A Generalization of Yoshida-Nicolaescu Theorem using Partial Signatures

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Consider X a closed oriented riemannian manifold *partitioned* by a hypersurface Y, that is, $X = X_+ \cup X_-$ where $X_\pm$ are connected manifolds with common boundary Y. If P is a Dirac operator defined on a vector bundle E over X, then the symbol of P in the transversal variable at Y gives a complex structure in the space $\mathrm{L}^2(E|_Y)$. This complex structure induces a symplectic structure in $\mathrm{L}^2(E|_Y)$ and the Cauchy data spaces $\mathrm{H}_\pm(P)$ corresponding to the operator P and the decomposition $X = X_+ \cup X_-$ form a Fredholm pair of lagrangian subspaces. These subspaces are, roughly speaking, formed by *restrictions* $u|_Y$ of solutions of the equation $Pu = 0$ in $X_\pm$.

T. Yoshida showed in 1990 that if $\{P(s)\}_{s\in[0,1]}$ is a curve of Dirac operators in a three dimensional manifold such that $P(0)$ and $P(1)$ are invertible, then the spectral flow of the family $\{P(s)\}$ coincides with the Maslov index of the pair of lagrangian curves $(\mathrm{H}_+(P(s)), \mathrm{H}_-(P(s)))$. In 1994, L. Nicolaescu extended this result to higher dimensions, under the same hypotesis.

In this article, we give an alternative proof of the Yoshida-Nicolaescu theorem which works in a quite general situation, both in the degenerate and the non-degenerate case. The technique used employs the notion of partial signatures, which makes the proof very simple and direct. We will assume that the restriction to the cut submanifold of the transversal symbol is constant and that it defines a complex structure on the Hilbert space of L^2-sections of the bundle over Y.