MAT216 – Cálculo Diferencial e Integral III
Respostas da Lista de Exercícios 2

1. (a) $(0,1)$ é ponto de mínimo absoluto. (b) $(0,0)$ é ponto de sela. (c) $(1,1)$ é ponto de sela. (d) $(0,0)$ é ponto de sela. (e) $(0,0)$ é ponto de mínimo absoluto e $(-1/4,-1/2)$ é ponto de sela.

3. (a) $\pm\sqrt{a^2+b^2}/ab$. (b) $a^2b^2/(a^2+b^2)$, mínimo global.

4. ± 3.

5. $(0,0,\pm 1)$.

6. $(\pm 1,0,0)$ e $(0,\pm 1,0)$.

7. $a^a b^b c^c/(a+b+c)^{a+b+c}$.

8. (a) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. (b) $\begin{pmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} \\ \frac{-y}{x^2+y^2} & \frac{x}{x^2+y^2} \end{pmatrix}$. (c) $\begin{pmatrix} 2x & 0 \\ 0 & 2y \end{pmatrix}$. (d) $\begin{pmatrix} \frac{x}{x^2+y^2} & \frac{y}{x^2+y^2} \\ \frac{-y}{x^2+y^2} & \frac{x}{x^2+y^2} \end{pmatrix}$. (e) $\begin{pmatrix} y^2 & 2xy \\ 2xy & x^2 \end{pmatrix}$. (f) $\begin{pmatrix} 3x^2 & -1 \\ 1 & 3y^2 \end{pmatrix}$.

9. (a) $\mathbf{R}^2$, se $ad-bc\neq 0$; nenhum ponto, se $ad-bc=0$. (b) Todo ponto do domínio, $x\neq 0$. (c) Se $x\neq 0$ e $y\neq 0$. (d) Todo ponto do domínio, $(x,y)\neq(0,0)$. (e) Se $x\neq 0$ e $y\neq 0$. (f) $\mathbf{R}^2$.

10. (a) 1. (b) $4x^3$. (c) $e^{2x/(x^2+y^2)}/(x^2+y^2)^2$.

11. (a) $Jf(r,\theta,\varphi)=\begin{pmatrix} \cos\theta\sin\varphi & -r\sin\theta\sin\varphi & r\cos\theta\cos\varphi \\ \sin\theta\sin\varphi & r\cos\theta\sin\varphi & r\cos\theta\cos\varphi \\ \cos\varphi & 0 & -r\sin\varphi \end{pmatrix}$. (b) $-r^2\sin\varphi$. (c) $u_r=\cos\theta\sin\varphi g_x+\sin\theta\sin\varphi g_y+\cos\varphi g_z$, $u_\theta=-r\sin\theta\sin\varphi g_x+r\cos\theta\sin\varphi g_y$, $u_\varphi=r\cos\theta\cos\varphi g_x+r\sin\theta\cos\varphi g_y-r\sin\varphi g_z$, onde as derivadas de g são calculadas em $f(r,\theta,\varphi)$. (d) $g_x=\frac{x}{\sqrt{x^2+y^2+z^2}}u_r-\frac{y}{x^2+y^2}u_\theta+\frac{zx/\sqrt{x^2+y^2}}{x^2+y^2+z^2}u_\varphi$, $g_y=\frac{y}{\sqrt{x^2+y^2+z^2}}u_r+\frac{x}{x^2+y^2}u_\theta+\frac{zy/\sqrt{x^2+y^2}}{x^2+y^2+z^2}u_\varphi$, $g_z=\frac{z}{\sqrt{x^2+y^2+z^2}}u_r-\frac{\sqrt{x^2+y^2}}{x^2+y^2+z^2}u_\varphi$, onde as derivadas de u são calculadas em $(r(x,y,z),\theta(x,y,z),\varphi(x,y,z))$. (e) $||\nabla g||^2=(u_r)^2+\frac{1}{x^2+y^2}(u_\theta)^2++\frac{1}{x^2+y^2+z^2}(u_\varphi)^2$, onde as derivadas de u são calculadas em $(r(x,y,z),\theta(x,y,z),\varphi(x,y,z))$.

12. (a) $Jf(x,y)=\begin{pmatrix} e^{x+2y} & 2e^{x+2y} \\ 2\cos(y+2x) & \cos(y+2x) \end{pmatrix}$ e $Jg(u,v,w)=\begin{pmatrix} 1 & 4v & 9w^2 \\ -2u & 2 & 0 \end{pmatrix}$. (b) $h(u,v,w)=(\exp(u-2u^2+4v+2v^2+3w^3),\sin(2u-u^2+2v+4v^2+6w^3))$. (c) $Jh(1,-1,1)=\begin{pmatrix} -3 & 0 & 9 \\ 0 & -6\cos 9 & 18\cos 9 \end{pmatrix}$.