MAT122 e MAT2116 – Álgebra Linear
Respostas da Lista de Exercícios 6

1. Temos

$$\begin{aligned} ||x+y||^2 &= (x+y)^t(x+y) \\ &= x^tx + x^ty + y^tx + y^ty \\ &= ||x||^2 + 2x^ty + ||y||^2 \\ &\leq ||x||^2 + 2||x||||y|| + ||y||^2 \qquad \text{(por Cauchy-Schwarz)} \\ &= (||x|| + ||y||)^2. \end{aligned}$$

Provamos que $||x+y||^2 \leq (||x|| + ||y||)^2$. Extraindo a raiz quadrada de ambos os membros, e observando que $||x+y||$ e $||x|| + ||y||$ são ambos positivos, obtemos o resultado desejado.

2. $(\frac{10}{3}, \frac{10}{3}, \frac{10}{3})$; $(\frac{5}{9}, \frac{10}{9}, \frac{10}{9})$.

3. $-\frac{1}{3}$; $109,471220634$ graus.

4. $\begin{pmatrix} \frac{4}{5} & -\frac{2}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{pmatrix}$

5. (a) $P_1 = \begin{pmatrix} \frac{1}{10} & \frac{3}{10} \\ \frac{3}{10} & \frac{9}{10} \end{pmatrix}$; $P_2 = \begin{pmatrix} \frac{9}{10} & -\frac{3}{10} \\ -\frac{3}{10} & \frac{1}{10} \end{pmatrix}$.
(b) $P_1 + P_2 = I$ e $P_1P_2 = 0$.

6. $\bar{x} = 2$.

7. $\bar{x} = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$, $p = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{pmatrix}$.

8. $b = \frac{61}{35} - \frac{36}{35}t$.

9. $\frac{1}{6}\begin{pmatrix} 5 & 2 & 1 \\ 2 & 2 & -2 \\ 1 & -2 & 5 \end{pmatrix}$

10. O espaço-coluna é S e o posto é k.

11. (a) $\{\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}\}$; (b) $P = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$; (c) $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$.

12. $\text{proj}_{a_1}b = 2a_1 = (\frac{4}{3}, \frac{4}{3}, -\frac{2}{3})$; $\text{proj}_{a_2}b = 2a_2 = (-\frac{2}{3}, \frac{4}{3}, \frac{4}{3})$; $\text{proj}_{\langle a_1, a_2\rangle}b = 2a_1 + 2a_2 = (\frac{2}{3}, \frac{8}{3}, \frac{2}{3})$.

13. Sejam A e B matrizes ortogonais de ordem n. Temos $A^tA = I$ e $B^tB = I$. Agora AB é quadrada de ordem n e $(AB)^t(AB) = (B^tA^t)(AB) = B^t(A^tA)B = B^tIB = B^tB = I$. Logo, AB é ortogonal.

14. Obtemos $q_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $q_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $q_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. Na forma $A = QR$:

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

15. $q_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$, $q_2 = \begin{pmatrix} \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{pmatrix}$, $q_3 = \begin{pmatrix} -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$;

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \sqrt{2} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{3}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & 0 & \frac{2}{\sqrt{3}} \end{pmatrix}.$$

16. $q_1 = (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0)$, $q_2 = (\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, 0)$, $q_3 = (\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}, -\frac{3}{2\sqrt{3}})$.

17. $q_1 = \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \\ -\frac{2}{3} \end{pmatrix}$, $q_2 = \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{pmatrix}$, $q_3 = \begin{pmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$; o núcleo à esquerda de A; $\bar{x} = (1, 2)$.