cancel{\frac{R^3}{2\sqrt{2}}} + \frac{2R^3}{3} - \frac{R^3}{\sqrt{2}} + \cancel{\frac{R^3}{3 \cdot 2\sqrt{2}}} \right)$$

$$= 16\left(R^3 - \frac{R^3}{\sqrt{2}} \right) = 16R^3 \underbrace{\left(1 - \frac{1}{\sqrt{2}}\right)}_{\approx 0.292}$$

$$= 16R^3 \underbrace{\frac{\sqrt{2} - 1}{\sqrt{2}}} = 8\sqrt{2} \cdot R^3 \cdot (\sqrt{2} - 1) = 8(2 - \sqrt{2})R^3$$

$$16R^3 \cdot \frac{1}{3}$$

Ex. 1, § 20.7 [Português]



Momento de inércia em rel eixo z ?

$$\begin{cases} 0 \le r \le \alpha \\ 0 \le \varphi \le \alpha \end{cases}$$

$$I_z = \iiint_R \underbrace{(\text{dist ao eixo } z)^2}_{= x^2+y^2} \delta \, dV \qquad \delta \text{ constante}$$

$$\begin{cases} x = r\cos\theta\sin\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\varphi \end{cases}$$

$$dV = r^2 \sin\varphi \, dr d\theta d\varphi$$

$$= \delta \int_0^{\alpha} \int_0^{2\pi} \int_0^{a} r^2 \sin^2\varphi \cdot r^2 \sin\varphi \, dr d\theta d\varphi$$

$$= \delta \int_0^{\alpha} \sin^3\varphi \, d\varphi \int_0^{2\pi} d\theta \int_0^{a} r^4 dr$$

$$= \delta \left[\frac{\cos^3\varphi}{3} - \cos\varphi \right]_0^{\alpha} 2\pi \frac{a^5}{5} = \frac{2\pi a^5 \delta}{5} \left(\frac{\cos^3\alpha}{3} - \cos\alpha + \frac{2}{3} \right)$$

$$\left(\frac{\cos^3\varphi}{3} - \cos\varphi \right)' = \cos^2\varphi(-\sin\varphi) + \sin\varphi$$
$$= \sin\varphi(1 - \cos^2\varphi) = \sin^3\varphi$$

$$I_z = \frac{2\pi a^5 \delta}{15}\left(2+\cos^3\alpha - 3\cos\alpha\right)$$

$$M = \delta V = \delta \frac{2}{3}\pi a^3 \left(1-\cos\alpha\right)$$

$$\frac{I_z}{M} = \frac{a^2}{5}\,\frac{2+\cos^3\alpha - 3\cos\alpha}{1-\cos\alpha} \qquad \alpha=0:\ \frac{0}{0}$$

Esfera solida : $\alpha = \pi$ $\qquad \cos\pi = -1$

$$\frac{I_z}{M} = \frac{a^2}{5}\,\frac{2-1+3}{1+1} = \frac{2a^2}{5}$$

Hemisfério solido : $\alpha = \frac{\pi}{2}$ $\qquad \cos\frac{\pi}{2} = 0$

$$I_z/M = \frac{2a^2}{5}. \qquad \frac{0}{0}$$

$$\lim_{\alpha\to 0^+} \frac{2+\cos^3\alpha - 3\cos\alpha}{1-\cos\alpha} \overset{L'H}{=} \lim_{\alpha\to 0^+} \frac{-3\cos^2\alpha\sin\alpha + 3\sin\alpha}{\sin\alpha}$$

$$= \lim_{\alpha\to 0^+} -3\cos^2\alpha + 3 = 0$$

$$\begin{array}{l|l} t^3-3t+2 & t-1 \\ \hline -t^3+t^2 & t^2+t-2 \end{array}$$

$t^2 - 3t + 2$

$$\boxed{M\frac{a^2}{5}\left(2-\cos\alpha-\cos^2\alpha\right)=I_z}$$

$-t^2+t$

$-2t+2$

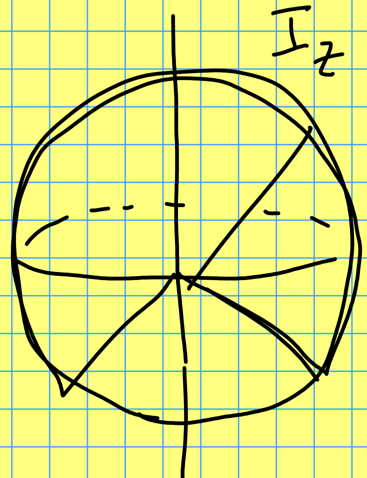
$$f(t)=2-t-t^2 \qquad f'(t)=-1-2t=0$$

$$\Leftrightarrow t=-\frac{1}{2}$$

$\cos\alpha=-\frac{1}{2}$ $\qquad \alpha=\frac{2\pi}{3}$ ponto de máximo

$$I_z=\frac{a^2}{5}M\left(2+\frac{1}{2}-\frac{1}{4}\right)=\frac{9}{4}\frac{a^2}{5}M$$

Valor máximo



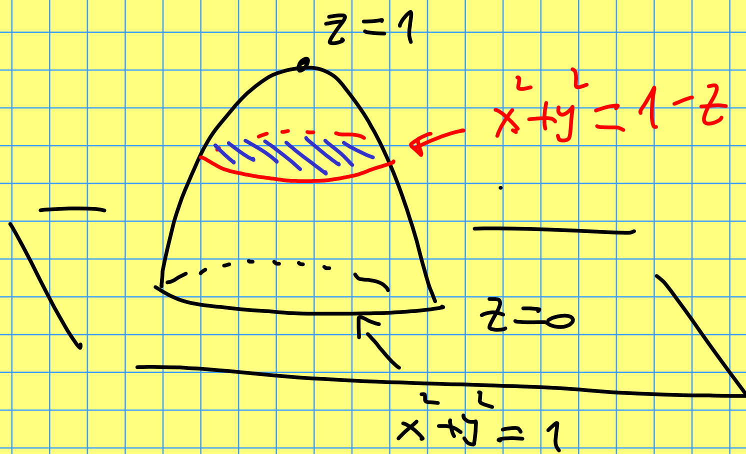
Ex. 15 § 20.5 [Port]

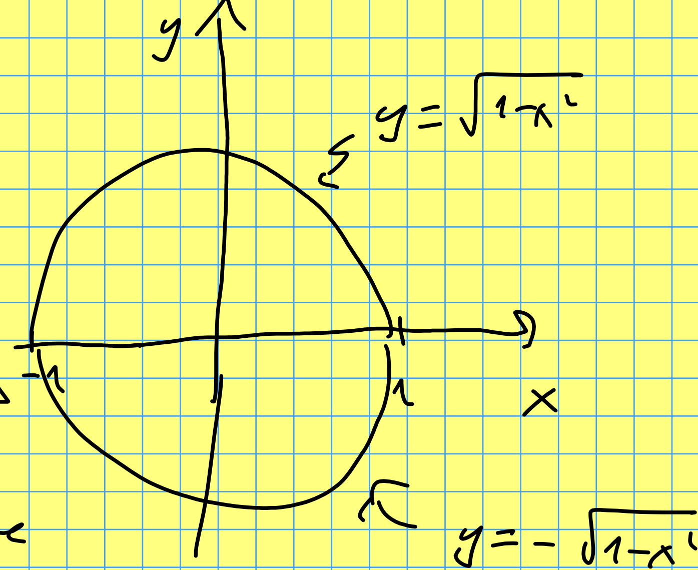
$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{1-x^2-y^2} f(x,y,z)\,dz\,dy\,dx=\iiint f(x,y,z)\,\underline{dx\,dy\,dz}$$

$$\begin{cases}-1\le x\le 1\\ -\sqrt{1-x^2}\le y\le\sqrt{1-x^2}\\ 0\le z\le 1-x^2-y^2\end{cases}$$

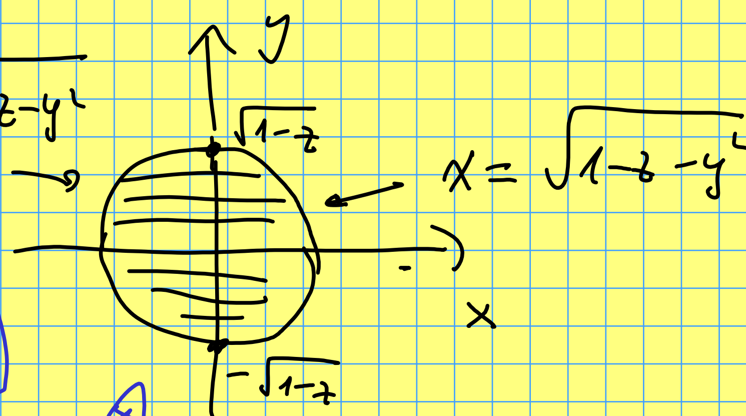
$z=1-x^2-y^2$

parabolóide de revolução





R é o interior do parabolóide que fica acima do plano $z=0$.



$$0 \leq z \leq 1$$

$$-\sqrt{1-z} \leq y \leq \sqrt{1-z}$$

$$-\sqrt{1-z-y^2} \leq x \leq \sqrt{1-z-y^2}$$

$$\int_0^1 \int_{-\sqrt{1-z}}^{\sqrt{1-z}} \int_{-\sqrt{1-z-y^2}}^{\sqrt{1-z-y^2}} f(x,y,z)\, dx\, dy\, dz$$