<u>Prop.</u>: (H) $\begin{cases} E, F, G, H \text{ espaços vetoriais} \\ \Phi : F \times G \longrightarrow H \text{ bilinear} \\ \omega \in A_p(E,F) \;,\; \eta \in A_q(E,G) \\ v \in E \end{cases}$

(T) $i_v(\omega \wedge_\Phi \eta) = i_v\omega \wedge_\Phi \eta + (-1)^p \omega \wedge_\Phi i_v \eta$

<u>Dem.</u>: Consideramos $S_{p+q-1} \subset S_{p+q}$ o subgrupo de S_{p+q} que fixa a primeira coordenada e permuta as $(p+q-1)$ últimas, e $S_1 = \{id\} \subset S_{p+q}$. Então, conforme vimos em aula, para cada $\sigma' \in S_{p+q}$, $\exists! \, \sigma \in Sh(1, p+q-1)$, $\exists! \, \alpha \in S_{p+q-1}$ tais que $\sigma' = \sigma\alpha$. Portanto, para todo $\xi = (\xi_1, \dots, \xi_{p+q-1}) \in E^{p+q-1}$:

$$i_v(\omega \wedge_\Phi \eta)(\xi) = (\omega \wedge_\Phi \eta)(v, \xi) = \frac{(p+q)!}{p!\,q!} \mathrm{Alt}[\Phi(\omega,\eta)](v,\xi) =$$

$$= \frac{(p+q)!}{p!\,q!} \cdot \frac{1}{(p+q)!} \sum_{\sigma \in S_{p+q}} \varepsilon(\sigma) [\sigma \cdot \Phi(\omega,\eta)](v,\xi) =$$

$$= \frac{1}{p!\,q!} \sum_{\sigma \in Sh(1,p+q-1)} \sum_{\alpha \in S_{p+q-1}} \varepsilon(\sigma)\varepsilon(\alpha) \underbrace{[\sigma\alpha \cdot \Phi(\omega,\eta)](v,\xi)}_{\Phi(\omega,\eta)[\overbrace{(v,\xi)\circ\alpha\circ\sigma}^{=(v,\xi_\alpha)\circ\sigma}]} =$$

$$= \frac{1}{p!\,q!} \sum_{i=1}^{p+q} \sum_{\alpha \in S_{p+q-1}} (-1)^{i-1} \varepsilon(\alpha) \underbrace{\Phi(\omega,\eta)(\xi_{\alpha_1}, \dots, \xi_{\alpha_{i-1}}, v, \xi_{\alpha_i}, \dots, \xi_{\alpha_{p+q-1}})}_{\doteq A} =$$

$$= \overbrace{\frac{1}{p!\,q!} \sum_{i=1}^{p} (-1)^{i-1} \sum_{\alpha \in S_{p+q-1}} \varepsilon(\alpha) A}^{\doteq A_1} + \overbrace{\frac{1}{p!\,q!} \sum_{i=p+1}^{p+q} (-1)^{i-1} \sum_{\alpha \in S_{p+q-1}} \varepsilon(\alpha) A}^{\doteq A_2} \quad (*)$$

$$A_1 = \frac{1}{p!\,q!} \sum_{i=1}^{p} (-1)^{i-1} \sum_{\alpha \in S_{p+q-1}} \varepsilon(\alpha)\, \Phi(\omega,\eta)(\xi_{\alpha_1}, \dots, \xi_{\alpha_{i-1}}, v, \xi_{\alpha_{i+1}}, \dots, \xi_{\alpha_{p+q-1}}) =$$

$$= \frac{1}{p!\,q!} \sum_{i=1}^{p} \sum_{\alpha \in S_{p+q-1}} (-1)^{i-1} \varepsilon(\alpha) (-1)^{i-1} \Phi(\omega,\eta)(v, \xi_{\alpha_1}, \dots, \xi_{\alpha_{p+q-1}}) =$$

$$= \frac{1}{p!\,q!} \sum_{i=1}^{p} \sum_{\alpha \in S_{p+q-1}} \varepsilon(\alpha)\, \Phi(i_v\omega, \eta)(\xi \circ \alpha) =$$

$$= \frac{1}{p!\,q!}\, p \cdot (p-1)!\, q!\, (i_v\omega \wedge_\Phi \eta)(\xi) = (i_v\omega \wedge_\Phi \eta)(\xi)$$

$$A_2 = \frac{1}{p!\,q!} \sum_{i=p+1}^{p+q} (-1)^{i-1} \sum_{\alpha \in S_{p+q-1}} \varepsilon(\alpha)\, \Phi(\omega,\eta)(\xi_{\alpha_1}, \dots, \xi_{\alpha_p}, \dots, \xi_{\alpha_{i-1}}, v, \xi_{\alpha_i}, \dots, \xi_{\alpha_{p+q-1}}) =$$

$$= \frac{1}{p!\,q!} \sum_{i=p+1}^{p+q} \sum_{\alpha \in S_{p+q-1}} (-1)^{i-1} \varepsilon(\alpha) (-1)^{i-(p+1)} \Phi(\omega,\eta)(\xi_{\alpha_1}, \dots, \xi_{\alpha_p}, v, \xi_{\alpha_{p+1}}, \dots, \xi_{\alpha_{p+q-1}}) =$$

$$= \frac{1}{p!\,q!} \sum_{i=p+1}^{p+q} (-1)^p \sum_{\alpha \in S_{p+q-1}} \varepsilon(\alpha)\, \Phi(\omega, i_v\eta)(\xi \circ \alpha) =$$

$$= (-1)^p \frac{1}{p!\,q!} \cdot q \cdot p!\,(q-1)!\, (\omega \wedge_\Phi \eta)(\xi) =$$

$$= (-1)^p (\omega \wedge_\Phi \eta)(\xi)$$

$$\therefore (*) = A_1 + A_2 = (i_v\omega \wedge_\Phi \eta + (-1)^p \omega \wedge_\Phi i_v \eta)(\xi)$$

donde $i_v(\omega \wedge_\Phi \eta) = i_v\omega \wedge_\Phi \eta + (-1)^p \omega \wedge_\Phi i_v\eta$ #