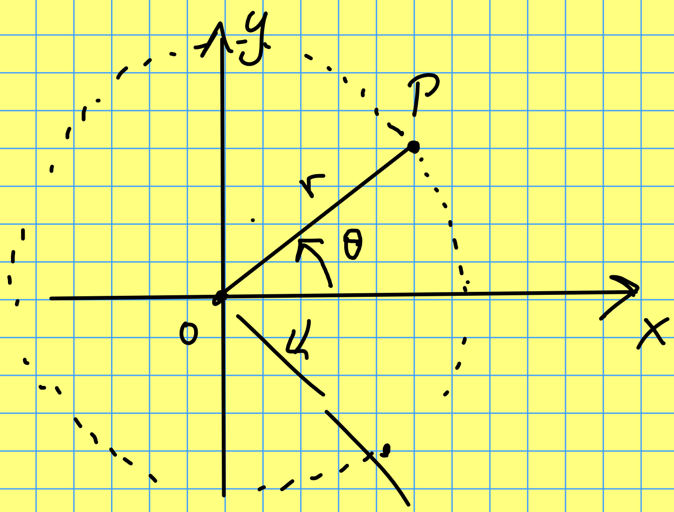
COORDENADAS POLARES

26/08/21

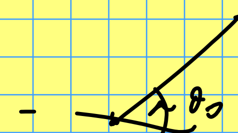


$r = d(O, P) = \|\vec{OP}\| > 0$ se $P \neq O$.

θ : medida do ângulo formado por $\vec{OP}$ e o eixo Ox, no sentido anti-horário

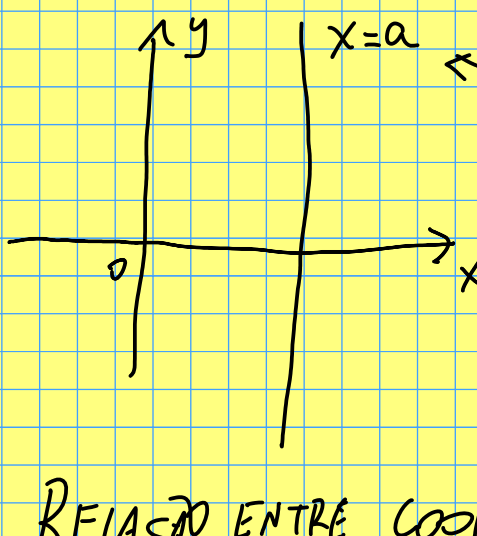
Exs.

- $r = r_0$ (constante) é a equação de um círculo de raio r_0 centrado na origem.



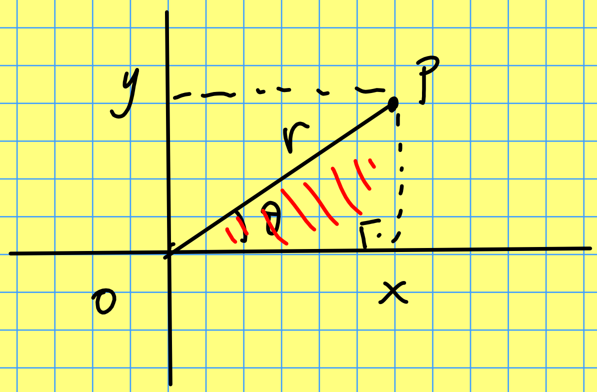
- $\theta = \theta_0$ (constante) é uma semi-reta pela origem. $(r > 0)$

- $\theta = \theta_0$, $r \in \mathbb{R}$ é uma reta pela origem.



← Equação em coordenadas polares?

RELAÇÃO ENTRE COORDS POLARES E COORDS CARTESIANAS



↳ $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ $\tan\theta = \dfrac{y}{x}$

$$\boxed{r = \sqrt{x^2+y^2}} \quad r^2 = x^2+y^2$$

$\rightarrow \theta = \arctan \dfrac{y}{x} \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ se $x > 0$

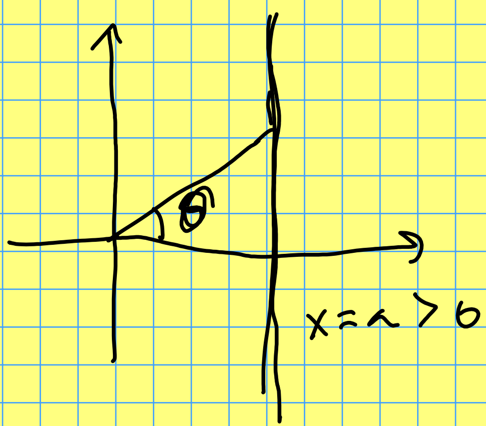
$(+2\pi k$

$k \in \mathbb{Z})$

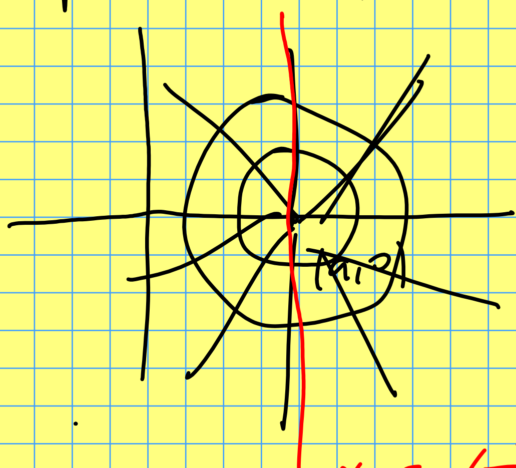
Voltando à reta $x=a$:

$a = r\cos\theta$

$$\boxed{\begin{gathered} r = \frac{a}{\cos\theta} \\ \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \end{gathered}}$$



É mais fácil tomar coords polares em relação ao ponto $(a,0)$:



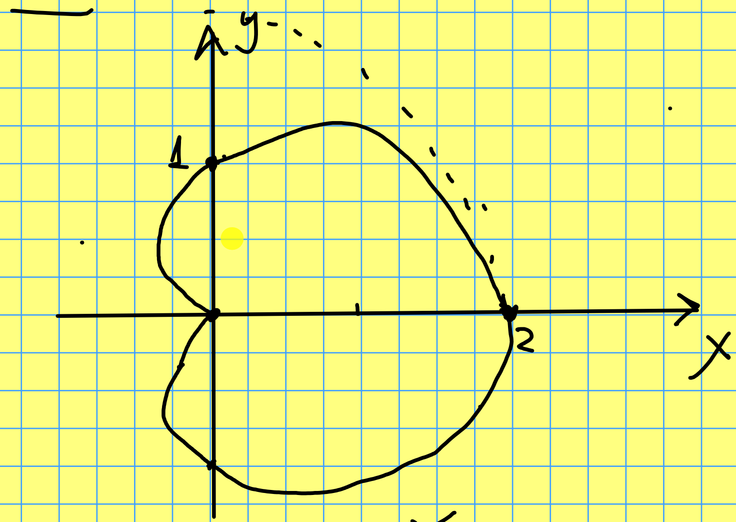
$\begin{cases} x - a = r\cos\theta \\ y = r\sin\theta \end{cases}$

$\begin{cases} x = a + r\cos\theta \\ y = r\sin\theta \end{cases}$

$x=a \iff \cos\theta = 0 \iff \theta = \dfrac{\pi}{2}$

$r \in \mathbb{R}$

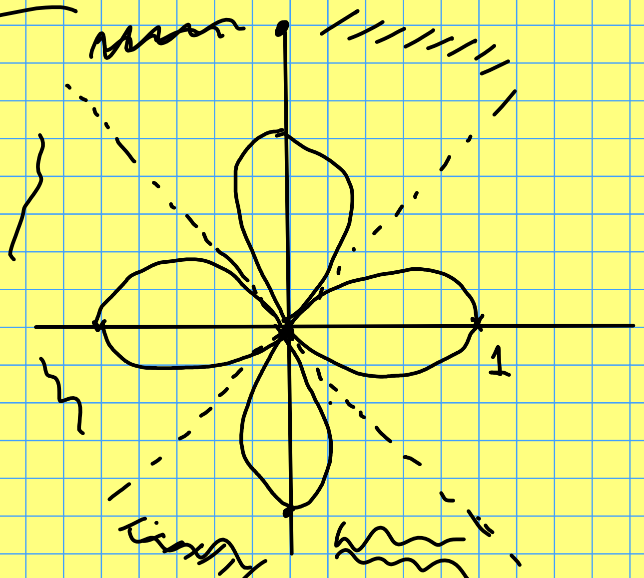
Ex. $r = 1 + \cos\theta$



θ	$\cos\theta$	$1+\cos\theta$
0	1	2
$\pi/6$	$\sqrt{3}/2$	$1+\sqrt{3}/2$
$\pi/4$	$\sqrt{2}/2$	$1+\sqrt{2}/2$
$\pi/3$	$1/2$	$3/2$
$\pi/2$	0	1
π	-1	0

CARDIÓIDE

Ex. $r = \cos 2\theta$

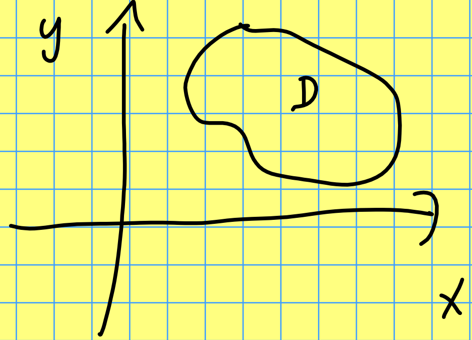


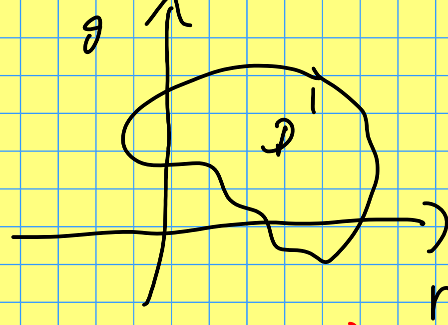
θ	$\cos 2\theta$
0	1
$\pi/12$	$\sqrt{3}/2$
$\pi/8$	$\sqrt{2}/2$
$\pi/6$	$1/2$
$\pi/4$	0 ←
$3\pi/8$	$-\sqrt{2}/2$
$\pi/2$	-1 ←
$3\pi/4$	0

ROSÁCEA DE QUATRO PÉTALAS

MUDANÇA PARA COORDS POLARES NA INTEGRAL DUPLA

$$\boxed{\iint_D f(x,y)\, dA = \iint_{D'} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta}$$

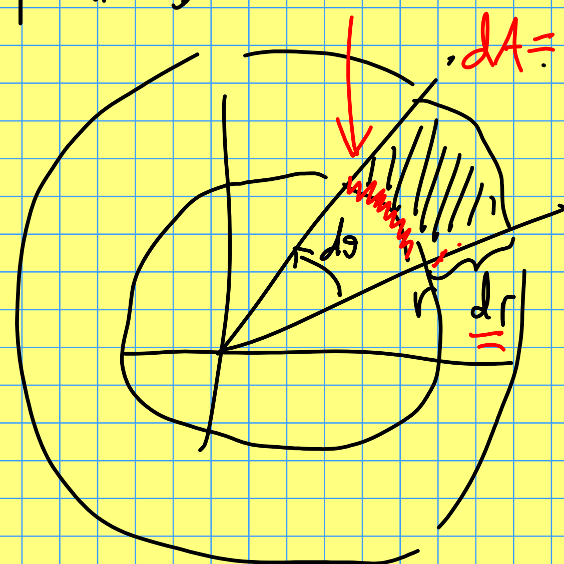




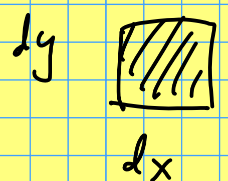
dA em coords polares?

comprimento é $rd\theta$

$.dA = rdrd\theta$



$$dA = dxdy$$



Raio R :  $.\pi R^2$

setor ângulo α

área αR^2

$$2\pi \;—\; \pi R^2$$

$$\alpha \;—\; \frac{\alpha}{2} R^2$$

dA:

Área do setor do círculo de raio $r+dr$ e ângulo $d\theta$ — Área do setor do círculo de raio r e ângulo $d\theta$

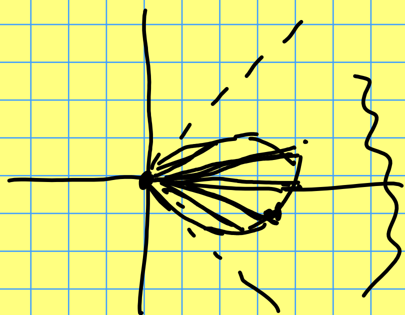
$$= \frac{1}{2} d\theta \left(r + dr \right)^2 - \frac{1}{2} d\theta\, r^2$$

$$= \frac{1}{2} d\theta \left(r^2 + 2rdr + (dr)^2 \right) - \frac{1}{2} d\theta\, r^2$$

$$= rdrd\theta + \cancel{\frac{1}{2} (dr)^2}$$

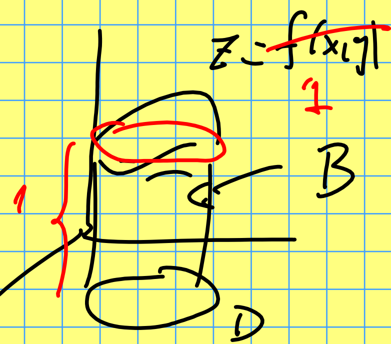
$$\boxed{dA = rdrd\theta} \leftarrow$$

Exemplo Calcular a área de uma pétala da rosácea de quatro pétalas $r = \cos 2\theta$.



$$A = \iint_D dA = \iint_{D'} r\,dr\,d\theta$$

$$D' : \begin{cases} -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4} \\ 0 \leq r \leq \cos 2\theta \end{cases}$$



$$Vol(B) = \iint_D f(x,y)\,dA$$

$$\text{Área}(D) = \iint_D dA$$

$$A = \int_{-\pi/4}^{\pi/4} \left(\int_0^{\cos 2\theta} r\,dr \right) d\theta$$

$$= \int_{-\pi/4}^{\pi/4} \left[\frac{r^2}{2} \right]_{r=0}^{r=\cos 2\theta} d\theta = \int_{-\pi/4}^{\pi/4} \frac{\cos^2 2\theta}{2}\, d\theta = 2\int_0^{\pi/4}$$

$$= \int_0^{\pi/4} \underbrace{\cos^2 2\theta}_{= \frac{1+\cos 4\theta}{2}}\, d\theta = \frac{1}{2} \int_0^{\pi/4} 1 + \cos 4\theta \, d\theta$$

$$= \frac{1}{2} \left[\theta + \frac{\cancel{\sin 4\theta}^{\,0}}{4} \right]_0^{\pi/4} = \frac{\pi}{8} \;//$$

Exemplo. Vamos usar coords polares para calcular

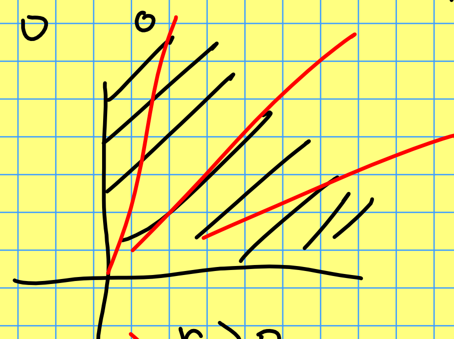
$$I = \int_0^{+\infty} e^{-x^2}\, dx.$$

$$I^2 = I \cdot I = \left(\int_0^{\infty} e^{-x^2} dx\right)\left(\int_0^{\infty} e^{-y^2} dy\right)$$

$$= \int_0^{\infty} \left(\int_0^{\infty} e^{-x^2} dx\right) e^{-y^2} dy$$

$$= \int_0^{\infty} \left(\int_0^{\infty} e^{-x^2} e^{-y^2} dx\right) dy = \int_0^{\infty}\left(\int_0^{\infty} e^{-(x^2+y^2)} dx\right) dy$$

$$= \int_0^{\infty}\int_0^{\infty} e^{-\underbrace{(x^2+y^2)}_{=r^2}} \underbrace{dx\,dy}_{=dA}$$



$\infty > r > 0$

$\theta \in [0, \frac{\pi}{2}]$

$$= \int_0^{\pi/2}\int_0^{\infty} e^{-r^2}\, r\, dr\, d\theta$$

$$= \int_0^{\pi/2} \left[-\frac{1}{2} e^{-r^2}\right]_0^{\infty} d\theta \qquad (e^{-r^2})' = -2r e^{-r^2}$$

$$= \int_0^{\pi/2} 0 - \left(-\frac{1}{2} e^0\right) d\theta = \frac{1}{2}\int_0^{\pi/2} d\theta = \frac{\pi}{4}$$

$$\therefore I = \frac{1}{2}\sqrt{\pi} \quad //.$$

§20.4