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BROADCASTING IN BUS INTERCONNECTION NETWORKS

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In most distributed memory MIMD multiprocessors, processors are connected by a point-to-point interconnection network, usually modeled by a graph where processors are nodes and communication links are edges. Since interprocessor communication frequently constitutes serious bottlenecks, several architectures were proposed that enhance point-to-point topologies with the help of multiple bus systems so as to improve the communication efficiency. In this paper we study parallel architectures where the communication means are constituted *solely* by buses. These architectures can use the power of bus technologies, providing a way to interconnect much more processors in a simple and efficient manner.

We present the *hyperpath*, *hypergrid*, *hyperring*, and *hypertorus* architectures, which are the bus-based versions of the well used point-to-point interconnection networks. Using (hyper) graph theoretic concepts to model inter-processor communication in such networks, we give optimal algorithms for broadcasting a message from one processor to all the others. For deriving high performance communication patterns we developed a new tool called *simplification*. The idea is to construct a graph, to be called *representative graph*, from the original hyper-topology, in such a way that it will become easy to describe and perform communication schemes to the former that will fit to the latter, because the simplification concept also allows us to partially use some already known communication algorithms for usual networks.

Keywords: Massively parallel architectures, multiple bus systems, global communication, communication models, hypergraphs and applications, broadcast.

1. Introduction

In most distributed memory MIMD multiprocessors, processors are connected by a point-to-point interconnection network. Since interprocessor communication frequently constitutes serious bottlenecks, several architectures were proposed to *enhance* point-to-point topologies with the help of multiple bus systems so as to improve the communication efficiency.

In this paper[a], we study massively parallel systems where processors are connected *solely* by buses. These architectures can use the power of bus technologies, providing a way to interconnect much more processors in a simple and efficient manner. This represents a major issue in the construction of parallel computers, because a multiprocessor system with a large number (on the order of thousands) of processors cannot be provided with full connectivity and need to operate by communication between processors. Furthermore, the multiple bus architecture has the desirable ability to expand. While a crossbar architecture needs a quadratic number of switches, a multiple bus system of P processors and B buses requires only BP switches, though with reduced connectivity [25]. Without the underlying point-to-point network, such architectures are feasible and suitable for VLSI implementation, being able to interconnect a lot more processors, while keeping the diameter of the network small. For instance, 8100 processors can compose a 2-dimensional hypergrid, as defined in the next section, where each processor is connected to at most four buses, each bus has 30 processors, and the distance between the farthest apart processors is as low as 10. Notice that this is better than a usual hypercube of the same size, and easier to build, since it requires only 900 buses.

Some parallel computers use bus systems to implement interprocessor communications. For example, the KSR design from Kendall Square Research presented bus-based hierarchical rings of processors, and the MCS from Matra has clusters formed by heterogeneous processors interconnected by buses. The clusters themselves are interconnected in a bus-based ring. The goal of the theoretical framework developed in this paper is to contribute in the design of novel highly parallel architectures.

1.1. *Previous work*

Common networks are usually modeled by a graph where processors are nodes and communication links are edges. On the other hand, *bus interconnection networks* (BINs, for short) are best modeled by a *hypergraph* [3], in which an edge is a subset of nodes. Each bus in the network is thus represented by an edge of the hypergraph [4, 5, 24]. Several papers used derived models to propose and study new classes of interconnection networks for massively parallel computers. In a companion paper, we proposed all-to-all communication procedures for BINs, showing that global communication procedures are not a drawback for the utilization of BINs. Other approaches include, for instance, the one in [22], where Scherson proposes the concept of *orthogonal graphs*, that can describe, among others, binary m-cubes

[a] A preliminary version appeared in [13].

or meshes with buses in rows and columns. He also studies the routing problem in such architectures. Further, Fiduccia [14] tackles the problem of pin optimality of interconnection networks, concluding that bused n-node hypercubes, where n buses run along the $\log n$ dimensions of the hypercube, are pin-optimal.

Interestingly, hypergraphs were used by Szymanski [24] to model some photonic networks, where processors communicate through optical fibers carrying laser beams that have different wavelengths. When using optical technology, to fully utilize the high bandwidth provided, a physical optical channel should be shared by multiple transmissions [18]. This induces the use of BINs instead of point-to-point interconnections. Bermond and Ergincan present an excellent survey on structural problems related to BINs [6]. For more applications of BINs in optical networks, we refer the interested reader to [9, 12].

Another approach for using buses in interconnection networks is to enhance existing point-to-point networks with multiple bus systems, and many papers can be found on this subject [1, 2, 7, 8, 10, 17, 19, 20, 21, 23, 25]. However, it is out of the scope of this paper, where point-to-point links are not considered.

1.2. *Main results of this paper*

In this paper, we propose broadcast schemes for different BINs, modeled by hypergraphs, motivated by the fact that global communication procedures are critical to the efficiency of parallel algorithms in all areas of applications [16]. We focus on the *hyperpath*, the d-dimensional *hypergrid*, the *hyperring*, and the d-dimensional *hypertorus* architectures, which are the bus-based versions of the well known point-to-point interconnection networks. Considering that underlying point-to-point interconnection networks lost most of their interest with the advent of massively parallel computers, we focus our attention on the use of multiple buses as the *only* means to implement inter-processor communication, showing that global communication procedures are very efficient and easy to implement.

For deriving efficient communication patterns we develop a new tool called *simplification*. The idea is to construct a graph, to be called *representative graph*, from the original hyper-topology, in such a way that it will become easy to describe and perform communication schemes to the former that will fit to the latter, because the simplification concept also allows us to use some known communication algorithms for usual networks. Our results are as follows.

- By generalizing the notion of a linear network, a BIN can be designed with the following features. Let $\mathcal{P}_{n,m}$ be a parallel computer having nm processors, $n-1$ buses, with $2m$ processors per bus, each processor being connected to at most two buses. In such an architecture, we show how to broadcast a message from one processor to every other processor in at most $n-1$ rounds.

- Let $\mathcal{R}_{n,m}$ be a parallel computer having nm processors, n buses, with $2m$ processors per bus, each processor being connected to exactly two buses. In such an architecture, given by the generalization of a ring network, we show how to broadcast a message from one processor to all the others in at most $\lceil n/2 \rceil$ rounds.

- If we start from a 2-d grid, the BIN will be denoted $\mathcal{G}_2 = \mathcal{P}^1_{n_1,m_1} \times \mathcal{P}^2_{n_2,m_2}$. This is a parallel computer with $n_1 m_1 n_2 m_2$ processors, $n_1 n_2$ buses, each bus with $2m_1$ or $2m_2$ processors, where each processor is connected to at most four buses. We show that, in $\mathcal{G}_2$, we can perform broadcasting in $n_1 + n_2 - 2$ steps.

- If we start from a 2-d torus, we will generate a $\mathcal{T}_2 = \mathcal{R}^1_{n_1,m_1} \times \mathcal{R}^2_{n_2,m_2}$. This is a parallel computer with $n_1 m_1 n_2 m_2$ processors, $n_1 n_2$ buses, each bus with $2m_1$ or $2m_2$ processors, where each processor is connected to four buses. In $\mathcal{T}_2$, we can perform broadcasting in no more than $\lceil n_1/2 \rceil + \lceil n_2/2 \rceil$ steps, if n_1 or n_2 is even.

All these schemes are optimal since they asymptotically match the lower bound given by the diameter of the network.

Our paper is organized as follows. In the next Section we discuss some preliminary material on hypergraphs. In Section three we suggest the models of communication best suited for BINs. Section four introduces our main tool, called simplification, to deal with global communication in such massively parallel systems. Then, in Section five, we present broadcasting schemes for hyperpath, hyperring, d-hypergrid, and d-hypertorus BINs. We close the paper with some concluding remarks and ways for further research in these very attractive architectures.

2. Preliminaries

2.1. *Hypertopologies*

A graph is denoted $G(V, E)$, where V is a set of nodes or vertices $\{v_1, \ldots, v_n\}$ and E is a set of edges $\{e_1, \ldots, e_m\}$, where each edge $e_i \subset V, |e_i| = 2$. A hypergraph shall be denoted $\mathcal{H}(V, E)$, where V is a set of nodes $\{v_1, \ldots, v_n\}$ and E is a set of hyperedges $\{e_1, \ldots, e_m\}$ where each hyperedge $e_i \subset V \mid \bigcup_{e_i \subset E} = V$. Notice that a hypergraph $\mathcal{H}(V, E)$ where each hyperedge $e \in E$ has cardinality 2, is a graph, and that every graph $G(V, E)$ where $\bigcup_{e \in E} = V$ is a hypergraph.

Given a hypergraph $\mathcal{H}$, denote by $D(\mathcal{H})$ its diameter, defined as for graphs. Define *transmission* as the sending of a message along a hyperedge from one node to one or more adjacent nodes, where two nodes are said to be adjacent if they share a hyperedge. For a question of uniformity, in this paper we use graphs and hypergraphs, with the analogy of processor–node, communication link–edge, and bus–hyperedge.

In the following we study parallel architectures where interprocessor communication is implemented by bus systems only. We concentrate on (hyper)-topologies derived from the most common point-to-point interconnection networks, namely paths, rings, grids, and tori. Informally, our hypertopologies are obtained as follows. Each edge in the point-to-point topology will correspond to a hyperedge in the BIN, and the adjacency between the vertices is maintained. To do this, each edge (with two vertices) will yield a hyperedge with $2m$ vertices.

2.1.1. *Hyperpaths*

Topology: A path of size n is defined as $P_n(V, E)$ where:

$$\begin{aligned} &V = \{0, \ldots, n-1\}; \\ &e \in E \Leftrightarrow e = \{v_1, v_2\} \mid v_1 = (v_2 - 1). \end{aligned}$$

Generalization: We define hyperpath $\mathcal{P}_{n,m}(V, E)$ of size n and order m by:

$$\begin{aligned} &V = \{0, \ldots, nm-1\}; \\ &e \in E \Leftrightarrow e = \{v_1, \ldots, v_m, v'_1, \ldots, v'_m\} \\ &\qquad \text{where} \begin{cases} v_i = v_j \bmod n \\ v'_i = v'_j \bmod n \\ v_i = (v'_i - 1). \end{cases} \end{aligned}$$

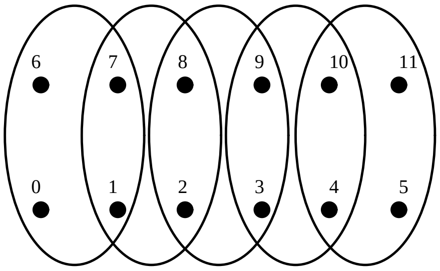

Figure 1: Hyperpath of size 6 and order 2 ($\mathcal{P}_{6,2}$).

2.1.2. *Hyperrings*

Topology: A ring of size n is defined as $R_n(V, E)$ where:

$$\begin{aligned} &V = \{0, \ldots, n-1\}; \\ &e \in E \leftrightarrow e = \{v_1, v_2\} \mid v_1 = (v_2 - 1) \bmod n. \end{aligned}$$

Generalization: We define hyperring $\mathcal{R}_{n,m}(V, E)$, of size n and order m as:

$$\begin{aligned} &V = \{0, \ldots, nm-1\}; \\ &e \in E \leftrightarrow e = \{v_1, \ldots, v_m, v'_1, \ldots, v'_m\} \\ &\qquad \text{where} \begin{cases} v_i = v_j \bmod n \\ v'_i = v'_j \bmod n \\ v_i = (v'_i - 1) \bmod n. \end{cases} \end{aligned}$$

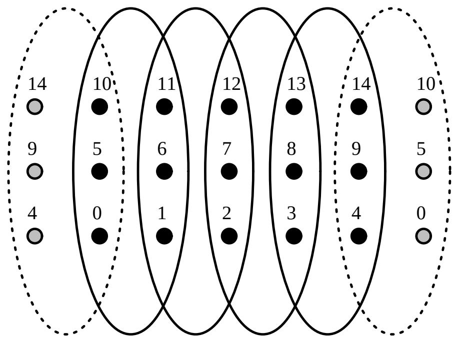

Figure 2: Hyperring of size 5 and order 3 ($\mathcal{R}_{5,3}$).

2.1.3. *Hypergrids*

Following the definition of grids by means of a product of paths, we can define a d-dimensional hypergrid (or simply, d-hypergrid) as $\mathcal{G} = \mathcal{P}^1_{n_1,m_1} \times \ldots \times \mathcal{P}^d_{n_d,m_d}$ where $\mathcal{P}^i_{n_i,m_i}$ are hyperpaths. For this a formal definition of the Cartesian Product for hypergraphs is needed. Below, we define 2-hypergrids, the definition of d-hypergrids being analogous [3].

Definition 1 *Let $\mathcal{P}^1_{n_1,m_1}$ and $\mathcal{P}^2_{n_2,m_2}$ be two hyperpaths as defined above. The 2-dimensional hypergrid $\mathcal{G}(V,E) = \mathcal{P}^1_{n_1,m_1} \times \mathcal{P}^2_{n_2,m_2}$ is as follows.*

$V = \{u_k\}, 0 \leq k < n_1m_1n_2m_2$, *such that* $u_k = (v_i, v'_j)$ *with* $0 \leq i < n_1m_1$ *and* $0 \leq j < n_2m_2$.

$E = \{e_q\}, \mid e_q \mid = 2m_1$, *or* $\mid e_q \mid = 2m_2$, *such that two vertices (a,b) and (c,d) belong to the same edge in* $\mathcal{G}$ *if and only if*

1. *$a = c$ and b, d belong to the same edge in $\mathcal{P}^2_{n_2,m_2}$, or*
2. *$b = d$ and a, c belong to the same edge in $\mathcal{P}^1_{n_1,m_1}$.*

Figure 3 shows an example.

2.1.4. *Hypertori*

Analogously to the generalization of grids to hypergrids, hypertori can be defined as the generalization of tori through the Cartesian product of hyperrings, as follows.

A d-hypertorus $\mathcal{T}^d$ is defined as $\mathcal{R}^1_{n_1,m_1} \times \ldots \times \mathcal{R}^d_{n_d,m_d}$ where $\mathcal{R}^i_{n_i,m_i}$ are hyperrings.

2.2. *Comparison with graphs*

We compare the defined topologies with the usual graph topologies. We consider the number of nodes, degree of nodes and edges, and diameter. In Table 1 $\mathrm{dgr}(v)$ and $\mathrm{dgr}(e)$ are respectively the maximum degree of the nodes and edges in a graph. We also have $\mathcal{G}^d(V,E) = \mathcal{P}^1_{n_1,m_1} \times \ldots \times \mathcal{P}^d_{n_d,m_d}$, and $\mathcal{T}^d(V,E) = \mathcal{R}^1_{n_1,m_1} \times \ldots \times \mathcal{R}^d_{n_d,m_d}$.

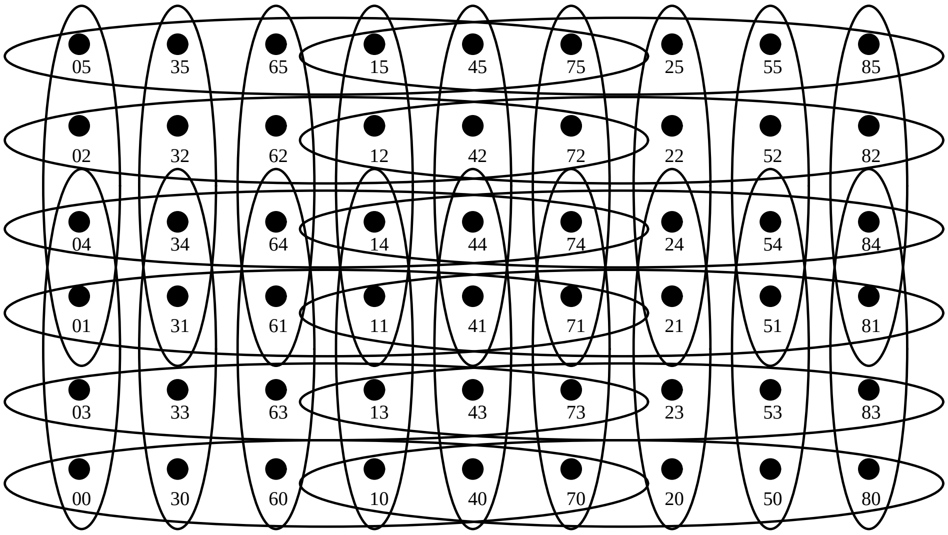

Figure 3: Hypergrid $\mathcal{G} = \mathcal{P}^1_{3,3} \times \mathcal{P}^2_{3,2}$.

Table 1: Comparative table.

Topology	Number of nodes	dgr(v)	dgr(e)	Diameter
$P_n(V,E)$	n	2	2	$n-1$
$R_n(V,E)$	n	2	2	$\lfloor \frac{n}{2} \rfloor$
$G_{n_1,\ldots,n_d}(V,E)$	$n_1 \ldots n_d$	$2d$	2	$\sum_{i=1}^{d}(n_i - 1)$
$T_{n_1,\ldots,n_d}(V,E)$	$n_1 \ldots n_d$	$2d$	2	$\sum_{i=1}^{d}(\lfloor \frac{n_i}{2} \rfloor)$
$\mathcal{P}_{n,m}(V,E)$	nm	2	$2m$	$n-1$
$\mathcal{R}_{n,m}(V,E)$	nm	2	$2m$	$\lfloor \frac{n}{2} \rfloor$
$\mathcal{G}^d(V,E)$	$\prod_{i=1}^{d}(m_i n_i)$	$2d$	$\max_{i=1}^{d}(2m_i)$	$\sum_{i=1}^{d}(n_i - 1)$
$\mathcal{T}^d(V,E)$	$\prod_{i=1}^{d}(m_i n_i)$	$2d$	$\max_{i=1}^{d}(2m_i)$	$\sum_{i=1}^{d}(\lfloor \frac{n_i}{2} \rfloor)$

We see that, for example, a topology with N nodes in a ring has diameter $\lfloor \frac{N}{2} \rfloor$. Whereas a topology with N nodes in a hyperring (with hyperedges of size s assuming $N \bmod s = 0$) has diameter s times smaller ($\frac{N}{2s}$).

In Table 2 we have the diameters of various topologies, by fixing the number of nodes to be 48, and assuming 6 nodes per hyperedge.

3. Modeling bus access and capacity

We first recall the usual notation used for modeling communications in point-to-point interconnection networks [15]. Given a graph $G(V,E)$, there exists a communication model for edges and another for nodes. Relative to the edges, when data can be transmitted in only one direction in each cycle, we have the *half-duplex* model (H). In the case where information can traverse in both directions simultaneously,

Table 2: Comparison of diameters among graph and hypergraph topologies.

Topology	Diameter	(hyper)topology	Diameter
Path	47	Hyperpath	7
Ring	24	Hyperring	4
Grid (2×24)	24	Hypergrid (2×4)	4
Grid (4×12)	14	Hypertorus (2×4)	3
Grid (6×8)	12		
Torus (2×24)	13		
Torus (4×12)	8		
Torus (6×8)	7		

we have the *full-duplex* model (F). If a node can use (send/receive) only one of its edges simultaneously, we have the 1-port model; if it can use k edges we have the k-port model. If a node can use all its edges in one same cycle, we have the $*$-port model. The notation E_k is used, where E is the model for edges and k is the model for nodes (*e.g.*, F_k stands for full-duplex, k-port, and H_1 stands for half-duplex, 1-port).

Our models for BIN's stem from the observation that, as far as VLSI implementations are concerned, an edge being half or full duplex means that a communication link can carry one or two different pieces of information, implying that one or two physical links have to be implemented between the two processors. Therefore, in order to model bus communications, we can assume that a bus, as a communication link, can carry more than one information. Theoretically speaking, a bus could support as much information transfer as the number of vertices it connects. A similar idea can be found in [24].

Definition 2 *Given a BIN $\mathcal{H}(V,E)$, inter-processor communications can be modeled as follows.*

Bus capacity: *if only one information can traverse in a bus in one cycle, we call it the* `1-capacity` *(or* `single-capacity`*) model; if k information can traverse at the same time, we call it the* `k-capacity` *model.*

Bus access: *if a processor can use (send/receive) simultaneously only one of its buses, we call it the* `1-port` *model; if it can use k (all) of its buses at the same time, we call it the* `k-port` *(*`*-port`*) model.*

3.1. *Notation*

We use the notation $M_{\alpha,\rho}$ where α is the model of bus capacity and ρ is the model for bus access.

Given a graph $G(V,E)$ and two adjacent nodes $u, v \in V$, we use the notation $u \leftarrow v$ to denote the transmission of node v to node u. Given a hypergraph $\mathcal{H}(V,E)$,

$v \in V$, $e \in E$, with $v \in e$ and given $N \subset e$, we denote by $N \Leftarrow v$ a transmission from node v to the set of nodes N.

Due to VLSI feasibility concerns, as well as to main features in global communication techniques, in this paper we focus our attention to broadcast (one-to-all) schemes for BINs, only under the models $M_{i,j}, i = 1, j = 1$ or *. Hence, buses can transport at most one information in a single time step, and nodes are able to use one or all of its buses in a single cycle.

4. Simplification

A parallel computer built according to the model proposed above has to prove its utility in overcoming communication bottlenecks. In the following, we develop a tool to help us carry out standard global communication operations, as broadcast, in the previously defined BINs.

The idea is to use a new concept called *simplification* in order to construct a graph, to be called *representative graph*, from the hypergraph defining the BIN, in such a way that it will become easy to describe and perform communication in the hypergraph[b]. Furthermore, we also want to make use of the already known communication algorithms for point-to-point interconnection networks.

The simplification operation is done basically by choosing nodes in the hypergraph, called *representative nodes*, to be the nodes of the representative graph $G(V', E')$. Adjacent representative nodes in the hypergraph will be adjacent in the representative graph. For each $u \in V'$ we will define the set of nodes represented by u, N_u.

Each $y_i \in N_u$ is called a *represented node* with regard to u. Let $N = \bigcup_{u \in V'} N_u$, we define $W = V \backslash (N \bigcup V')$, the set of nodes without representative. The details will be given henceforth.

A transmission to a representative node in the representative graph also means transmission to nodes it represents in the hypergraph. That is, $u \leftarrow v$ in the representative graph also implies $N_u \Leftarrow v$ and $N_v \Leftarrow v$ in the hypergraph.

It is important to point out that the simplification operation depends not only on the hypergraph, but also on the communication model adopted. Below we show the main forms for simplification of hyperpaths, hyperrings, d-dimensional hypergrids and d-dimensional hypertori.

- Simplification for the model $M_{i,j}, (i \leq j)$. We use the simplification in the hypergraphs in such a way that it will be easy to emulate the communication in the hypergraph under the model $M_{i,j}, (i \leq j)$ through the representative graph under the model H_j, when $i = 1$, or under the model F_j, if $i = 2$. The main idea is to choose a representative node in the intersection of hyperedges.

[b] We remark that "representative graph" does not have here the same meaning as in [3], where it denotes the line graph of a hypergraph.

- Simplification for the model $M_{2,1}$. In this case we want to model the communication in the hypergraph through a representative graph under the full-duplex model, where each edge can simultaneously carry information in both directions. Instead of picking only one representative node in each intersection of hyperedges, we now choose two of them.

- *-Simplification for model $M_{1,1}$. With this simplification operation, as well as the next one, we make better use of the characteristics of hypergraphs. For this purpose, we choose more than one representative per intersection, to be seen later.

- *-Simplification for model $M_{2,1}$. Here we assume that a node, under model $M_{x,1}$ (as in model H_1) can send at most one message and receive several messages in a same cycle. That is, we assume that the model adopted limits only the number of transmissions per node. This simplification will be used in such a way that we can work with a graph in model F_*. Details will be given later.

4.1. *Simplification for the model* $M_{i,j}, (i \leq j)$

- **Representative graph $G(V', E')$ of a hyperpath $\mathcal{P}_{n,m}(V, E)$.**
  We elect $n-1$ representatives $V' = \{u_1, \ldots, u_{n-1}\}$, with $u_i \in V$ such that:

$$\begin{cases} u_1 \bmod n = 0 \\ u_{i+1} = u_i + 1, \text{ for } 1 \leq i \leq n-2. \end{cases}$$

  The nodes of G are adjacent if and only if they are adjacent in $\mathcal{P}_{n,m}$. In this way, we will have one representative for each intersection of hyperedges. In addition to these, we have one representative in *one* of the extremes of $\mathcal{P}_{n,m}$. The sets of represented nodes are such that, for each $u \in V', N_u = \{y_1, y_2, \cdots, y_{m-1}\}$,

$$\begin{cases} (y_i - u) \bmod n = 0 \\ y_i \neq u, 1 \leq i \leq m-1. \end{cases}$$

  An example can be seen in Figure 4.

- **Representative graph $G(V', E')$ of a hyperring $\mathcal{R}_{n,m}(V, E)$.**
  We elect n representatives $V' = \{u_1, \ldots, u_n\}$, with $u_i \in V$ such that:

$$\begin{cases} u_1 \bmod n = 0 \\ u_{i+1} = u_i + 1, \text{ for } 1 \leq i \leq n-1. \end{cases}$$

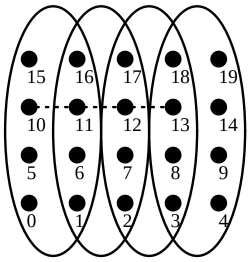

Figure 4: One possible representative graph for the hyperpath $\mathcal{P}_{5,4}$ has the nodes $\{10, 11, 12, 13\}$, the edges are in dotted lines. We have also: $N_{10} = \{0, 5, 15\}$, $N_{11} = \{1, 6, 16\}$, $N_{12} = \{2, 7, 17\}$, $N_{13} = \{3, 8, 18\}$ and $W = \{4, 9, 14, 19\}$.

The nodes of G are adjacent if and only if they are adjacent in $\mathcal{R}_{n,m}$. In this way, we will have one representative node for each intersection of hyperedges. The sets of represented nodes are such that, for each $u \in V', N_u = \{y_1, y_2, \dots, y_{m-1}\}$,

$$\begin{cases} (y_i - u) \bmod n = 0 \\ y_i \neq u, 1 \leq i \leq m-1. \end{cases}$$

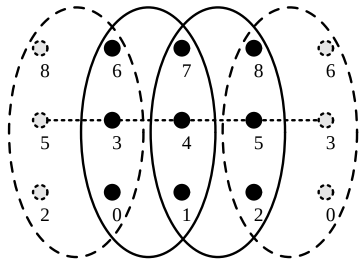

Figure 5: Representative graph (dotted lines) of the hyperring $\mathcal{R}_{3,3}$.

- **Representative graph $G(V', E')$ of a d-hypergrid $\mathcal{G} = \mathcal{P}^1_{n_1,m_1} \times \ldots \times \mathcal{P}^d_{n_d,m_d}$.**
  We perform simplification by dimensions. We first choose the representative nodes for the hyperpath $\mathcal{P}^1_{n_1,m_1}$. Let $G_1(V'^{(1)}, E'^{(1)})$ be its representative graph, with its representative nodes $V'^{(1)} = \{u^{(1)}_1, \dots, u^{(1)}_{n_1-1}\}$. Let $\mathcal{G}^{(1)} = G_1 \times \mathcal{P}^2_{n_2,m_2} \times \ldots \times \mathcal{P}^d_{n_d,m_d}$. We denote the nodes of $\mathcal{G}^{(1)}$ by *representative nodes of first order*. We choose the representative nodes for the hyperpath $\mathcal{P}^2_{n_2,m_2}$, and we will have the representative graph $G_2(V'^{(2)}, E'^{(2)})$, with its representative

nodes $V'^{(2)} = \{u_1^{(2)}, \ldots, u_{n_2-1}^{(2)}\}$. The nodes of $\mathcal{G}^{(2)} = G_1 \times G_2 \times \mathcal{P}^3_{n_3,m_3} \times \ldots \times \mathcal{P}^d_{n_d,m_d}$ will be called *representative nodes of second order*. By doing the same for the remaining paths, we will have $\mathcal{G}^{(d)} = G_1 \times \ldots \times G_d$. Observe that each $G_i, 1 \leq i \leq d$, is a graph, and therefore $\mathcal{G}^{(d)}$ is the representative graph G of $\mathcal{G}$, more specifically a d-dimensional grid. For the sake of uniformity in notation, we define also $\mathcal{G}^{(0)} = \mathcal{G}$.

For each node x of $\mathcal{G}^{(j)}$ we define the set $N_x^{(j)} = \{y_1^{(j)}, \ldots, y_{m_j-1}^{(j)}\}$, that are the nodes in $\mathcal{G}^{(j-1)}$ such that:

$$\begin{cases} (x)_l = (y_i^{(j)})_l, l \neq j, 1 \leq l \leq d \\ (y_i^{(j)})_j - (x)_j \bmod n_j = 0 \\ x \neq y_i^{(j)}, 1 \leq i \leq m_j - 1 \end{cases}$$

where $(\ldots)_j$ means the value in the j-th dimension. The vertices in $N_x^{(j)}$ are equal to x in all coordinates, except coordinate j, and in coordinate j those vertices are equal to x $(\mathrm{mod} n_j)$.

In each hypergraph $\mathcal{G}^{(j)}, 1 \leq j \leq d$, we define the set $W^{(j)}$ of *nodes without representative*, in $\mathcal{G}^{(j-1)}$, that is, $v \in W^{(j)} \Leftrightarrow v \in \mathcal{G}^{(j-1)}, v \notin \mathcal{G}^{(j)}$, and v is not represented by any node in $\mathcal{G}^{(j)}$. For each j, the set $W^{(j)}$ is formed by the nodes without representative corresponding to the simplification of all the $(n_1 - 1) \ldots (n_{j-1} - 1)(n_{j+1}m_{j+1}) \ldots (n_d m_d)$ hyperpaths on this dimension in $\mathcal{G}^{(j-1)}$. Let

$$W^{(j)}_{i_1,\ldots,i_d} \begin{cases} 0 \leq i_s < n_s - 1, s < j \\ i_j = 1 \\ 0 \leq i_s < n_s m_s, s > j, \end{cases}$$

be defined as follows:

$$v \in W^{(j)}_{i_1,\ldots,i_d} \Leftrightarrow \begin{cases} v \in \mathcal{G}^{(j-1)} \\ (v)_s = \begin{cases} u^{(j)}_{i_s+1}, s < j \\ i_s, s > j \end{cases} \\ (v_j \bmod n_j) = n_j - 1. \end{cases}$$

We see that $W^{(j)}$ is the union of the sets $W^{(j)}_{i_1,\ldots,i_d}$. Figure 6 shows an example.

In Figure 6 we have steps 1 and 2, that correspond to $\mathcal{G}^{(1)}$ and $\mathcal{G}^{(2)}$. For each node $x^{(1)}$ of $\mathcal{G}^{(1)}$, we have their represented nodes $(N_{x^{(1)}})$ in $\mathcal{G}^{(0)}$, for example, for $x = (0, 5)$ we have $N_{x^{(1)}} = \{(3, 5), (6, 5)\}$. The set of nodes without representative of $\mathcal{G}^{(1)}$ is

$$W^{(1)} = \bigcup_{i=0}^{5} \{(2, i), (5, i), (8, i)\}.$$

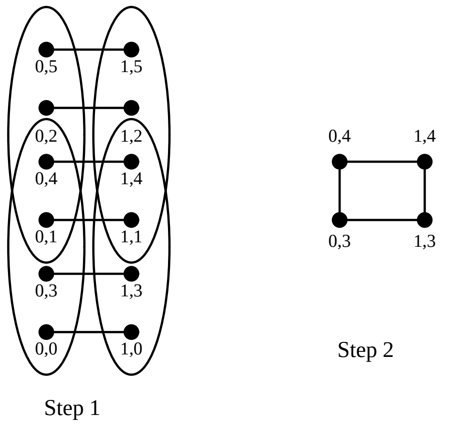

Figure 6: The 2 steps of simplification for the 2-hypergrid $\mathcal{G} = \mathcal{P}^1_{3,3} \times \mathcal{P}^2_{3,2}$.

It is easy to see that $|W^{(1)}| = m_1(n_2m_2) = 18$. We see that $W^{(1)} = \bigcup_{i=0}^{5} W_{1,i}$ e $W_{1,i} = \{(2,i),(5,i),(8,i)\}$, $0 \le i \le 5$.

For each node x of $\mathcal{G}^{(2)}$, we have their represented nodes ($N_{x^{(2)}}$) in $\mathcal{G}^{(1)}$, for example, for $x = (0,4)$ we have $N_{x^{(2)}} = \{(0,1)\}$. The set of nodes without representative of $\mathcal{G}^{(2)}$ is $W^{(2)} = \{(0,2),(0,5),(1,2),(1,5)\}$, $|W^{(2)}| = n_1m_2 = 4$. We see that $W^{(2)} = \bigcup_{i=0}^{1} W_{i,1}$ and that $W_{0,1} = \{(0,2),(0,5)\}$ and $W_{1,1} = \{(1,2),(1,5)\}$.

- **Representative graph of a d-dimensional hypertorus $\mathcal{T}^d = \mathcal{R}^1_{n_1,m_1} \times \ldots \times \mathcal{R}^d_{n_d,m_d}$.**
We perform simplification by dimensions. We first choose the representative nodes for the hyperring $\mathcal{R}^1_{n_1,m_1}$. Let $G_1(V'^{(1)}, E'^{(1)})$ be its representative graph, with its representative nodes $V'^{(1)} = \{u_1^{(1)}, \ldots, u_{n_1}^{(1)}\}$. Let $\mathcal{T}^{(1)} = G_1 \times \mathcal{R}^2_{n_2,m_2} \times \ldots \times \mathcal{R}^d_{n_d,m_d}$. We denote the nodes of $\mathcal{T}^{(1)}$ by *representative nodes of first order.* We choose the representative nodes for the hyperring $\mathcal{R}^2_{n_2,m_2}$, and we will have the representative graph $G_2(V'^{(2)}, E'^{(2)})$, with its representative nodes $V'^{(2)} = \{u_1^{(2)}, \ldots, u_{n_2}^{(2)}\}$. The nodes of $\mathcal{T}^{(2)} = G_1 \times G_2 \times \mathcal{R}^3_{n_3,m_3} \times \ldots \times \mathcal{R}^d_{n_d,m_d}$ will be called *representative nodes of second order.* By doing the same for the remaining rings, we will have $\mathcal{T}^{(d)} = G_1 \times \ldots \times G_d$. Observe that each $G_i, 1 \le i \le d$, is a graph, and therefore $\mathcal{T}^{(d)}$ is the representative graph G of $\mathcal{T}$, more specifically a d-dimensional torus. For the sake of uniformity in notation, we define also $\mathcal{T}^{(0)} = \mathcal{T}$.

For each node x of $\mathcal{T}^{(j)}$ we define the set $N_x^{(j)} = \{y_1^{(j)}, \ldots, y_{m_j-1}^{(j)}\}$, that are the nodes in $\mathcal{T}^{(j-1)}$ such that:

$$\begin{cases} (x)_l = (y_i^{(j)})_l, l \neq j, 1 \leq l \leq d \\ (y_i^{(j)} - x)_j \bmod n_j = 0 \\ x \neq y_i^{(j)}, 1 \leq i \leq m_j - 1 \end{cases}$$

where $(\ldots)_j$ means the value in the j-th dimension.

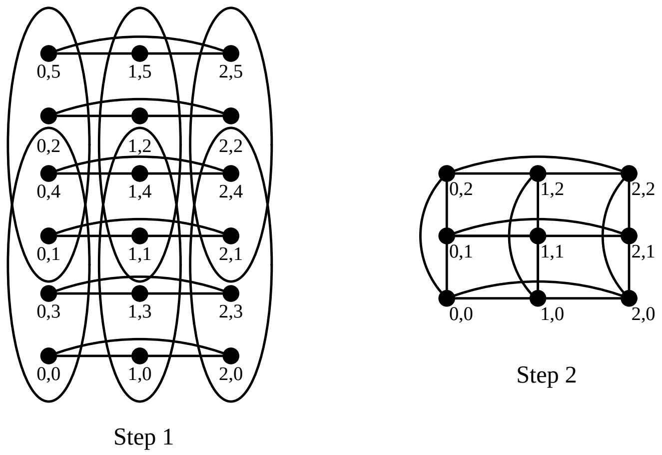

Figure 7: The 2 steps of simplification for the 2-hypertorus $\mathcal{T} = \mathcal{R}^1_{3,3} \times \mathcal{R}^2_{3,2}$.

Notice that a simplification of the given hypergraphs resulted in a representative graph, of the same family from which the hypergraph was generalized.

Let now the relation $(G_1, \lambda) \prec (G_2, \varphi)$ indicate that a communication scheme, excluding the trivial communication of a node to itself, in graph G_1 under model λ can be implemented within the same number of steps in graph G_2 under model φ.

For example suppose that we have the following cycle, in the model H_1, in the simplified graph of figure 6: $(0,4) \leftarrow (1,4), (1,3) \leftarrow (0,3)$, this can be done in the original hypergraph (figure 3), in the model $M_{1,1}$. With the cycle

$$\begin{cases} \{(0,4),(3,4),(6,4),(4,4),(7,4)\} \Leftarrow (1,4) \\ \{(1,3),(4,3),(7,3),(3,3),(6,3)\} \Leftarrow (0,3). \end{cases}$$

Claim 1 *Given a hypergraph $\mathcal{H}$ and its representative graph G under the model $M_{i,j}, i \leq j$. If $i = 1$ then $(G, H_j) \prec (\mathcal{H}, M_{1,j})$. If $i = 2$ then $(G, F_j) \prec (\mathcal{H}, M_{2,j})$.*

4.2. *Simplification for the model $M_{2,1}$*

For the following definitions of representative graphs, we assume, without loss of generality, that there exist as many nodes in hyperedge intersections as necessary.

- **Representative graph $G(V', E')$ of a hyperpath $\mathcal{P}_{n,m}(V, E)$.**

  We elect $2(n-1)$ representative nodes, $V' = \{u_1, \ldots, u_{n-1}, u_n, \ldots, u_{2(n-1)}\}$, such that:

  $$\begin{cases} (u_1 \bmod n) = (u_n \bmod n) = 0 \\ u_{i+1} = u_i + 1, 1 \leq i \leq n-2, n \leq i \leq 2n-3 \\ u_1 \neq u_n. \end{cases}$$

  Nodes of G are adjacent if and only if they are adjacent in $\mathcal{P}_{n,m}$. We define the set of represented nodes N_z, for each $z \in V', N_z = \{y_1, \ldots, y_m\}$, such that $(y_i - z) \bmod n = 0, 1 \leq i \leq m$.

  Note that now z belongs to N_z and that for a given $i, N_{u_{i+(n-1)}} = N_{u_i}$. Then, we say in this case that u_i and $u_{i+(n-1)}$ are *equivalent*. Let $N = \bigcup_{u \in V'} N_u$, we define the set of nodes without representative $W = V \backslash N$.

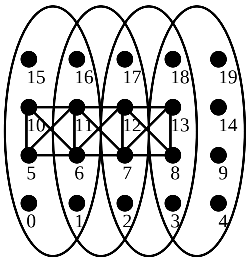

Figure 8: Representative graph for the model $M_{2,1}$ of the hyperpath $\mathcal{P}_{5,4}$.

- **Representative graph $G(V', E')$ of a hyperring $\mathcal{R}_{n,m}(V, E)$.** We elect $2n$ representative nodes, $V' = \{u_1, \ldots, u_n, u_{n+1}, \ldots, u_{2n}\}$, such that:

  $$\begin{cases} (u_1 \bmod n) = (u_n \bmod n) = 0 \\ u_{i+1} = u_i + 1, 1 \leq i \leq n-1, n < i \leq 2n-1 \\ u_1 \neq u_n. \end{cases}$$

  We define analogously the sets $N_u, u \in V'$, of the represented nodes, in the same manner as for the hyperpah.

- **Representative graph $G(V', E')$ of a d-dimensional hypergrid $\mathcal{G} = \mathcal{P}^1_{n_1,m_1} \times \ldots \times \mathcal{P}^d_{n_d,m_d}$** is constructed similarly, again dimension after dimension. As before, we can define similarly the sets of represented nodes, and the sets of nodes without representative. See Figure 9 for an example.

- **Representative graph of a d-dimensional hypertorus $\mathcal{T}^d = \mathcal{R}^1_{n_1,m_1} \times \ldots \times \mathcal{R}^d_{n_d,m_d}$.**
  Analogous to the previous one.

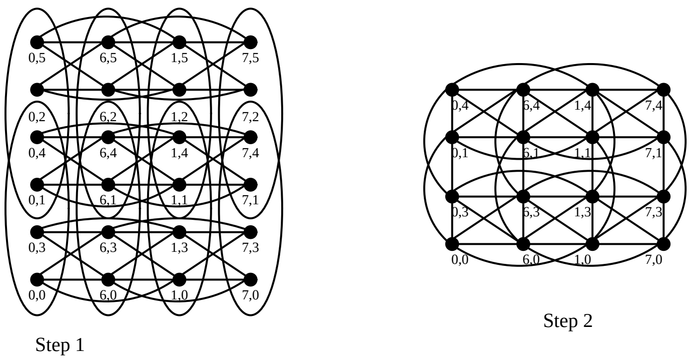

Figure 9: Simplification for the model $M_{2,1}$ of the 2-hypergrid $\mathcal{G} = \mathcal{P}^1_{2,3} \times \mathcal{P}^2_{2,2}$.

Proposition 1 $(G, H_1) \prec (\mathcal{G}, M_{2,1})$.

Proof. Given two adjacent nodes u, v em $\mathcal{G}$ it is easy to verify that the isolate transmission $u \leftarrow v$ can be realized in $\mathcal{G}$. Suppose that there exists a cycle T realizable in G in model H_1, that cannot be executed in $\mathcal{G}$ in model $M_{2,1}$. We can decompose T into $t_1, \ldots, t_d$, where t_i contains the transmissions on dimension i.

It follows that there exists a $j, 1 \leq j \leq d$, such that t_j cannot be realized in $\mathcal{G}$ in model $M_{2,1}$. Observing that on dimension j, we have two nodes of G per hyperedge of $\mathcal{G}$, and that each edge can carry out two transmissions, we come to a contradiction $\Box$.

4.3. **-Simplification for model* $M_{1,1}$

- *Representative graph* **of a** d**-dimensional hypergrid** $\mathcal{G}^d(V, E) = \mathcal{P}^1_{n_1,m_1} \times \ldots \times \mathcal{P}^d_{n_d,m_d}$. Let $G_1(V'_1, E'_1)$ be the representative graph of $\mathcal{P}^1_{n_1,m_1}$. As in the following, let the representative nodes be $V'_1 = \{u^{(1)}_1, \ldots, u^{(1)}_{(n_1-1)2d}\}$, such that:

$$\begin{cases} u_{1+j(n_1-1)} \bmod n_1 = 0 & \\ u_{i+1+j(n_1-1)} = u_{i+j(n_1-1)} + 1 & 1 \leq i \leq n-1 \text{ and } 0 \leq j \leq 2d-1 \\ u_{1+l(n_1-1)} \neq u_{1+k(n_1-1)}, \text{ if } k \neq l. & \end{cases}$$

where the nodes of G_1 are adjacent, if and only if they are adjacent in $\mathcal{P}^1_{n_1,m_1}$. In the same way we define the representative graphs $G_i(V'_i, E'_i)$ corresponding to the hyperpaths $\mathcal{P}^i_{n_i,m_i}$, $2 \leq i \leq d$.

Let $\mathcal{G}^{(1)} = G_1 \times \mathcal{P}^2_{n_2,m_2} \times \ldots \times \mathcal{P}^d_{n_d,m_d}$, we denote the nodes of $\mathcal{G}^{(1)}$ as representative nodes of first order. The nodes of $\mathcal{G}^{(2)} = G_1 \times G_2 \times \mathcal{P}^3_{n_3,m_3} \times \ldots \times \mathcal{P}^d_{n_d,m_d}$

will be denoted representative nodes of second order. Continuing like this, we will have $\mathcal{G}^{(d)} = G_1 \times \ldots \times G_d$, that will be a graph, whose nodes will be the representative nodes of d-th order (Figure 10).

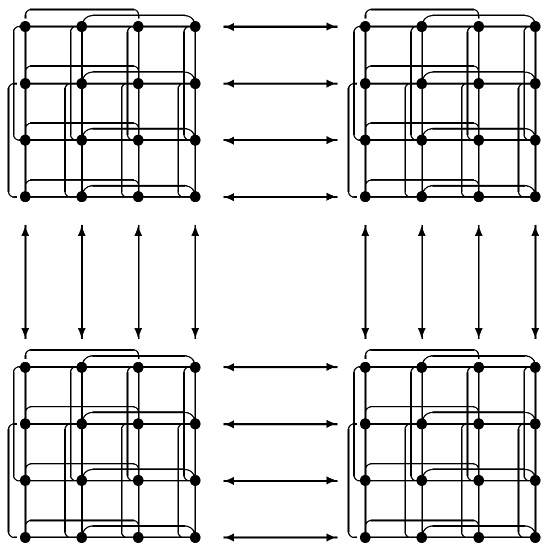

Figure 10: *-representative graph of a bidimensional hypergrid $\mathcal{G}^2 = \mathcal{P}_{3,m_1} \times \mathcal{P}_{3,m_2}$, $(m_1, m_2 \geq 4)$. The arrows indicate that all the nodes of a row or column are adjacent.

For each $x \in \mathcal{G}^{(j)}$ we define the set $N_x = \{y_1^{(j)}, \ldots, y_{m_j}^{(j)}\}$ belonging to $\mathcal{G}^{(j-1)}$ such that:

$$\begin{cases} (x - y_i^{(j)})_l = 0, l \neq j \\ (x - y_i^{(j)})_j \bmod n_j = 0. \end{cases} \quad 1 \leq i \leq m_j$$

We also define the sets $W_{i_1,\ldots,i_d}^{(j)}, 1 \leq j \leq d$, of the nodes without representative analogously to the definition for d-hypergrid in model $M_{1,1}$.

Claim 2 $(\mathcal{G}^{(d)}, H_1) \prec (\mathcal{G}^d, M_{1,1})$.

Idea: A transmission from a set of equivalent nodes to all its neighbors (in a bidimensional grid) is realized according to Figure 11 by the nodes that gave rise to v in $\mathcal{G}^{(d)}$ (The arrows indicate the direction of transmission) $\square$.

4.4. **-Simplification for model* $M_{2,1}$

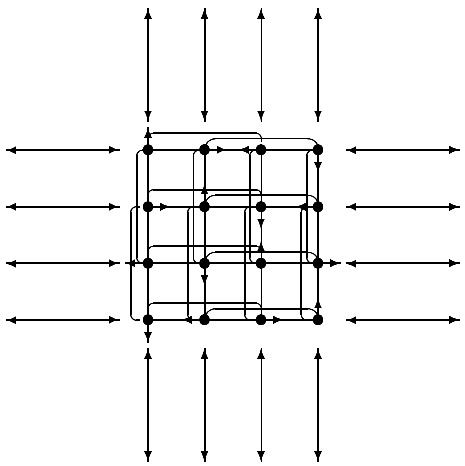

Figure 11: Simulation of a transmission in model H_* in the representative graph of $\mathcal{G}^{(d)}$. The arrows indicate the direction of transmission.

Following the same steps to derive the representative graph G of a hypergrid $\mathcal{G}$ in the *-simplification for model $M_{1,1}$, we have:

Claim 3 *A communication algorithm in $\mathcal{G}^{(d)}$, in model F_1, can be simulated in $\mathcal{G}^d$ in model $M_{2,1}$.*

The procedures for the hypertorus are analogous.

5. Communication patterns

Given a hypergraph $\mathcal{H}(V', E')$ and its representative graph $G(V, E)$, we denote by $broadcast_M(G)$ the broadcast operation, under model M, in graph G. Each transmission $v \leftarrow u$ in graph G represents a transmission from u to all the nodes of the hyperedge that contains u and v in $\mathcal{H}$.

Henceforth, when we define a representative graph, we also implicitly define the sets of representative nodes, represented nodes, and nodes without representative. We denote by $D(G)$ the diameter of G.

5.1. *Broadcast*

Recall that for broadcast it suffices to analyze models $M_{1,i}$ in any hypergraph, for the following reason. There exists, for each step of broadcast, a bipartition of nodes, those which contain the information and those that do not, and the information should be transmitted from one partition to another. Under the model of transmission assumed (transmission time independent of the message size), the capacity of the buses that connect the same two nodes between partitions is meaningless. It follows that the cases in which the buses are k-capacity are included in the ones where buses are single-capacity.

We assume, without loss of generality, that if the node that initially contains the message is candidate to become a representative node, it is selected as one such node during the simplification phase. Furthermore, for all topologies we consider the worst case, that is, we suppose that the source is the node for which the broadcasting time is maximal.

Hyperpath $\mathcal{P}_{n,m}$.

Consider the weakest model $M_{1,1}$, and let $G(V', E')$ be the representative graph, $V' = \{u_1, \ldots, u_{n-1}\}$ the set of representative nodes, $N_u, u \in V'$, the set of represented nodes and W the set of nodes without representative. The representative node that contains the message initiates the broadcast sending the message to a neighbor representative node in graph G. Note that sending a message to a representative means also transmission to all its represented nodes. In this way, the message is disseminated in $\mathcal{P}_{n,m}$. At the end, the nodes of $\mathcal{P}_{n,m}$ without representatives should receive the message from the representative node adjacent to them. In a number of steps less or equal to $D(G) + 1 = D(\mathcal{P}_{n,m})$ (which is the trivial lower bound) the broadcast will complete. Therefore, we have the following theorem.

Theorem 1 *The complexity on the number of steps for broadcasting in a hyperpath $\mathcal{P}_{n,m}$ is $D(\mathcal{P}_{n,m})$, which is optimal.*

Algorithm:

1. $broadcast_{H_1}(G)$
2. $W \Leftarrow u_{n-1}$

Hyperring $\mathcal{R}_{n,m}$.

Consider the model $M_{1,x}$, $x = 1, 2$, and let $G(V', E')$ be the representative graph, $V' = \{u_1, \ldots, u_n\}$ the set of representative nodes, $N_u, u \in V'$, the set of represented nodes. The representative node that contains the message initiates the broadcast sending the message to a neighbor representative node in graph G. Note that sending a message to a representative means also transmission to all its represented nodes. In this way, the message is disseminated in $\mathcal{R}_{n,m}$. The description of the algorithm could not be simpler:

Algorithm:

1. $broadcast_{H_x}(G)$

Theorem 2 *The complexity of the broadcast operation for the hyperring $\mathcal{R}_{n,m}$, in models $M_{1,1}$ and $M_{1,2}$, is optimal.*

Proof. For the model $M_{1,2}$ this is obvious, since the source can send its message over its two buses simultaneously. The time complexity of the algorithm under model $M_{1,1}$ depends on the parity of n. If n is even, then the algorithm completes broadcast in $D(\mathcal{R}_{n,m})$ cycles, implying its optimality. Otherwise, the algorithm takes $D(\mathcal{R}_{n,m})+1$ cycles. Notice, however, that n being odd implies that there are two nodes in different hyperedges of $\mathcal{R}_{n,m}$ at distance D (diameter) from the source. Then $D+1$ is a lower bound in this case. $\square$.

Hypergrid $\mathcal{G} = \mathcal{P}_{n_1,m_1} \times \ldots \times \mathcal{P}_{n_d,m_d}$.

There will be d phases under the weakest model $M_{1,1}$. First we disseminate the message on the path $\mathcal{P}_{n_1,m_1}$ according to the algorithm already seen, in at most n_1-1 cycles. Next, all the paths of the second dimension perform a broadcast. In this way, until dimension d, we complete the broadcast in at most $n_1+\ldots+n_d-d = D(\mathcal{G})$ cycles.

Theorem 3 *The complexity on the number of steps for broadcasting in a hypergrid $\mathcal{G} = \mathcal{P}_{n_1,m_1} \times \ldots \times \mathcal{P}_{n_d,m_d}$ is $D(\mathcal{G})$, which is optimal.*

Hypertorus $\mathcal{T} = \mathcal{R}_{n_1,m_1} \times \ldots \times \mathcal{R}_{n_d,m_d}$.

As previously, this problem is reduced to broadcast in the representative graph of $\mathcal{T}$, which is in its turn a d-torus. Then, the algorithms from [11, 15] apply directly, as well as their analyses:
Algorithm:

1. $broadcast_{H_x}(G)$

Theorem 4 *The complexity on the number of steps for broadcasting in a d-dimensional hypertorus is $D+\max\{0, k-1\}$, where k is the number of odd dimensions in $\mathcal{T} = \mathcal{R}_{n_1,m_1} \times \ldots \times \mathcal{R}_{n_d,m_d}$.*

The above results imply that for the broadcast operation, only the diameter of the BIN has influence on the communication time. That is, the time to complete a broadcast operation independs on the number of nodes on each bus of the BIN.

6. Conclusion

The need for massively parallel systems in order to deal with real size applications is exposing the weakness of point-to-point interconnection networks, due mostly to VLSI implementation related issues. Bus interconnection networks could be an efficient and reliable way to interconnect thousands of processors, while ensuring that the distances among processors are kept small.

We presented communication patterns for broadcasting on BINs topologies, which are simple and very efficient. Below we summarize our results for broadcast on some BINs.

Broadcast	$M_{1,1}$	$M_{1,2}$	diameter
$\mathcal{P}_{n,m}(V,E)$	$n-1$	$n-1$	$n-1$
$\mathcal{R}_{n,m}(V,E)$	$\lceil \frac{n}{2} \rceil$	$\lfloor \frac{n}{2} \rfloor$	$\lfloor \frac{n}{2} \rfloor$
$\mathcal{G}^d(V,E) = \mathcal{P}^1_{n_1,m_1} \times \ldots \times \mathcal{P}^d_{n_d,m_d}$	$\sum_{i=1}^d (n_i - 1)$	$\sum_{i=1}^d (n_i - 1)$	$\sum_{i=1}^d (n_i - 1)$
$\mathcal{T}^d(V,E) = \mathcal{R}^1_{n_1,m_1} \times \ldots \times \mathcal{R}^d_{n_d,m_d}$	$\sum_{i=1}^d (\lceil \frac{n_i}{2} \rceil)$	$\sum_{i=1}^d (\lfloor \frac{n_i}{2} \rfloor)$	$\sum_{i=1}^d (\lfloor \frac{n_i}{2} \rfloor)$

Regarding further research, BINs represent a very interesting way of thinking massive parallelism. Besides designing new algorithms and refining the bus access and capacity models, issues like whether or not the total lack of point-to-point communication will be a major drawback for efficiently using such systems, should be studied. Moreover, as noted in [12, 24], communication algorithms in such a formal model can find applications in the area of emerging optical technologies.

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